DETECTION MODELS AND TARGET INFORMATION PROCESSING

by

Dr. James W. Bond
Naval Ocean Systems Center
San Diego, Ca., USA

ABSTRACT

The passive sonar equation leads in a natural manner to sensor detection performance contours. Useful information can be extracted from observed fractional holding time on non-cooperative targets by use of these contours. The derived information could aid in multiple sensor contact data association and the localization and tracking of poorly held targets. Target source level and sensor processor recognition differential need not be input to the prediction model to prioritize candidate contact data associations. A surprising example shows that use of predictions to estimate the range to initially detected targets is not likely to work. An idealized but reasonable example shows that the probability of a target remaining undetected at further ranges and first detected at a given range can be independent of the range of the target from the sensor. The theory of stochastic differential equations applied to tracking a target may alleviate the difficulty. The paper closes with recommendations for the development of model capabilities to provide more useful outputs for information processing.

INTRODUCTION

Detection models for passive acoustic sensors have been used for operational research, sensor trade-off studies, and planning sensor deployments. Detection models have been little used for real-time information processing. The purpose of this paper is to describe several applications of models to real-time information processing. The use of detection models to aid in contact data association and target localization and tracking will be investigated.

To provide a context for the discussion, postulate a processing center designed to handle non-cooperative targets through the reduction and analysis of passive acoustic sensor data. The target data consists of detections of signals, whose frequency, bearing relative to the sensor, and signal level relative to ambient noise are measured. The environmental data consists of directional ambient noise and clutter levels estimated within the processing center as well as externally provided data to the center.

Targets held consistently by several sensors pose no intrinsic information handling problems while targets held infrequently pose very difficult handling problems. For infrequently held targets, detection models appear useful in two ways: (1) to extract meaning from detections and non-detections and (2) to supplement measurement data on a target with predicted data. Applications using models in each of these ways will be discussed.

In section one an operationally observable measure, fractional holding time, is defined. Previous results obtained by the author comparing operational fractional holding time with predicted seasonal detection performance contours are briefly reviewed. These operations analysis results provide, in particular, a characterization of the variability of fractional holding time. The variability of the observable is especially important because it provides the basis for an assessment of the operational usefulness of detection performance predictions.

In section two, a general approach to using detection models to aid in contact data association is identified. The model is used to refine geographic consistency algorithms now commonly used for data association by accounting for the probabilities of observing given fractional holding times conditioned on the association. The predictions required need not use an input value of target source level, thus avoiding the often very difficult problem of estimating this parameter.

In section three the use of detection models for single sensor localization is first discussed. The inference of target range from fractional holding time is investigated. An example indicates that a most likely range for an initial contact need not exist. A more general approach to target tracking appears to be required. The use of stochastic differential equations seems to offer some chance of overcoming these difficulties with the added advantage that use can be made of sensor non-holding information (so-called negative information) as well as better use of sensor holding information (positive information).

Section four contains a brief discussion of the directions that passive acoustic sonar modeling might take to better aid in information processing. It is suggested that the predictions should more fully utilize measurement data available in the processing center and should aim to predict additional observable features of detected signals.

1 SINGLE SENSOR DETECTION PREDICTIONS

The basis for passive acoustic sensor detection performance predictions is the passive sonar equation (in decibels)

$$SE_t = SL - RD - PI + \varepsilon_t, \ t \geq 0 \ . \qquad (1)$$

From the viewpoint of the user of the predictions, source level (SL) summarizes the relevant target characteristics, while recognition differential (RD) summarizes the relevant signal processor characteristics. The performance index (PI) is a function of range, seasonal, and directional dependent transmission loss (TL), season and directional dependent ambient noise (AN), and directional dependent array gain (AG) in the following manner:

$$PI = TL + AN - AG .$$

The performance index therefore summarizes all of the non-target and non-processor relevant properties of the environment and sensor whose performance is being predicted. Signal excess SE_t varies as a function of time t.

One possible representation of the stochastic behavior of SE_t is the λ-σ jump model which is a simple step function of time. The family of step functions are conditioned by assuming

(1) ε_t is normally distributed with zero mean and variance σ^2

(2) ε_s and ε_t are different with probability $1 - \exp[\lambda(s-t)]$ and equal with probability $\exp[-\lambda(s-t)]$, $\lambda > 0$, $s > t$.

These criteria define a commonly used stochastic process for SE_t which has a normal distribution with variance σ^2 and an exponential autocorrelation function with a relaxation time constant of $1/\lambda$.

A natural manner to summarize expected performance over a period of time T is to predict the probability p that the instantaneous signal excess SE_t is non-negative. For example, if the expected value of signal excess were zero, then SE_t would be expected to be non-negative half of the time and p would be $\frac{1}{2}$. Furthermore, it is assumed that the operational measure of holding performance, the fraction of time that contact is held, fractional holding time (FHT) is an unbiased statistical estimate of p. Values of FHT will therefore be the observables which will be compared to predicted values of p.

From (1) it follows that

$$p = \text{Prob}\{SE_t \geq 0\} = \text{Prob}\{\varepsilon_t \geq PI - SL + RD\}$$

$$= 1 - \Phi\left(\frac{PI - SL + RD}{\sigma}\right)$$

with

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \exp[-\tfrac{1}{2}y^2]\, dy . \tag{2}$$

Another formulation of equation (2) is

$$\Phi^{-1}(1 - p) = (PI - SL + RD)/\sigma. \tag{3}$$

Equation (3) exhibits $\phi^{-1}(1 - p)$ as linearly related to PI with the slope of the line inversely proportional to the standard deviation σ of signal excess and with intercept $(SL - RD)/\sigma$.

The observable to be compared with the predictions is fractional holding time, FHT, defined as a sum of holding intervals of a particular target acoustic source over the sum of opportunity time in a region bounded by two PI contours $1\frac{1}{2}$ dB apart as illustrated in Figure 1. The FHT for such a region will be compared with the probability p for the average value of the two PI values. The expected variability of the observable FHT is a function of the true probability of non-negative SE_t in the PI bin and the total target opportunity time T in the bin. Let f(t) = 1 if the source is detected and 0 if a source is not detected by the sensor. Then, from FHT = $1/T \int_0^T f(t)\,dt$ and the fact that ε_t, $t \geq 0$ is a zero mean λ-σ stationary process it follows

$$E[FHT] = p \tag{4}$$

$$VAR[FHT] = 2p(1 - p)[(e^{-\lambda T}-1+\lambda T)/(\lambda T)^2] \,. \tag{5}$$

The term involving λT converges very rapidly to $1/\lambda T$ as $\lambda T \to \infty$ and therefore equation (5) simplifies to

$$Var[FHT] \simeq 2p(1 - p)\ 1/\lambda T \,. \tag{6}$$

The variation in SE induced by target position changes and target source level changes as well as sound propagation and ambient noise variation can be estimated by regression of FHT against PI. For this regression each observation $\Phi^{-1}(1\text{-}FHT)$ should be weighted according to its expected variance. The formulas used for the results presented in this paper are derived from equation (6) by use of the first two terms of the Taylor expression of Φ^{-1} evaluated at 1 - p and resulted in

$$Var\,\Phi^{-1}(1\text{-}FHT) \simeq \left[\dot{\Phi}(x)\right]^{-2} Var\ FHT = (2\pi)e^{X^2}Var[FHT] \tag{7}$$

where $1 - p = \Phi(x)$ and $x = E[FHT]$ using known properties of Φ.

Data was pooled for several targets with no a priori assumptions as to source level or recognition differential. The standard deviation of signal excess was also assumed to be unknown but dependent on season but independent of target. Figure 2 presents an example of the regression results obtained. Data is presented for seven targets deployed during the winter season. The left scale is in terms of the transformed variable $\Phi^{-1}(1\text{-}FHT)$ and the right-hand scale is natural. The overall σ results obtained are summarized in Table 1. The noteworthy implication of these results is that summer estimates of σ are from 4 to 7 db higher than winter estimates. This result lends credibility to the estimates because greater variability would be expected during the summer than the winter due to propagation effects. The fact that propagation paths are more sharply focused during the summer would lead to greater variation of received signal level as a function of target position.

The variability in signal excess implied by the values of σ in Table 1 exceeds that estimated due to statistical variability by equation (7). Some of this increased variability is induced by the use of fixed transmission loss estimates as a function of angle for each sensor and season and the use of an average direction dependent ambient noise for each sensor.

TABLE 1. EXAMPLE OF σ ESTIMATES OBTAINED FROM A REGRESSION ANALYSIS OF FHT ON REAL TARGETS.

	Summer Season	Winter Season
SENSOR 1		*
Number of Targets	5	7
Degrees of Freedom	26	41
Point Estimate of σ	17.1	10.2
90% Fiducial Bounds of σ	12.3 27.9	8.2 13.6
SENSOR 2		
Number of Targets	4	7
Degrees of Freedom	29	14
Point Estimate of σ	10.1	5.9
90% Fiducial Bounds of σ	8.1 13.5	4.4 8.7

*Case presented in Figure 2.

Figure 3 shows an example of the variability observed for a nominal case, not the best case nor the worst case. The solid lines are 90% confidence bounds on the regression line and the dotted lines are 90% confidence bounds on the observations. In this example, even though the regression line is estimated with reasonable confidence, the variability of the observations is around 7 dB. Predictions which use real-time estimates of transmission should reduce this loss or ambient noise variability. However, at this time an analysis to assess the variability of observed fractional holding time relative to real-time predictions has not been undertaken to the author's knowledge.

The operational analysis results imply two conclusions relative to the use of models for real-time information processing:

(1) predictions for real-time use should be based on real-time data; and

(2) although regression analysis can estimate σ and SL-RD for a large data base, these estimates will normally not be available for real-time applications.

2 USE OF PREDICTIONS FOR CONTACT DATA ASSOCIATION

The fundamental problem in data association is to calculate the probability that a given set of observations occurs. The calculation can be factored into the calculation of two probabilities:

(1) the probability that the observed line characteristics, e.g., frequency, bandwidth, stability, would be associated with the acoustic sources of a single target; and

(2) the probability that the available operator heuristic information would have been generated if they were on the same target.

If detections are associated for different sensors, then an additional factor becomes important:

(3) the probability of the observed detections occurring on specific bearings for some sensors and not occurring on specific bearings for other sensors for each target acoustic source.

There appears to be a real possibility of exploiting detection models to improve the estimation of the third factor and little possibility of their use in the estimation of the other factors. Therefore, we focus our attention on the two sensor contact association problem. It is convenient to introduce the following notation to aid in a concise formulation of the problem:

The observations consist of fractional holding times FHT_i (f_j) for a specified opportunity time T, for sensors i = 1, 2, ..., M, and acoustic sources with nominal frequencies $f_1, f_2, \ldots, f_N$.

The sensor indices i = 1 and 2 are chosen so that the contact association consists of FHT_1 (f_j) and FHT_2 (f_k).

The FHT_i (f_j) are all supposed to be appropriately calculated, i.e., the holding time on a sensor i for target source with frequency f_j has been on a single target. (The reader will note that it would be straightforward to extend the results presented to the case when these single sensor, single source associations had been made correctly with probability P_{ij} over the opportunity time T.)

To interpret the observed fractional holding times it is necessary to associate PI values to them. A value of T of 24 hours or larger appears to be reasonable. To associate a PI value to an observed FHT define an opportunity region as in Figure 4 compatible over time with the bearing information observed on the sources f_j and f_k for sensors 1 and 2, respectively, during the opportunity interval T. Such a region would have lengths on the order of 2 VT where V is the average speed-of-advance of the target. View the region defined in log R coordinates and take the centroid (or a point approximating it) as the position for which PI predictions are compared with the observed FHT.

The PI predictions for the centroid should be as accurate as possible and therefore should be real-time predictions using sensor measured directional ambient noise and the best available sound velocity depth profiles for transmission loss predictions.

Equation (3) exhibits the probability of detection p as a function of PI, with parameters σ, RD, and SL. The value of σ can be empirically estimated by sensor based on an analysis of historical data. The value of RD can be updated in real time from the outputs of passive acoustic sensors through the use of an automatic detection algorithm to calibrate operator responses. A priori estimates of SL are generally not available, so a posteriori estimates will be used. Toward this end, view p as a function of SL. Suppose that the opportunity time T is long enough and the target maneuvers such that the average value of SL observed by the sensors during the opportunity time T is independent of sensor. Then equation (3) implies that for sensors 1, 2, ..., M the probabilities $p_1(SL)$, $p_2(SL)$, ..., $p_M(SL)$ are derivable from the sensor PI values PI_1, PI_2, ..., PI_M for the centroid of the opportunity region as a function of a single unknown SL.

A natural measure of consistency between $p_i(SL)$ and FHT for a given acoustic source and the M sensor's is the weighted mean-square error E(SL) defined by

$$E(SL) = \sum_{i=1}^{M} (FHT_i - p_i(SL))^2/(Var[FHT_i] + Var[p_i(SL)]) \ . \tag{8}$$

The smaller E(SL) can be made the more consistent are the observations. Furthermore, the value of SL which minimizes E(SL) provides the a posteriori estimate of SL as previously mentioned.

The value of Var[FHT] used in equation (8) can be determined using equation (5) with the value of λ estimated empirically from an analysis of historical operational data. (Such an analysis is now in progress at NOSC by the author.)

The value of Var[p] can be estimated by differential approximation (refer to Figure 5). It follows that

$$\Delta p \simeq \dot{\Phi}\left(\frac{SE}{\sigma}\right) \frac{\Delta PI}{\sigma} \ , \tag{9}$$

where SE = -PI + SL - RD and σ is the standard deviation of signal excess with $\dot{\Phi}(x) = \frac{e^{-x^2/2}}{\sqrt{2\pi}}$ by the fundamental theorem of calculus. Assume the value of Δp to match the observed holding time as a function of time is uniformly distributed. It then follows that

$$Var[p] = \frac{1}{3} \Delta p^2 \ . \tag{10}$$

The desired approximation for Var[p] is obtained by substitution of (9) into (10).

The inclusion of the variances in equation (8) appears to be important. It helps ensure correct behavior of (8) for small p when FHT = 0. This is because it can be shown that

$$p/(\mathrm{Var}[FHT] + \mathrm{Var}[p]) \to \text{constant as } p \to 0.$$

The expression in (8) could clearly be used to rank different associations for the same M sensors. It is not so clear how to compare E(SL) values when the sensors used for the calculations differ. An alternative approach to the calculation of consistency, albeit more complicated, would be to calculate the probability of the observations for an optimum choice of SL. For this approach the random variable

$$(FHT - p(SL)) \,/\, \sqrt{\mathrm{Var}[FHT] + \mathrm{Var}[p]}$$

is assumed to be normally distributed with unit variance. Then the probability of observing Y satisfying $|Y-p(SL)| \leq |FHT-p(SL)|$ is given by

$$1-2\Phi(-|FHT-p(SL)| \,/\, \sqrt{\mathrm{Var}[FHT] + \mathrm{Var}[p]}) \qquad (11)$$

and the probability of the observations $FHT_1, \ldots, FHT_M$ is given by

$$P(SL) = \sum_{i=1}^{M} (1-2\Phi(-|FHT_1-p_i(SL)| \,/\, \sqrt{\mathrm{Var}[FHT_i] + \mathrm{Var}[p_i(SL)]})) \qquad (12)$$

for a choice of SL leading to the normal distributions of the random variables. This choice of SL is assumed to be given by the value which maximizes P(SL).

For either approach, it is suggested that SL be estimated iteratively. A good first choice would be a sensor for which the data association was most straightforward, as long as it was not held all the time. The initial choice for SL can then be obtained by solving equation (3) for SL when the observed fractional holding time is substituted for p.

The value of E(SL) or P(SL) thus obtained would be used in the ranking of contact data associations. The estimates of SL would of themselves be of great value since target source level of a non-cooperative target is not observable using the usual non-calibrated sensors in an incompletely characterized environment. The method provides a recursive procedure for estimating SL for all acoustic sources of all targets being tracked.

At the present time this methodology is being investigated by studying the SL estimates obtained from processing of historical data associated to operational targets with known source level.

3 USE OF PREDICTIONS FOR TARGET LOCALIZATION AND TRACKING

Another important potential use of models is to provide range estimates when cross-fixes are not available as illustrated in Figure 6. Due to uncertainties in the predictions and the expected variability in fractional holding time it appears unlikely that range estimates obtained from comparing predicted probabilities as a function of range with observed holding time would provide useful information when cross-fixes are available during the time frame T associated with the fractional holding time observation. It is natural then to consider the case when an isolated detection has occurred by a single sensor.

Suppose that a sensor first detects a target. Figure 7 summarizes the performance of the sensor. Suppose further that the target is moving toward the sensor so that it moves through regions of poorer coverage toward regions of better coverage. Then the probability of being first detected p in region K is the product of the probability of detection for region K and the probability of non-detection for the regions 1, 2, ..., k-1, i.e.,

$$P_k = p_k(1-p_{k-1})(1-p_{k-2}) \ . \ . \ . \ (1-p_1) \ . \tag{13}$$

Several examples indicated that P_k tended to vary slowly with K. A fellow worker at NOSC, Stefen Hui, found the following important example:

Let

$$p_k = p_1/(1-(k-1)p_1) \tag{14}$$

with $p_1 \leq 1/N$ to assure that the p_k as defined satisfy $0 \leq p_k \leq 1$. Then $P_1 = P_1 = P_2 = \ldots = P_N$.

To see that this is the case, proceed by mathematical induction. $P_1 = p_1$. Suppose $P_k = p_1$. Then,

$$P_{k+1} = p_{k+1}(1-p_k) \ \ldots \ (1-p_1)$$

$$= \frac{p_{k+1}}{p_k} (1-p_k) \ P_k \ ,$$

and it suffices to verify that $p_{k+1}(1-p_k)/p_k = 1$.

Using (14) it follows that

$$\frac{p_{k+1}}{p_k} (1-p_k) = \frac{1-(k-1)p_1}{1-kp_1} \left(1 - \frac{p_1}{1-(k-1)p_1}\right) = 1 \tag{15}$$

by cancellation after placing the second factor under a common denominator. Furthermore, whenever (15) holds, i.e., whenever $p_{k+1} = p_k/(1-p_k)$, it follows that $P_{k+1} = P_k$. For example, if $p_k = 1/n$, then $p_{k+1} = \frac{1}{n} \cdot \frac{n}{n-1} = \frac{1}{n-1}$. The example is not farfetched and suggests that in practice it will be difficult to estimate range from predicted probabilities of detection for a sensor. Indeed, the example shows that for a target of known track (radial toward sensor) and known probabilities of detection for the sensor as a function of range, but unknown time of opportunity for the sensor, there need be no range information associated with an initial detection event. To try to overcome this difficulty, the past history of the target needs to be considered, i.e., the use of the model for tracking rather than localization should be considered.

The contact data association calculation suggests that SL estimates may not be necessary for the tracking application. The ocean could be partitioned into regions with some boundaries associated for the sensors holding and others defined from environmental considerations. For each region a calculation of the probability of observing the given FHT_i for sensors $i = 1, 2, \ldots, M$ could be made and the calculation of P(SL) defined by equation (12) made as described in Section 2. The target track could be assumed to have random course/speed changes. The suggested technical approach would then attempt to apply the theory of stochastic differential equations to the problem.

Stochastic differential equations have been successfully used for modeling target motion and to represent a more general approach than Kalman filtering (reference 1). The interesting observation, I believe, is that a model using stochastic differential equations might provide a vehicle for using negative information as well as positive information. Here, using negative information simply means using the information that a sensor did not gain contact on a target.

Consider a charged particle moving through a field of charge particles of the same and of opposite charge. The particle is attracted by charges of opposite polarity and repelled by charges of like polarity. Furthermore, while moving through the field the particle experiences random external forces which tend to perturb its track.

Suppose the charged particle is the target. The like charged particles are non-detecting sensors whose repulsion forces are described by the predicted probability of non-detection as a function of the range and bearing of the target from the sensor. The oppositely charged particles are detecting sensors whose attraction force is also described by predicted probabilities of detection. The random forces are other factors leading to course and speed changes of the target. The analogy breaks down because of bearing information which requires that the target be along given bearings at given times. The problem is worth pursuing and it is our intent to study the problem in the future.

The goal of using stochastic differential equations with model inputs to allow use of negative information as well as better use of holding information is not to obtain extremely accurate localization information but to provide information useful for planning the allocation of mobile sensors or re-processing of data recorded in real time.

4 THE NATURE OF PREDICTIONS DESIRED FOR INFORMATION PROCESSING

Performance predictions to be used in information processing can always be based on measured or recently measured ambient noise and clutter.

When a detection has occurred, signal-to-noise estimates are available as a function of time during detection periods. Autocorrelation could be performed on the received signal during these periods. It might be possible to glean multi-path data from the resulting ambiguity surfaces.

It is rare that predictions for a sensor not holding contact will be required unless some other sensor is holding contact. For the case when the sensors and targets are all in a homogeneous environment, it seems reasonable to try to use what was observed by the detecting sensor to generate better predictions for the non-detecting sensor. To my knowledge, such conditional predictions have not been investigated.

It is my feeling that high operational payoff might result if modeling emphasis shifted from predictions based on historical or synopic data to predictions based on information derivable from directional noise measurements for the sensor and related available measurements for sensors whose outputs can be spectrum analyzed or coherently processed.

The durations of holding and non-holding intervals, signal-to-noise ratios during holding, bearing behavior, all may contain useful clues concerning propagation and hence target range or the probability that detections occur or do not occur on several sensors simultaneously or near simultaneously. The prediction of observables more general than probabilities of detection should be the direction that future acoustic modeling takes if it is to be a major tool in information processing of passive acoustic sensor data.

REFERENCES

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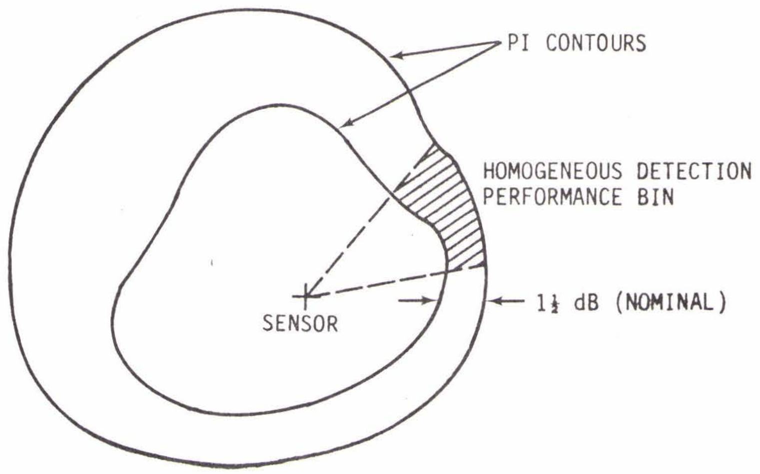

FIG. 1 TYPICAL HOMOGENOUS DETECTION PERFORMANCE BIN

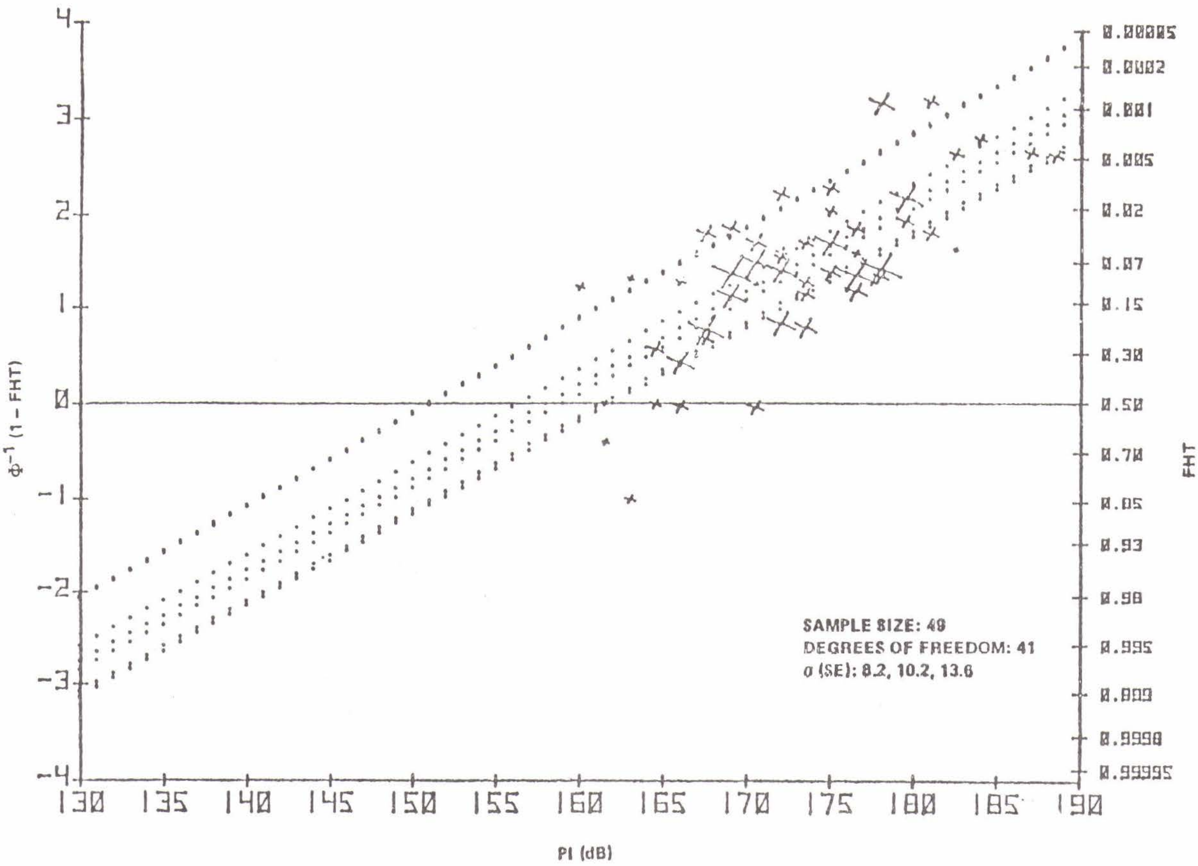

FIG. 2 POOLED TARGET REGRESSION SUMMARY PLOT DURING THE WINTER SEASON

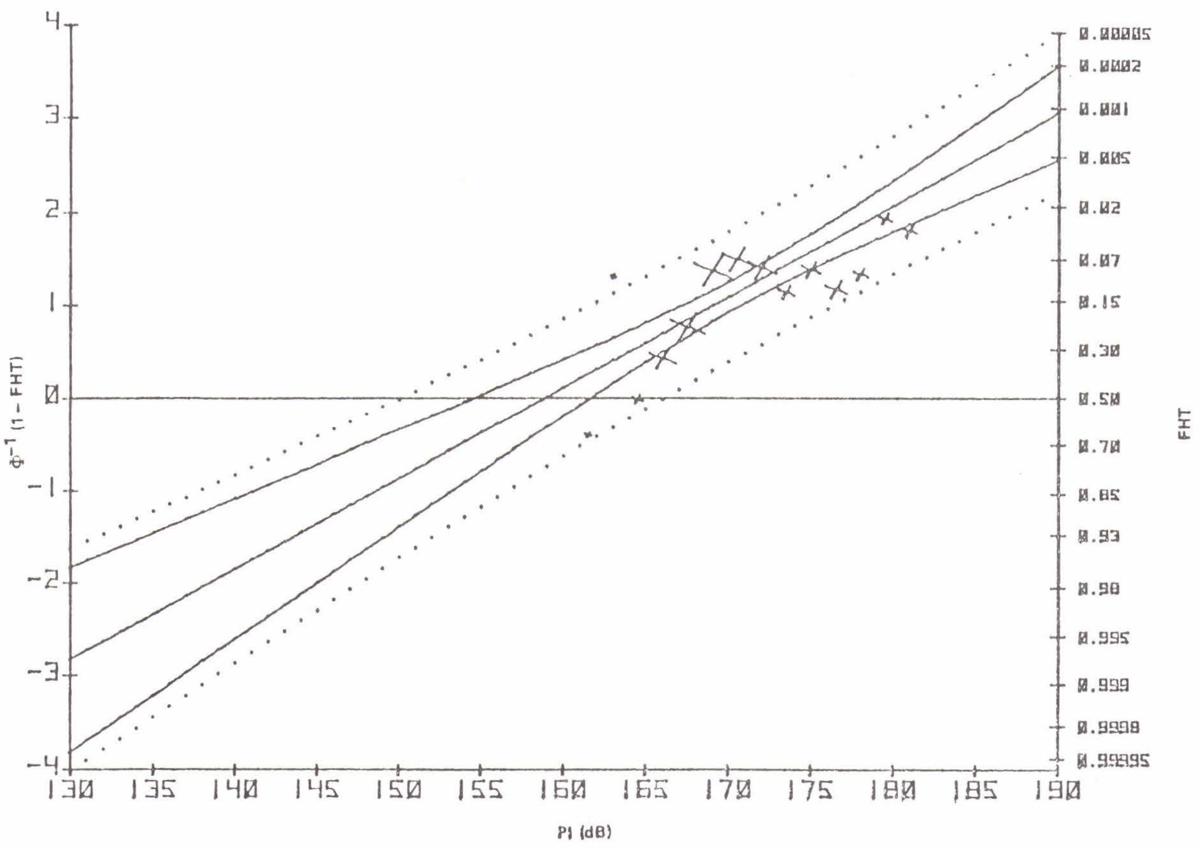

FIG. 3 POOLED REGRESSION RESULTS FOR DURING THE WINTER SEASON

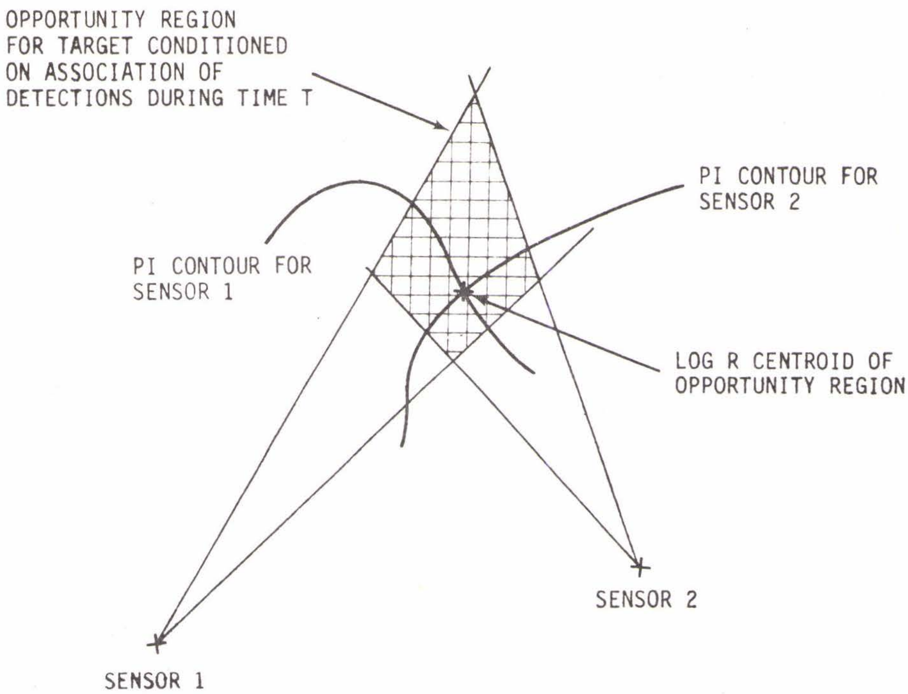

FIG. 4 OPPORTUNITY REGION FOR TWO SENSOR CONTACT DATA ASSOCIATION

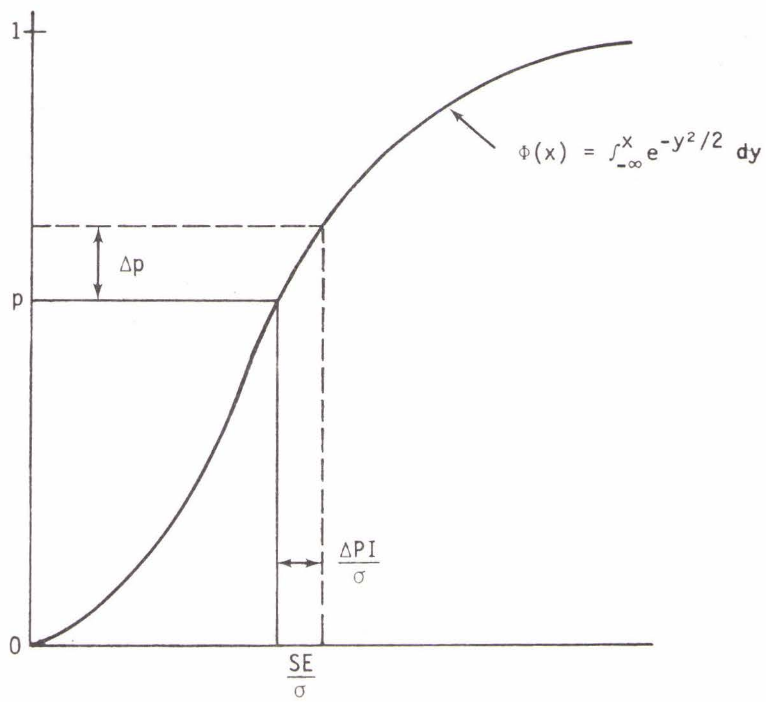

FIG. 5 ESTIMATION OF Var[p]

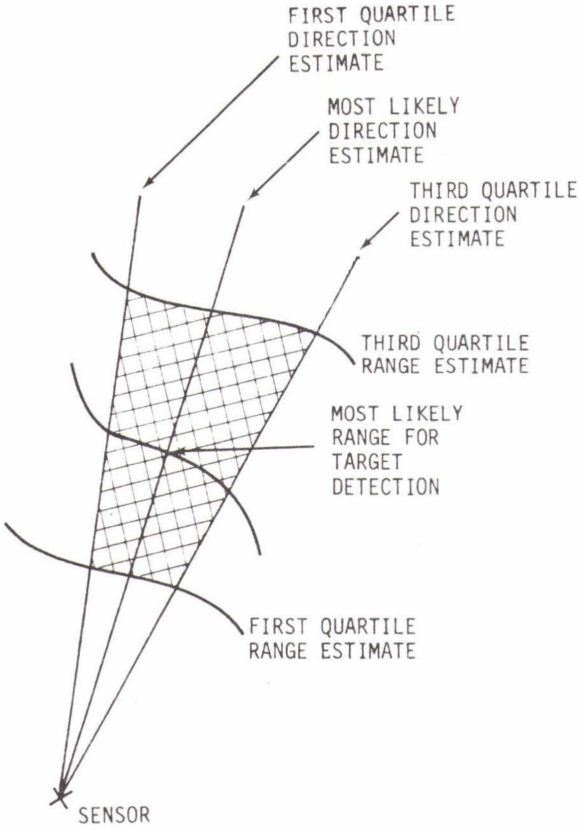

FIG. 6

SINGLE SENSOR RANGE ESTIMATION USING PREDICTIONS

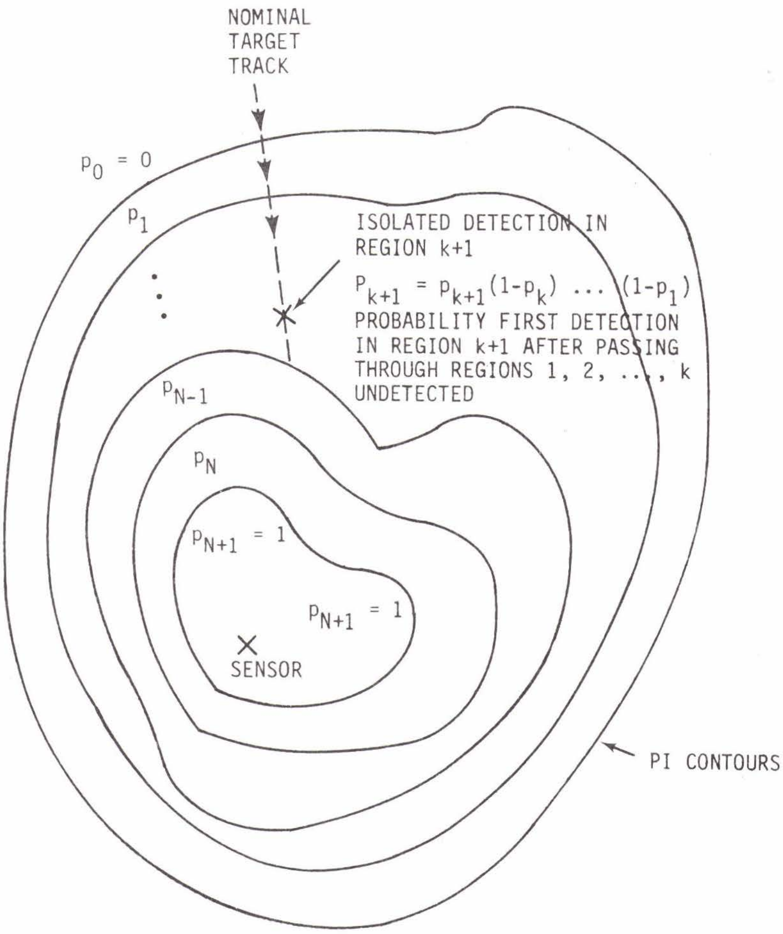

FIG. 7

INITIAL DETECTION SCENARIO