THE PREDICTION OF TEMPORAL STATISTICS OF DIRECTIONAL AMBIENT SHIPPING NOISE

by

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ABSTRACT

A technique for predicting the probability density function (PDF) of the ambient-noise power received by a narrow-band system in a modelled shipping and wind noise field is described. The densities of the shipping noise are obtained from a mathematical model for spectral-line components. The importance of the probability of occurrence of combinations of spectral-line components in the frequency band of interest is emphasized. It is shown that the uncertainty in receiving a particular combination of lines in the frequency band of interest leads to the definition of the unconditional PDF of the noise. Its prediction consists of the addition of numerically convolved densities in conjunction with the probability of occurrance of a line combination. The results show that the shape of the resulting PDF depends on the processing bandwidth and the statistics of the wind/background noise level. Suggestions are made to extend the technique to use it for closer shipping.

INTRODUCTION

The object of this study is the prediction of the statistics of directional ambient shipping noise with a mathematical-numerical technique. This reported work is an extension of the already existing deterministic model RANDI II <1> to which random theory is added.

The technique for predicting the statistics is based on the statistical model published by Dyer <2> and gives a simple analytical expression for the probability density function (PDF) of the shipping-noise power as a function of a combination of received spectral line components. The total short-time averaged noise power expected to be received in the frequency band of interest depends on the processing bandwidth relative to the assumed distribution of the lines in the noise spectrum. For a "narrow"-band system – narrow in the sense that the bandwidth is smaller than the average line spacing in a single-ship spectrum – the number of lines present in the band will be on the average less than the number of sources. The position of the lines in the spectrum is assumed to be random; consequently the appearance of a combination of lines in the frequency band of interest will be random. As all these combinations are mutually exclusive it can be shown that the unconditional PDF of the noise due to all possible combinations of spectral lines is a weighted sum of the conditional PDFs of the individual combinations; with their probability of occurrence as weighting factors. Results of the prediction of the statistics of shipping (plus wind noise) from a modelled shipping noise field from RANDI II are presented and discussed. RANDI II and the prediction technique are developed to be used if the beam of a sensor is steered in the direction of a shipping noise field that is kept static during the modelled observation time. This situation may be achieved for distant shipping. Experimental results from closer shipping show a highly variable shipping-noise field due to the moving of shipping in and out of the beam. This is also observed in a simulated shipping-noise field from RANDI II. To solve this problem an extension of the technique is suggested.

Section 1 describes the modelled physical situation and gives typical results using Dyer's Statistical Model. Section 2 proposes a spectral model and describes the uncertainty about the spectral line reception, which is the major reason for defining the unconditional PDF. A block diagram shows how Dyer's Statistical Model, the spectral model, and RANDI II relate to its numerical calculation. Results are presented in Section 3 and Section 4 discusses the moving shipping problem.

1 MODELLED PHYSICAL SITUATION

The medium and system modelled by the prediction system are shown in Fig. 1. The noise due to shipping propagates through multipaths to the sensor. The shipping field in the model is assumed to be static in the sense that the ships do not move during the measurement observation time. The sensor is an array of hydrophones. The signals received in the beams are passed through a band filter. The filter output is squared, short-time-averaged and yields the beam-power output. Figure 1 also shows the most important models used in the prediction system. Dyer's statistics of distant shipping noise <2> models distant shipping noise analytically as phase-random line components arriving from a number of independent sources. The model is applicable for cases where distinct assumptions about the random propagation mechanism <3> and the processing system are satisfied. In order to allow the addition of power intensities in most spectral-line combinations, the averaging time-bandwidth product must be much larger than unity (see App. A).

Dyer's PDFs and mathematical formulas for different spectral line receptions are shown in Fig. 2, in which μ_x is the long-time average intensity of each line and ℓ is the number of lines. The standard deviation (SDEV) of the level of one line is constant, about 5.6 dB. If more lines are combined, the PDF becomes narrower and the standard deviation lower. This is caused by the logarithmic transformation of the power intensities. The prediction technique uses an algorithm that calculates the PDF for a combination of lines with different long-time average intensities (see App. B).

2 THE SPECTRAL MODEL

In the spectral model it is assumed that the radiated noise of merchant ships consists of spectral-line components with a constant line spacing <2,4>. We consider only those pass bandwidths that are smaller than the line spacing (line-spacing/bandwidth ratio greater than 1); no more than one line per ship is expected to be present in the band, as shown in Fig. 3. To characterize the reception of a distinct combination of spectral lines, assuming N merchant ships in the beam of the array, we need to know the probability of receiving this distinct number of lines in the band. If we assume that both the position of the lines in the spectrum and the number of lines in the band of interest are random, we define the conditional probability of ℓ spectral lines given N ships: $P\{\ell/N\}$. As the lines are received from independent sources, we can use the binomial distribution theory to calculate this probability (see App. C).

An indication for the reception of different numbers of spectral lines in the band is obtained by observing the results from SACLANTCEN's towed-array experiments <4>. Figure 4 gives the beam-noise level output as a function of azimuth angle for three adjacent frequency bands with widths of 2.5 Hz. The mean and standard deviation of the noise from 50 data samples are plotted. The centre frequencies of the bands are 148, 151.5 and 153 Hz. Although in general the mean values from the three bands differ little within one beam, the standard deviations differ in some beams by more than 3 dB. Referring to Dyer's PDFs in Fig. 2, this difference is presumably caused by a different combination of spectral-line components in the bands of these beams. Because it is unknown which combination of lines, out of all the possible combinations, will be present in the band, we have defined the unconditional PDF of the noise. The formula is shown in Fig. 5. This PDF is the sum of all the conditional PDFs of every possible combination of lines, weighted by the probabilities of receiving a particular combination of lines in the band. Appendix D proves that the weighted summation of PDFs is allowed because the line combinations, with which the PDFs are connected, occur mutually and exclusively.

The block diagram, Fig. 6, shows the relation of the numerical calculation of the unconditional PDF by means of the different models. Three flows of information are extracted from the RANDI II model and directed to the submodels:

a) The power intensities of the line arrivals of N ships go to Dyer's statistical model,

b) The power intensities of the wind/background noise and the processing time-bandwidth product go to the wind statistics model,

c) The number of ships and processing bandwidth go to the spectral model.

The values of f_{ℓ_k} corresponding to the PDFs of all combinations of ℓ lines, are summed and normalized. The resulting PDF, f_ℓ, is convolved with the PDF of the wind noise f_w; this gives the PDF, f, which is weighted and summed over all possible numbers of lines.

3 RESULTS FROM THE PREDICTION MODEL

The prediction model has been run with shipping data acquired by maritime patrol aircraft during one of SACLANTCEN's towed-array experiments. The data set is given in Appendix D <4>. A typical result of the unconditional PDF of the noise from the shipping-noise field modelled in RANDI II with the above shipping data is shown in Fig. 7. This figure shows:

- The wind noise level and the noise levels of the 7 spectral lines from 7 ships.
- The conditional probabilities P $\{\ell/7\}$ ($\ell = 0,..7$)
- The weighted PDF curves for different combinations of lines ℓ. ($\ell = 0,..7$).
- The unconditional PDF curve.

In this example the shape of the resulting PDF is bimodal. This is because the wind noise (and if the wind is not present, the background noise) is always present in every combination of spectral lines. Every weighted PDF curve is either that of the wind/background noise alone or the result of a convolution of PDFs of wind noise and line noise. It is clear that the influence of the wind/background noise statistics on the unconditional PDF of ambient noise will be substantial, especially for higher frequencies.

Figures 8, 9, 10 and 11 show the results of predicting the unconditional PDF of noise from the modelled shipping noise field for different beam outputs and various time-bandwidth products (TBP) and line-spacing/ bandwidth ratios (LBR).

The modelled beam power response level vs azimuth angle and the unconditional PDFs with probabilities of line reception are plotted for some beams. It needs to be emphasized that no conditions are made about the combination of received lines in the band of each beam. It is therefore not possible to compare these results directly with the temporal statistics of real data. The TBP of the processing system influences the statistics of the wind/background noise intensity. A high TBP gives a narrow PDF of the wind noise. For narrow bandwidths the LBR will be high. Figures 8 to 11 demonstrate that the influence of the wind-noise level relative to the total shipping noise level is substantial, especially in beams with few ships. In most of the beams the shape of the PDFs is bimodal. The range of standard deviation is large.

We can draw the following general conclusions from the above results:

1) The unconditional PDF of the noise level is non-gaussian for most of the beams.

2) In the case of narrow bands the influence of the wind-background noise will be substantial in beams with few ships, because of the high probabilities of receiving no lines in the band.

3) Low-level wind-background noise leads to higher standard deviations than high-level wind-background noise.

4) Variation of the bandwidth influences the amplitude of the resulting PDF more than its standard deviation.

5) Wind-background noise with a narrow PDF dominates the unconditional PDF, particularly for narrow bandwidths.

4 THE MOVING SHIPPING PROBLEM

The above proposed technique is developed to predict the statistics of ambient shipping noise from distant shipping. If the shipping is closer, the influence of moving shipping will have substantial effect on the statistics of the ambient shipping noise. The varying shipping noise field can be observed in the experimental data and can be modelled with RANDI II. Figures 12a and b plot the beam noise level outputs for three series of successive and overlapping noise samples from towed-array measurements. Three values of mean and standard deviations of the noise are plotted against azimuth angle for 150 and 502 Hz. Beam outputs with large variation in the three mean values have, in general, high standard deviations, which might be explained by ships moving in and out of these beams during the measurement period. Another indication for moving shipping is simulated with RANDI II. Figure 13a shows the shipping noise field calculated from the shipping surveillance data (see App. D <4>) collected by maritime patrol aircraft during the measurements shown in Fig. 12. The spikes are shipping-noise levels in one-degree sectors. The curve is the noise field convolved with an unambiguous beam of 5°. Figure 13a shows the field at the start of an observation time period and Fig. 13b shows the field at the end of the period. A highly variable noise field is observed due to moving shipping. The impact on the corresponding modelled beam noise responses is shown in Figs. 14a and b.

In order to handle moving shipping with a prediction technique a dynamic shipping-noise field should be used. The statistics of the noise could be calculated out of noise samples from a varying shipping-noise field. This solution would be beyond the scope of this study, in which an analytical calculation is required. An extension of the prediction technique is suggested with the use of unconditional statistics (Fig. 15). We propose the unconditional PDF of the noise due to all possible combinations of M ships in the beam during the observation time period. This PDF is the weighted sum of the unconditional PDFs due to all possible combinations of spectral lines in the band, given a particular combination of N ships in the beam of the sensor. The weighting factors are the probabilities of occurrence of a particular combination of ships in the beam. With this extension, which is under development, the predicted PDFs are expected to be much wider, with much higher standard deviations. This will be caused by the extra uncertainty about the shipping covered by the beam.

APPENDIX A

JUSTIFICATION OF THE USE OF POWER ADDITION IN THE PROPOSED PREDICTION TECHNIQUE

If the output of the band-pass filter in Fig. 16 is represented by the signal

$$V(t) = \sum_{k=1}^{\ell} A_k \cos(\omega_k t + \Theta_k)$$

as a sum of line signals with amplitude A_k, frequency $f_k = \omega_k/2\pi$, and phase Θ_k, it can be shown that the squared and short-time averaged (interval I seconds) signal v(t) becomes:

$$P(t) = \frac{1}{2} \sum_{k=1}^{\ell} \sum_{m=1}^{\ell} \frac{A_k A_m}{I} \int_{t-I/2}^{t+I/2} du \cos[(\omega_k - \omega_m)u + \Theta_k - \Theta_m]$$

$$= \frac{1}{2} \sum_{k=1}^{\ell} A_k^2 + \text{off-diagonal terms}$$

These off-diagonal terms are $\approxeq 0$ if

$$\min_{k \neq m} \left| \omega_k - \omega_m \right| > \frac{2\pi}{I}$$

and P(t) is a sum of line-power intensities.

If we define the minimum relative line spacing Δ as

$$\Delta = \min_{k \neq m} \left| f_k - f_m \right| \quad \text{we get the condition } I > 1/\Delta$$

If we use a time-bandwidth product much larger than 1, then

$$IW \gg 1 \qquad \text{(Eq. A.1)}$$

For a bandwidth that is not too small and a few spectral lines received in the band, it is allowable to use the addition of spectral line power levels if the condition A.1 is fulfilled. If we use a line-spacing/bandwidth ratio of 5 and a maximum possible reception of 7 spectral lines in the pass band, a relative line spacing less than the minimum Δ will have a probability of occurrence of less than 10%.

APPENDIX B

The derivation of the PDF for unequal long-time average intensities is shown in <2>. An analysis of the characteristic functions of PDFs is used. An evaluation by the calculus of residues and transformation to the logarithmic domain yields

$$f(x) = \sum_{m=1}^{\ell} \mu_{\ell}^{-1} P_{\ell}^{-1} \exp[x - \exp(x/\mu_m)] \quad ,$$

where

$$P_m = \prod_{j \neq m}^{\ell} (1 - \mu_j/\mu_m) \quad ,$$

with μ_m the long-time average intensity of the m^{th} line and ℓ the number of lines that are combined. The property $\sum_{m=1}^{\ell} P_m^{-1} = 1$ is used in the prediction technique as a quality measure for the numerical calculation.

APPENDIX C

We assume N lines in a spectral line space divided into m bands, where m is the line-spacing/bandwidth ratio. If one line is randomly present in the space, the probability that this line is present in a marked band is 1/m and the probability that it is not present in this band is (m-1)/m.

Since all the N lines are independent and not labelled, we use the binomial distribution to calculate the probability that ℓ lines out of N are present in the band of interest:

$$P\{\ell/N\} = \binom{N}{\ell} \left(\frac{m-1}{m}\right)^{N-\ell} \left(\frac{1}{m}\right)^{\ell}$$

APPENDIX D

PROOF WEIGHTED SUMMATION OF DENSITIES

Suppose v_i is the event that the value of the random variable (r.v) X is greater than or equal to x; and less than or equal to x_{i+1}, the probability of event v_i is:

$$P\{v_i\} = P\{x_i \leq X \leq x_{i+1}\} = \int_{x_i}^{x_{i+1}} f(x)dx \quad ,$$

with f(x) being the PDF of r.v. X.

Further, suppose that c_j is the event that the j^{th} combination of spectral lines is received in the band of interest, with probability $P\{c_j\}$.

If v_i is the consequence of c_j, then the probability of event $v_j c_j$ is

$$P\{v_i c_j\} = P\{v_i / c_j\} P\{c_j\} \qquad \text{(Eq. D.1)}$$

The events in the set $v_i c_1, v_i c_2, \ldots, v_i c_{k_T}$ are mutually exclusive because $c_1, c_2, \ldots, c_{k_T}$ are mutually exclusive. The subscript k_T is defined as the total number of combinations of lines.

From above and Eq. D.1 we obtain

$$P\{v_i c_1 + v_i c_2 + \ldots + v_i c_{k_T}\} = \sum_{j=1}^{k_T} P\{v_i / c_j\} P\{c_j\}$$

or

$$P\{v_i (c_1 + c_2 + \ldots + c_{k_T})\} = \sum_{j=1}^{k_T} \int_{x_i}^{x_{i+1}} f_{c_j}(x)\, dx\, P\{c_j\}$$

or

$$\int_{x_i}^{x_{i+1}} f_{c_1 + c_2 + \ldots + c_{k_T}}(x)\, dx = \sum_{j=1}^{k_T} \int_{x_i}^{x_{i+1}} f_{c_j}(x)\, dx\, P\{c_j\} \qquad \text{(Eq. D.2)}$$

with $f_{c_j}(x)$ being the probability density of X due to the j^{th} combination of lines, and $f_{c_1+c_2+\ldots+c_{k_T}}(x)$ being the probability density of X due to "sum" of all the possible combinations of lines.

In numerical calculations for Δx sufficiently small, we obtain

$$P\{v_i\} = P\{x_i \le X \le x_i + \Delta x\} \cong f(x_i)\Delta x$$

From the above and Eq. D.2 we conclude that

$$f_{c_1+c_2+\ldots+c_{k_T}}(x_i)\Delta x = \sum_{j=1}^{k_T} f_{c_j}(x_i)\ \Delta x\ P\{c_j\}$$

or

$$f_{c_1+c_2+\ldots+c_{k_T}}(x_i) = \sum_{j=1}^{k_T} f_{c_j}(x_i)P\{c_j\} \qquad \text{(Eq. D.3)}$$

Hence a sample of the total PDF of the noise-power level X is the weighted sum of the PDFs due to all possible combinations of spectral lines with the probability of occurrence of the different combinations of lines as weighting factors.

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3. DYER, I. Statistics of sound propagation in the ocean. Journal of the Acoustical Society of America, 53, 1970: 337-345.

4. URICK, R.J. Principles of Underwater Sound for Engineers. New York, N.Y., McGraw-Hill, 1967.

5. BLUY, O.Z. Underwater ambient noise: Directionality and other statistics. Vol. II: Measurements near Gibraltar, September 1980. SACLANTCEN SR-58, NATO CONFIDENTIAL. La Spezia, Italy, SACLANT ASW Research Centre, 1982.

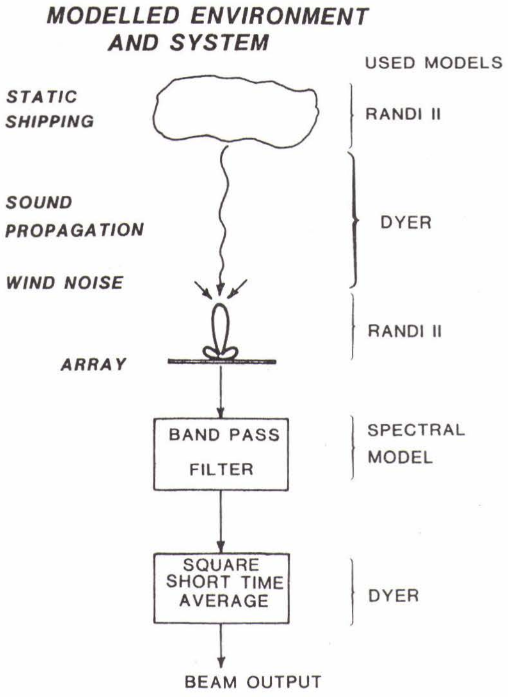


FIG. 1

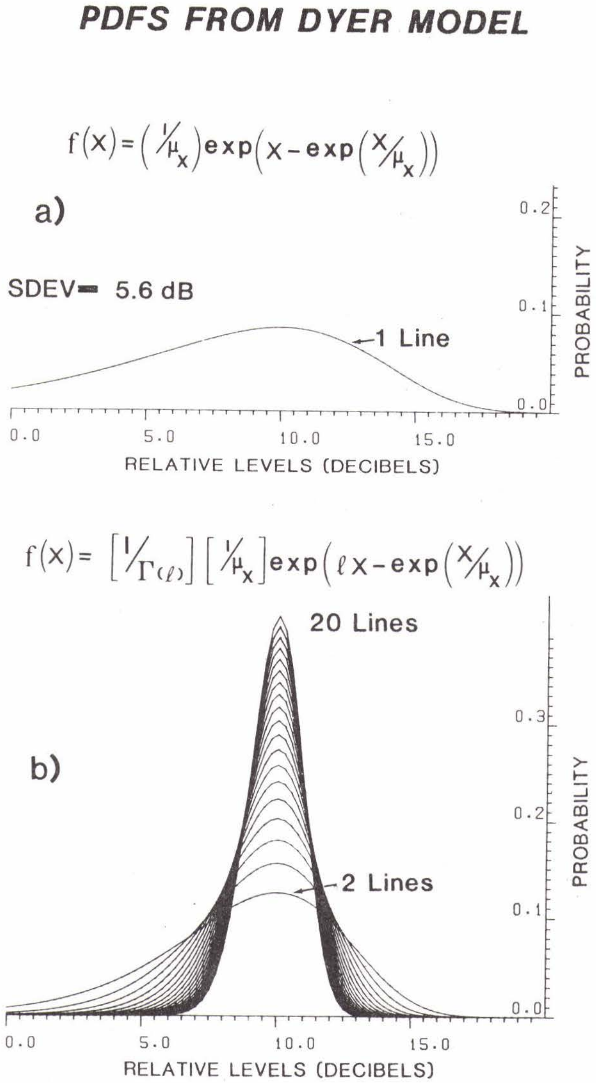


FIG. 2

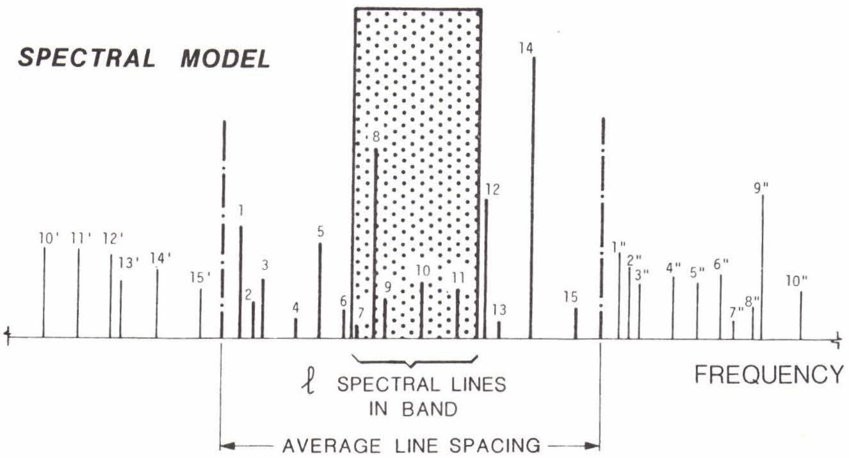


FIG. 3

BEAM NOISE OUTPUT

FOR THREE FREQUENCY BANDS

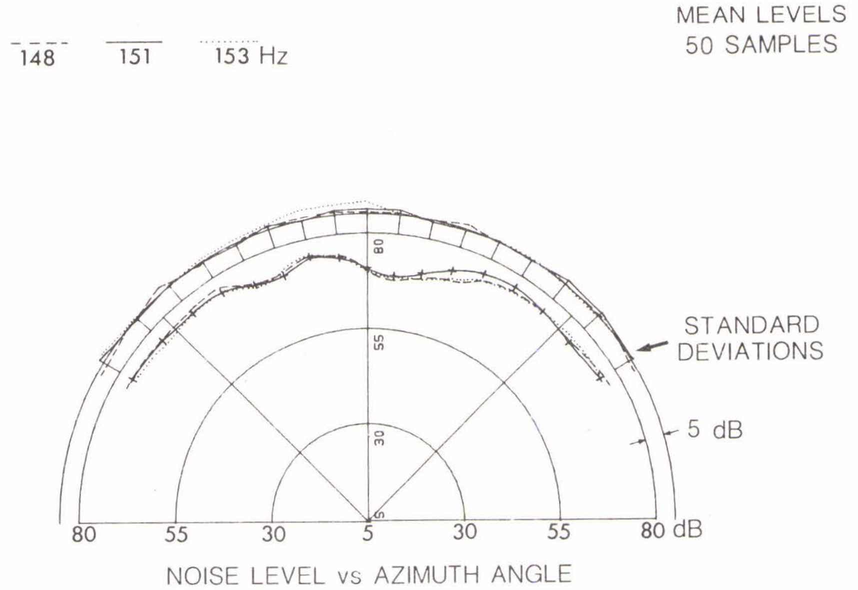


FIG. 4

$P\{\ell/N\}$: PROBABILITY OF ℓ SPECTRAL LINES GIVEN N SOURCES

$K = \binom{N}{\ell}$: POSSIBLE COMBINATION OF ℓ LINES OUT OF N

$K_T = \sum_{\ell=0}^{N} \binom{N}{\ell}$: ALL POSSIBLE COMBINATIONS OF LINES OUT OF N

$f_T(x)$: PROBABILITY DENSITY DUE TO ALL POSSIBLE COMBINATIONS OF LINES + WIND/BACKGROUND NOISE

$$\boxed{f_T(x) = \sum_{\ell=0}^{N} \underbrace{\frac{P\{\ell/N\}}{K_T}}_{\text{PROBABILITY}} \underbrace{\sum_{i=1}^{K} f_{\ell_i}(x)}_{\text{DENSITY}}}$$

i^{th} COMBINATION OF ℓ LINES

FIG. 5

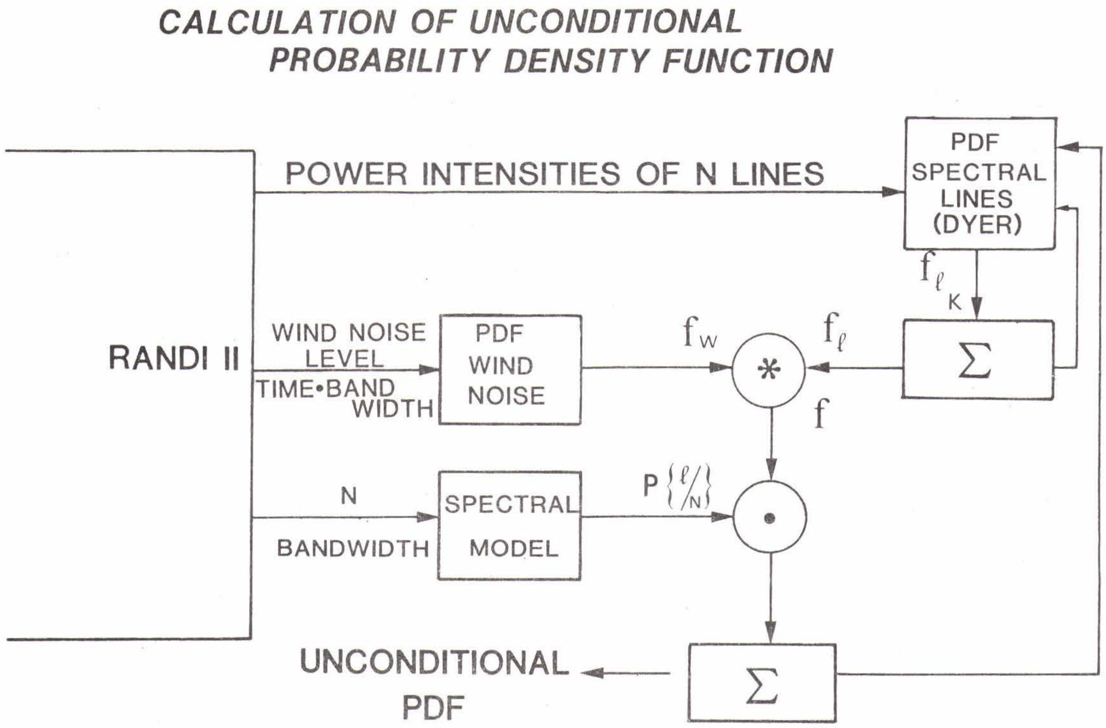

CALCULATION OF UNCONDITIONAL
PROBABILITY DENSITY FUNCTION
POWER INTENSITIES OF N LINES
PDF
SPECTRAL
LINES
(DYER)
RANDI II
WIND NOISE
LEVEL
TIME•BAND
WIDTH
PDF
WIND
NOISE
N
BANDWIDTH
SPECTRAL
MODEL
UNCONDITIONAL
PDF

FIG. 6

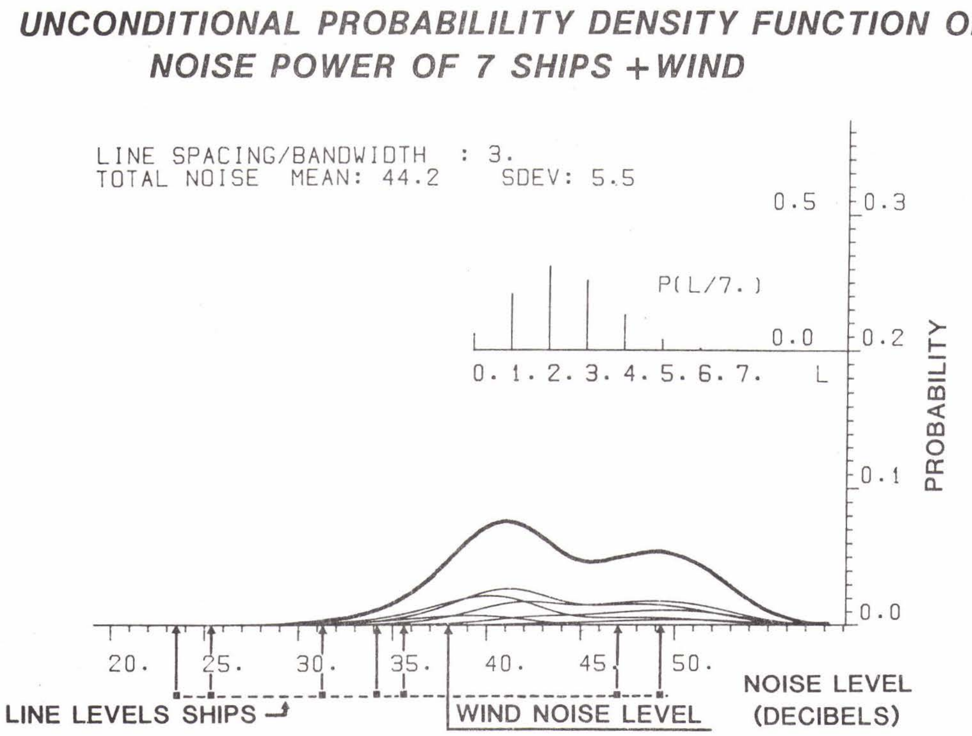

UNCONDITIONAL PROBABILILITY DENSITY FUNCTION OF
NOISE POWER OF 7 SHIPS +WIND
LINE SPACING/BANDWIDTH : 3.
TOTAL NOISE MEAN: 44.2 SDEV: 5.5
P(L/7.)
0. 1. 2. 3. 4. 5. 6. 7. L
PROBABILITY
LINE LEVELS SHIPS
WIND NOISE LEVEL
NOISE LEVEL
(DECIBELS)

FIG. 7

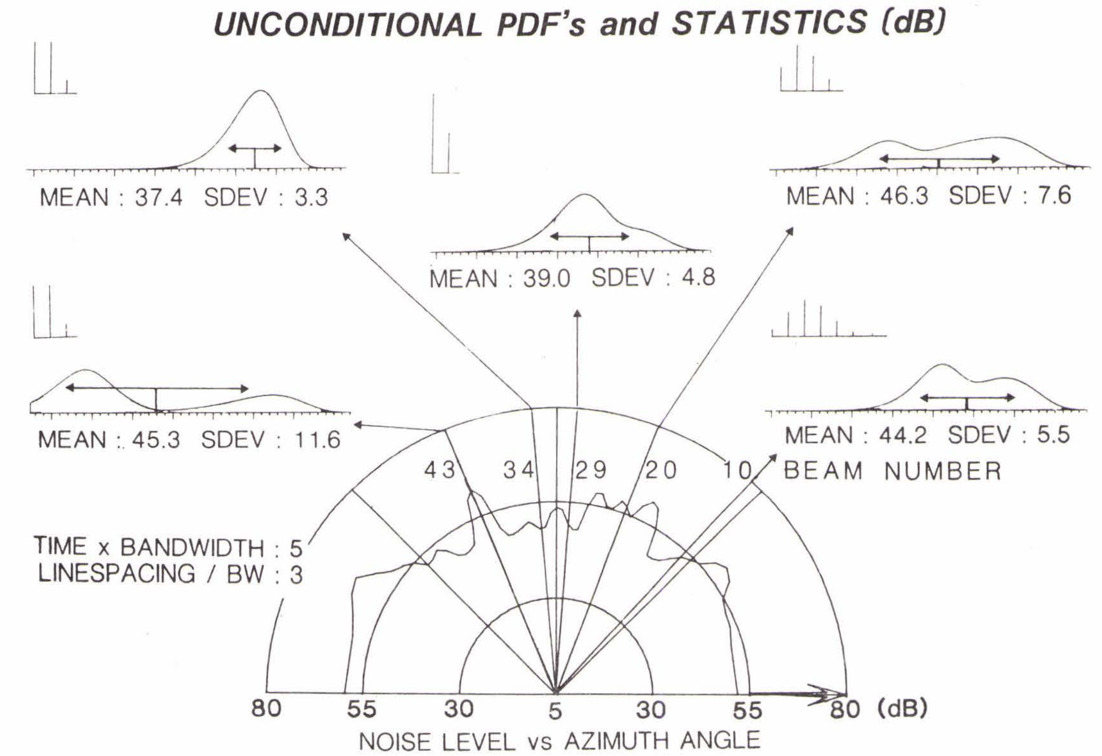

UNCONDITIONAL PDF's and STATISTICS (dB)
MEAN : 37.4 SDEV : 3.3
MEAN : 39.0 SDEV : 4.8
MEAN : 46.3 SDEV : 7.6
MEAN : 45.3 SDEV : 11.6
MEAN : 44.2 SDEV : 5.5
43 34 29 20 10
BEAM NUMBER
TIME x BANDWIDTH : 5
LINESPACING / BW : 3
80 55 30 5 30 55 80 (dB)
NOISE LEVEL vs AZIMUTH ANGLE

FIG. 8

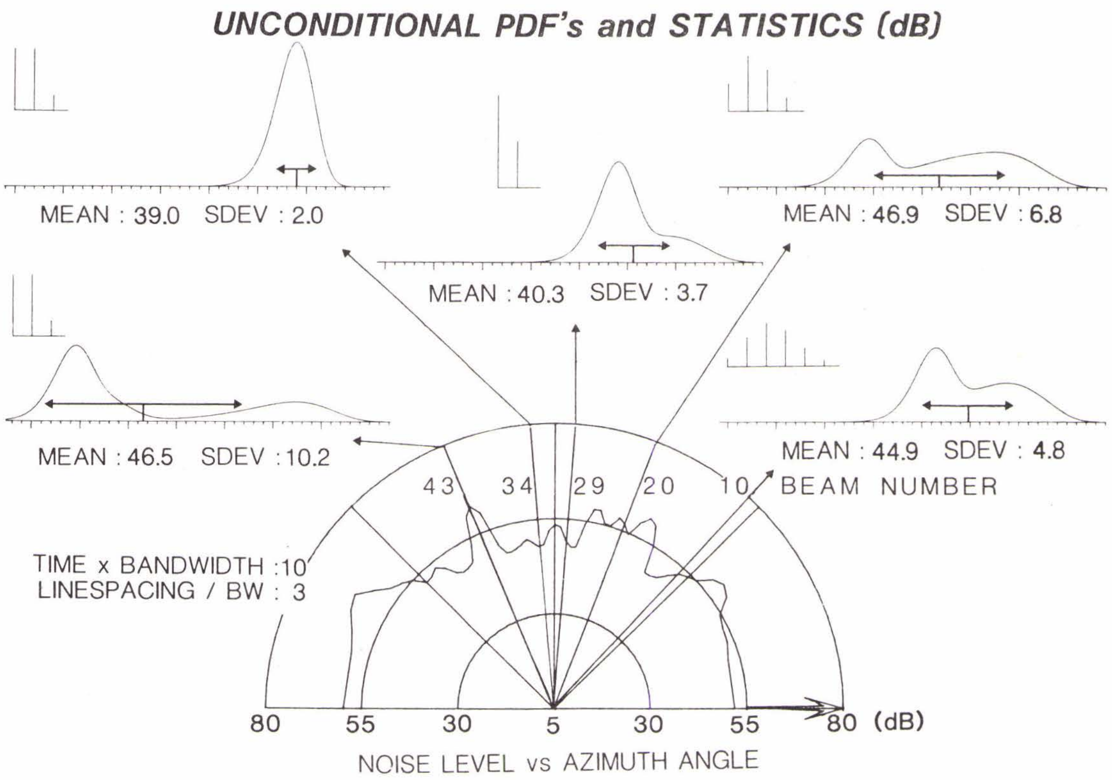

UNCONDITIONAL PDF's and STATISTICS (dB)
MEAN : 39.0 SDEV : 2.0
MEAN : 40.3 SDEV : 3.7
MEAN : 46.9 SDEV : 6.8
MEAN : 46.5 SDEV : 10.2
MEAN : 44.9 SDEV : 4.8
43 34 29 20 10
BEAM NUMBER
TIME x BANDWIDTH :10
LINESPACING / BW : 3
80 55 30 5 30 55 80 (dB)
NOISE LEVEL vs AZIMUTH ANGLE

FIG. 9

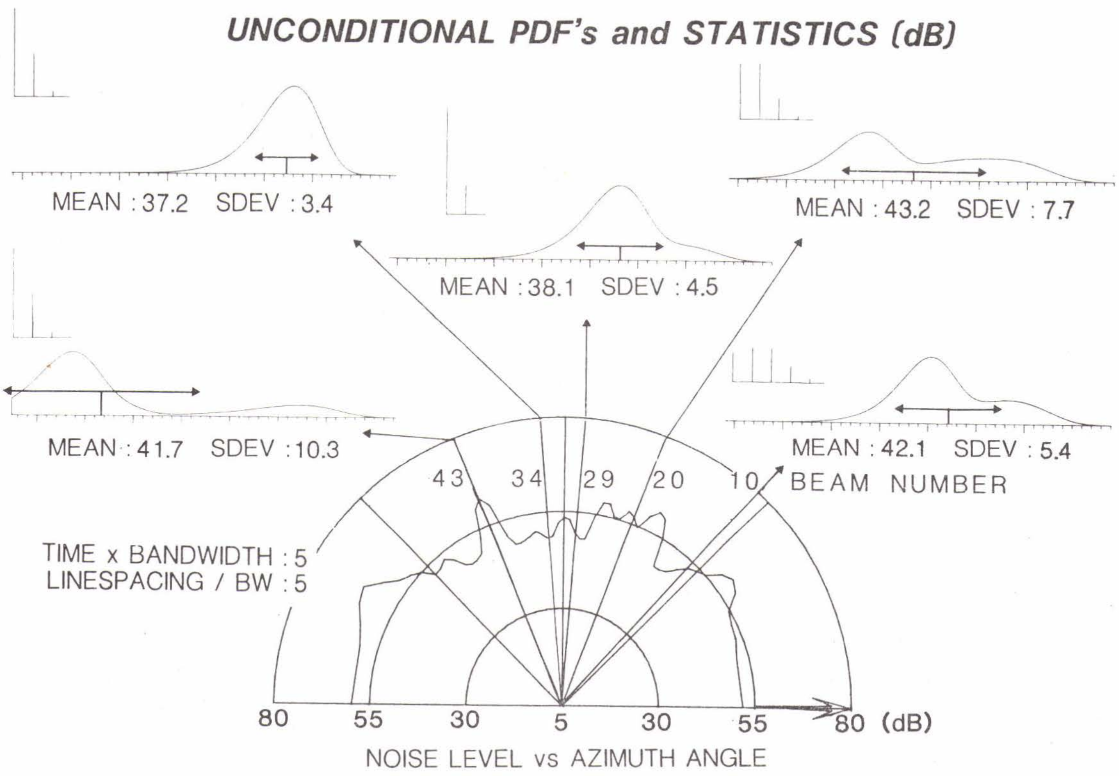

UNCONDITIONAL PDF's and STATISTICS (dB)
MEAN : 37.2 SDEV : 3.4
MEAN : 38.1 SDEV : 4.5
MEAN : 43.2 SDEV : 7.7
MEAN : 41.7 SDEV : 10.3
MEAN : 42.1 SDEV : 5.4
43 34 29 20 10
BEAM NUMBER
TIME x BANDWIDTH : 5
LINESPACING / BW : 5
80 55 30 5 30 55 80 (dB)
NOISE LEVEL vs AZIMUTH ANGLE

FIG. 10

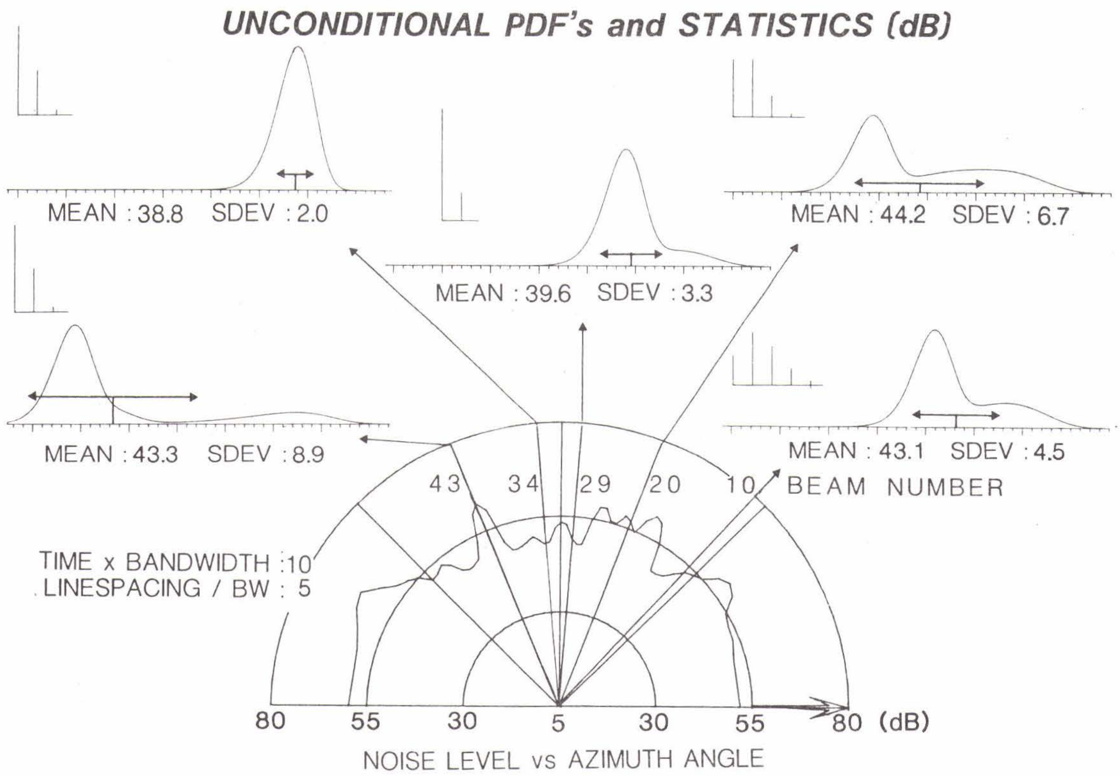

UNCONDITIONAL PDF's and STATISTICS (dB)
MEAN : 38.8 SDEV : 2.0
MEAN : 39.6 SDEV : 3.3
MEAN : 44.2 SDEV : 6.7
MEAN : 43.3 SDEV : 8.9
MEAN : 43.1 SDEV : 4.5
43 34 29 20 10
BEAM NUMBER
TIME x BANDWIDTH : 10
LINESPACING / BW : 5
80 55 30 5 30 55 80 (dB)
NOISE LEVEL vs AZIMUTH ANGLE

FIG. 11

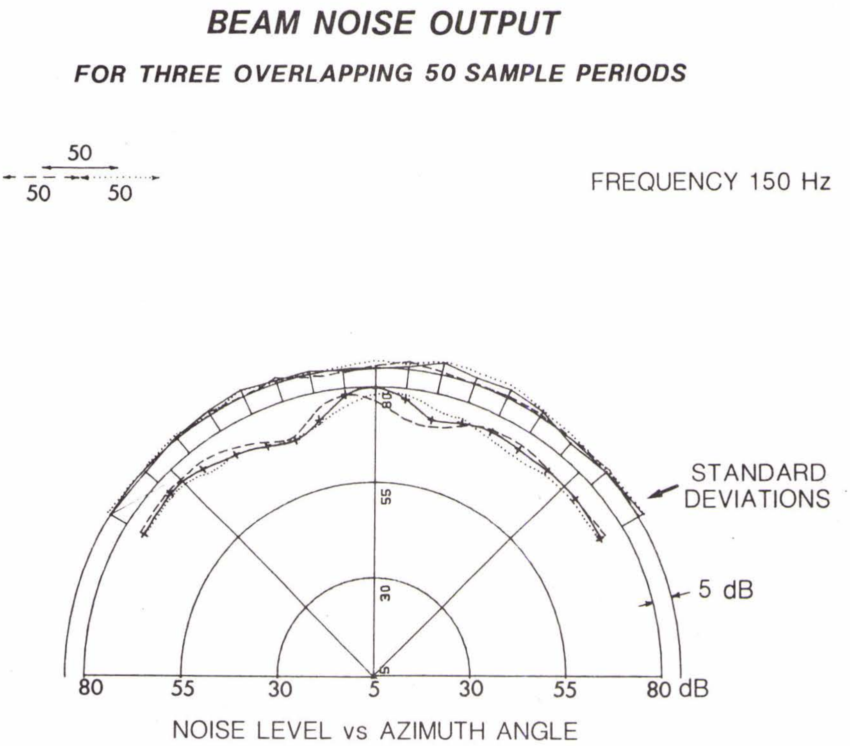


FIG. 12a

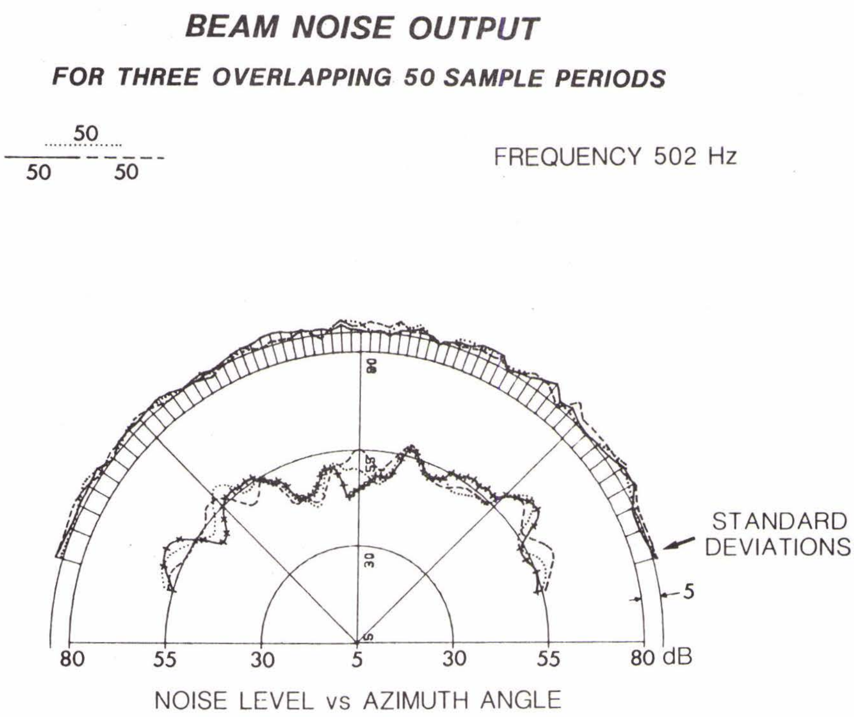
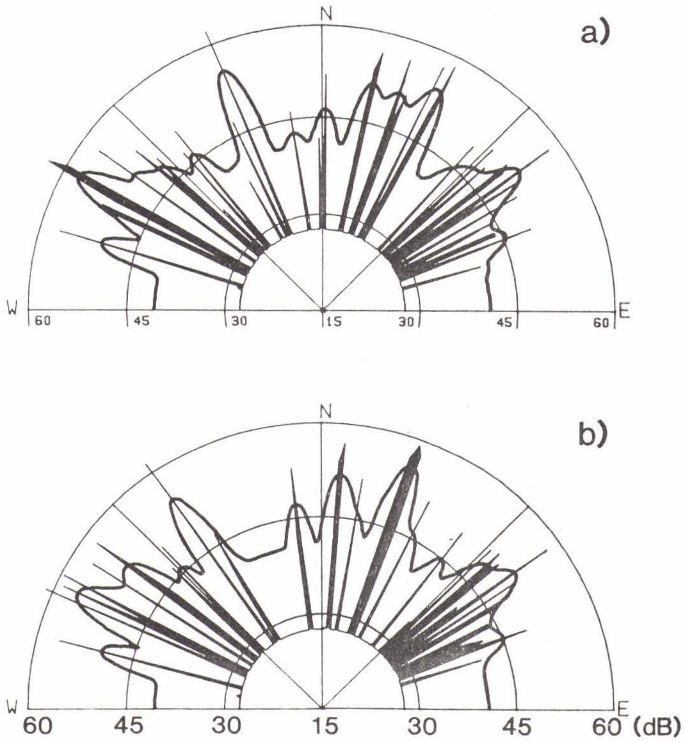


FIG. 12b

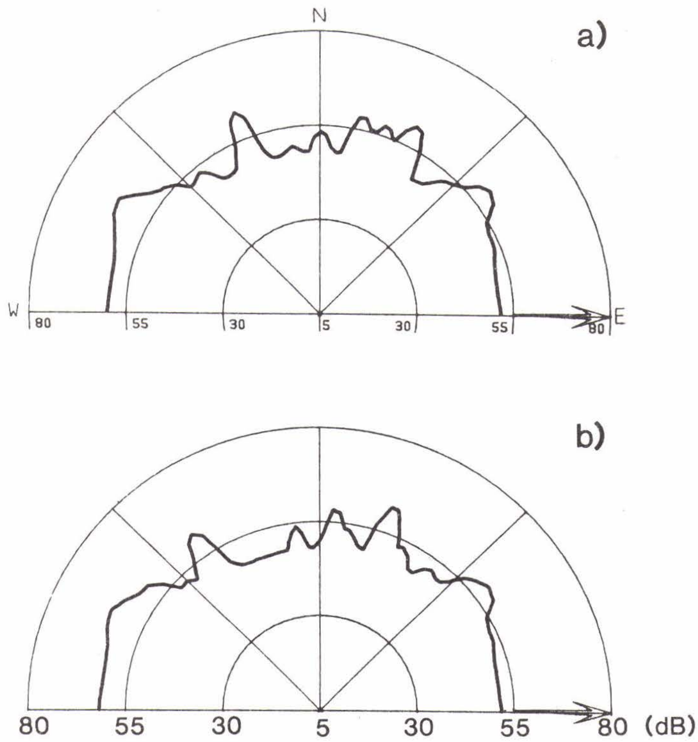
MODELLED SHIPPING NOISE FIELD



NOISE LEVEL vs AZIMUTH ANGLE

FIG. 13

MODELLED BEAM RESPONSE



NOISE LEVEL vs AZIMUTH ANGLE

FIG. 14

$P\{N/M\}$: PROBABILITY OF N SHIPS GIVEN M POSSIBLE SHIPS IN BEAM PER OBSERVATION TIME PERIOD

$I = \binom{M}{N}$: POSSIBLE COMBINATION OF N SHIPS OUT OF M

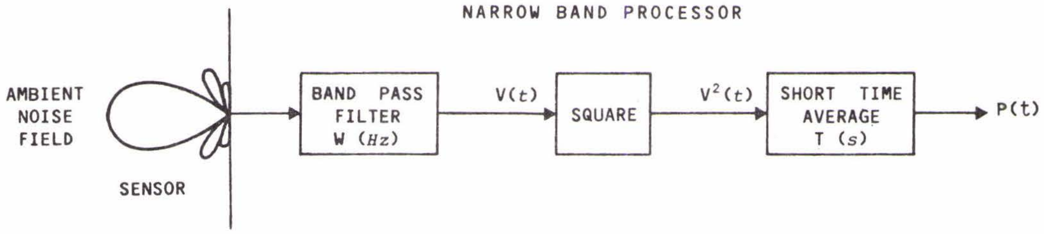
$I_T = \sum_{N=o}^{M} \binom{M}{N}$: ALL POSSIBLE COMBINATIONS OF SHIPS OUT OF M

$f\ (x)$: UNCONDITIONAL DENSITY DUE TO ALL POSSIBLE COMBINATIONS OF SHIPS IN BEAM

$$\boxed{f\ (x) = \sum_{N=o}^{M} \underbrace{\frac{P\{N/M\}}{I_T}}_{\text{PROBABILITY}} \sum_{i=1}^{I} \underbrace{f_{N_i}(x)}_{\text{DENSITY}}}$$

i^{th} COMBINATION OF ℓ LINES

FIG. 15



$V(t)$ = BAND PASS FILTER OUTPUT VOLTAGE ,

$P(t)$ = NOISE POWER OUTPUT PROCESSOR.

FIG. 16