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Dynamic Underwater Sensor Network for Sparse Field Estimation

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***Abstract*—A coordinated dynamic network of autonomous underwater gliders to estimate 3D environmental sparse fields is proposed and tested. Field spatial sparsity is exploited in the estimation algorithm. Moreover field measurements are acquired by compressive sensing devices. Integration with a network of surface relay nodes and asynchronous consensus are used to distribute local information and achieve the global field estimate. Tests on simulated data demonstrate the feasibility of the approach with relative error performance within 10%.**

***Keywords*—dynamic sensor network; underwater glider; random demodulator; SAKF;consensus protocols.**

I. INTRODUCTION

The latest development on autonomous underwater vehicles, called *gliders*, has opened a wide range of possibility for sampling the ocean efficiently and persistently at a feasible cost [1]. At the same time, this kind of vehicles and especially networks of gliders, pose new challenging problems regarding the managing and the automatic control of such networks.

This work describes an approach for optimally estimating constant or slowly-varying environmental spatial field in both a centralized and distributed fashions by a fleet of gliders integrated with a network of relay nodes. Moreover, we focus our attention on the estimation of *sparse fields*, i.e. that fields can be represented by a number of informative coefficients that is much lower than the total number of dictionary base functions used to represent it. If the field is not sparse, it is required to be at least compressible i.e. its coefficients, once ranked in descending order, decay with a given hyperbolic law. Examples of these fields are temperature and optical properties of the seawater, acoustic noise distribution and pollutant concentration. The contribution of the paper is then twofold. First we provide a compressive sampling scheme, based on the Random Demodulator (RD) [2], and a compressed estimation algorithm, based on the Sparsity Aware Kalman Filter (SAKF) [3], in order to: *i*) simplify the sampling hardware of the analog front end of the gliders and *ii*) fully exploit the sparsity property of the field to be estimated. Second, we propose an adaptive and automatic ocean sampling strategy along with centralized and decentralized estimation protocols. More specifically, the path of each glider (sensor) of the network is optimized in such a way that the sensors are forced to move into the most informative region of the field, i.e. the measurements are collected in those areas where the estimate is more inaccurate. Moreover, a centralized and a decentralized estimation protocol are discussed and compared in terms of field estimation error. In the case of a glider network, the centralised version is not feasible as it would require the field measurements or the field estimates from each sensors available at a Fusion Centre (FC) all at the same time, at each time step. Instead, due to physical and practical operational constraints, communications between gliders and the FC are sporadic and asynchronous, typically happening when the glider is at surface, through a satellite link [1]. Nevertheless, the centralised field estimation is taken as reference for testing the performance of the proposed protocol.

This work follows the seminal papers [4] and [5] on spatial field distributed estimation for static and dynamic cases. However, in [4] and [5], neither the field sparsity nor the possibility to have intermittent communication links (as in the underwater scenario) are considered.

This paper is organized as follows. Section 2 provides the overview of the system. In particular, the field decomposition is first introduced, then the field estimation algorithm based on a centralized SAKF architecture is described and the RD device model is specified. Finally, the adaptive control law is provided. In Section 3 the distributed architecture is discussed and the consensus protocol is detailed. Section 4 provides simulation results while Section 5 ends the paper drawing conclusions and highlighting future work.

II. OVERWIEV OF THE CENTRALIZED ALGORITHM

This section provides an overview of the centralized estimation algorithm. Particular attention will be paid to the methods exploited to promote the sparsity of the estimated field. The estimation procedure relyes on the expansion of the spatial field on an overcomplete base of known spatial functions weighted by unknown coefficients which are in general time variant. The spatial field $g(\mathbf{r},t)$ to be estimated can be written as [4]:

$$g(\mathbf{r},t) = \boldsymbol{\psi}(\mathbf{r})^T \mathbf{c}(t) = \sum_{j=1}^{L} c_j(t)\psi_j(\mathbf{r}) \tag{1}$$

where $\mathbf{r}$ is the coordinate vector of the 2 or 3-dimensional region of interest, t is the time variable, $\mathbf{c}(t)=[c_1(t),\ldots,c_L(t)]^T$ is the (possibly time varying) real coefficient vector and $\boldsymbol{\psi}(\mathbf{r})=[\psi_1(\mathbf{r}),\ldots,\psi_L(\mathbf{r})]^T$ is the vector of the base functions that depends only on the position vector $\mathbf{r}$. Given the base vector $\boldsymbol{\psi}(\mathbf{r})$, the problem of estimating the scalar field $g(\mathbf{r},t)$ is equivalent to estimate the coefficient vector $\mathbf{c}(t)$. Two hypotheses on $\mathbf{c}(t)$ are assumed in this work: *i*) the coefficient vector is constant or slowly time-varying, i.e. $\mathbf{c}(t)\approx\mathbf{c}$, *ii*) it is sparse, i.e. the number of non-zero entries K is much smaller of its dimension L, or compressible with ranked coefficients decaying with hyperbolic law. Both the hypotheses are reasonable for the fields of our interest. In fact, ocean fields need a long period in time to change significantly their values, and they present very smooth variation in space also. Given the field expansion in eq. (1) and the sparsity hypothesis, the coefficient vector $\mathbf{c}$ can be estimated using the SAKF introduced in [3]. Moreover, in order to reduce the energy consumption, the data storage load and the hardware complexity, the network agents considered in this work are equipped with a sparse sensing acquisition device such as the RD [2]. The coefficient vector then can be sequentially estimated at a sampling rate lower than the Nyquist rate by including in the measurement equation of the SAKF the sparse device model. In this work, the base functions in (1) are of the radial type. In particular, a set of Gaussian-like radial basis functions (RBF), $\psi_j(\mathbf{r})=\exp(-\|\mathbf{r}-\mathbf{r}_j\|^2/\beta)$, is chosen with a given spread parameter β (constant for all the functions) and centers $\mathbf{r}_j$ located on a regular grid in the spatial region of interest.

A. The Centralized estimation algorithm

The centralised solution here considered is based on the fusion at the estimation level rather than at sensor measurement level. Suppose to have a network of N agents or sensors moving in the area of interest. Each sensor sends its own local estimate of the coefficient vector to a FC at each time step k. The FC processes the local estimates by combining them to obtain the global field estimate. In each local agent, the estimation of the coefficient vector is performed using a SAKF. Under the slowly time-varying assumption, the state update equation of each SAKF can be expressed as:

$$\mathbf{c}_{i,k} = \mathbf{c}_{i,k-1} + \mathbf{q}_{i,k} \tag{2}$$

where i is the index of the particular agent and $\mathbf{q}_{i,k}$ is a white Gaussian forcing noise with zero mean and diagonal covariance matrix $\mathbf{Q}_{i,k} = \mathrm{diag}(\sigma^2_{1,i,k},\ldots,\sigma^2_{L,i,k})$. The matrix $\mathbf{Q}_{i,j}$ is a free parameter that can be tuned to adjust the speed of convergence of the filter. At each time step k, the i^{th} sensor collects a measurement $z_{i,k}$ that can be modeled as:

$$z_{i,k} = \boldsymbol{\psi}(\mathbf{p}_{i,k})^T \mathbf{c}_{i,k} + e_{i,k} \tag{3}$$

where $\boldsymbol{\psi}(\mathbf{p}_{i,k})$ is the vector of the spatial base function evaluated at the position vector $\mathbf{p}_{i,k}$ of the i^{th} sensor at time step k, $e_{i,k}$ is the scalar Gaussian measurement noise, independent from $\mathbf{q}_{i,k}$, with zero-mean and variance $\rho^2_{i,k}$. Each sensor runs its own SAKF to provide the sequential local estimate of the coefficient vector $\hat{\mathbf{c}}_{i,k}$ and its error covariance matrix $\mathbf{C}_{i,k}$ to the FC. By defining the local information matrix as $\mathbf{D}_{i,k} = \mathbf{C}^{-1}_{i,k}$, the FC fuses the local estimates performing the following two steps:

$$\mathbf{D}_k = \sum\nolimits_{i=1}^{N} w_i \mathbf{D}_{i,k} \ , \tag{4}$$

$$\hat{\mathbf{c}}_k = \mathbf{D}_k^{-1} \sum\nolimits_{i=1}^{N} w_i \mathbf{D}_{i,k} \hat{\mathbf{c}}_{i,k} = \mathbf{C}_k \sum\nolimits_{i=1}^{N} w_i \mathbf{D}_{i,k} \hat{\mathbf{c}}_{i,k} \ , \tag{5}$$

where the weights, w_i, can be constant such as $w_i=1/N$ $i=1,\ldots,N$, as in the simple average case. The FC broadcasts the global estimates, $\hat{\mathbf{c}}_k$ and $\mathbf{C}_k$, to the sensors that use them to figure out the next position vector as described later.

B. Sensor model and CS measurement equation

CS provides both the theoretical framework and the practical tools to efficiently approach the sampling and the reconstruction of (analog) sparse signals, i.e. the number of significant frequency components is much smaller than the band limit permits [2]. For this class of signals, informative components can be retrieved by far less than the number of samples acquired at the Nyquist rate. Different sub-Nyquist sample devices have been recently proposed in literature. An example of this devices is the RD proposed in [2].

The RD device consists in first modulating the analog input signal, in our case the spatial field $g(\mathbf{r},t)$, with an analog random sequence, $\varepsilon_c(t)$, of impulses at Nyquist rate of, namely, W hertz (the chip sequence) with amplitude that takes ± 1 equiprobable values. The modulator is followed by an integrator and a “sampling and hold” device that works at a lower sampling rate than the Nyquist rate. In particular, by defining the compression ratio $B=W/M$, where W is the number of Nyquist samples acquired in a given observation interval and M the number of CS samples acquired in the same interval, the discrete compressed sequence $y_{i,mB}$ can be obtained as:

$$y_{i,mB} = B^{-1} \int_{mB-B}^{mB} g(\mathbf{p}_i(t),t)\varepsilon_c(t)dt + n_{i,mB} \tag{6}$$

where $n_{i,mB}$ is a noise term and $m=1,\ldots,M$. By substituting eq. (1) in eq. (6) we have:

$$y_{i,mB} = B^{-1} \left(\int_{mB-B}^{mB} \boldsymbol{\psi}(\mathbf{p}_i(t))\varepsilon_c(t)dt \right)^T \mathbf{c}_{mB} + n_{i,mB} \ . \tag{7}$$

Finally, by exploiting the sampling property of the chip sequence, the linearity of the integral operator, eq. (7) can be rewritten as a vector matrix product (the integral is discretized at the Nyquist rate between $k=mB-B$ and $k=mB$):

$$y_{i,mB} = B^{-1} \boldsymbol{\varepsilon}_i^T \mathbf{H}_{i,mB} \mathbf{c}_{mB} + n_{i,mB} \ , \tag{8}$$

where:

$$\mathbf{H}_{i,mB} = \begin{bmatrix} \boldsymbol{\psi}_{i,mB-B+1} & \cdots & \boldsymbol{\psi}_{i,mB-1} & \boldsymbol{\psi}_{i,mB} \end{bmatrix}^T , \tag{9}$$

$$\boldsymbol{\psi}_{i,mB-b} = \begin{bmatrix} \psi_1(\mathbf{p}_{i,mB-b}) & \cdots & \psi_L(\mathbf{p}_{i,mB-b}) \end{bmatrix}^T , b=B\text{-}1,\ldots,0 \tag{10}$$

$n_{i,mB} = B^{-1}\boldsymbol{\varepsilon}_i^T \mathbf{e}_{i,mB}$, $\boldsymbol{\varepsilon}_i=[\varepsilon_{i,1},\ldots,\varepsilon_{i,B}]^T$, $\varepsilon_{i,j}=\pm 1$ is the random chip sequence of the i^{th} sensor end $\mathbf{e}_{i,mB}=[e_{i,mB-B+1},\ldots,e_{i,mB}]^T$ is the noise vector whose entries are obtained by sampling the noise process in eq. (3) at $k=mB-B+1,\ldots,mB$. As a consequence, $n_{i,mB}$ is a zero-mean Gaussian random variable with variance $\rho_{i,k}^2$.

C. Sparsity Aware Kalman Filter

In order to take advantage of the sparse structure of the coefficient vector, the local Kalman filter has to be properly modified. The SAKF proposed in [3] has been derived with the aim of enforcing sparsity in the KF estimate following a Weighted Least Square Approach. In the following, a description of the application of the SAKF is provided. For notation simplicity, we define the column vector $\mathbf{v}_{i,mB} = (B^{-1}\boldsymbol{\varepsilon}_i^T\mathbf{H}_{i,mB})^T$, then the state update equation in (2) and the measurement equation in (8) can be rewritten as:

$$\mathbf{c}_{i,mB} = \mathbf{c}_{i,(m-1)B} + \mathbf{q}_{i,mB}, \tag{11}$$

$$y_{i,mB} = \mathbf{v}_{i,mB}^T\mathbf{c}_{mB} + n_{i,mB}. \tag{12}$$

Following [3], the joint WLS estimate with sparsity constraint for $\hat{\mathbf{c}}_{i,mB|mB}$ and $\mathbf{C}_{i,mB|mB}$ can be obtained as:

$$\hat{\mathbf{c}}_{i,mB|mB} = \arg\min_{\mathbf{c}_{i,mB}}\left\{J(\mathbf{c}_{i,mB})\right\}, \text{ with:} \tag{13}$$

$$\begin{aligned} J(\mathbf{c}_{i,mB}) &= \left\|\hat{\mathbf{c}}_{i,mB|(m-1)B} - \mathbf{c}_{i,mB}\right\|^2_{\mathbf{C}^{-1}_{i,mB|(m-1)B}} \\ &+ \left\|y_{i,mB} - \mathbf{v}_{i,mB}^T\mathbf{c}_{i,mB}\right\|^2_{\rho^{-2}_{i,mB}} + \lambda\left\|\mathbf{c}_{i,mB}\right\|_{\ell} \end{aligned} \tag{14}$$

$$\begin{aligned} \mathbf{C}_{i,mB|mB} &= \mathbf{C}_{i,mB|(m-1)B} + \left(\left\|\mathbf{v}_{i,mB}\right\|^2_{\mathbf{C}_{i,mB|(m-1)B}} + \rho^2_{i,mB}\right)^{-1} \times \\ &\times \mathbf{C}_{i,mB|(m-1)B}\mathbf{v}_{i,mB}\mathbf{v}_{i,mB}^T\mathbf{C}_{i,mB|(m-1)B}, \end{aligned} \tag{15}$$

where $\left\|\mathbf{x}\right\|^2_{\mathbf{A}} \triangleq \mathbf{x}^T\mathbf{A}\mathbf{x}$, $\left\|\cdot\right\|_{\ell}$ is a norm that promote the sparsity, λ controls the sparsity-bias trade-off and

$$\mathbf{C}_{i,mB|(m-1)B} = \mathbf{C}_{i,(m-1)B|(m-1)B} + \mathbf{Q}_{i,mB}. \tag{16}$$

The first two terms of $J(\mathbf{c}_{i,mB})$ are two quadratic forms, then differentiable. The sparsity-promoting term is in general not differentiable (e.g. if the ℓ_1 norm is used). In order to have a closed form expression of the gradient of $J(\mathbf{c}_{i,mB})$, i.e. $\nabla J(\mathbf{c}_{i,mB})$, the smoothed ℓ_0 norm (SL0) is used. The SL0 is a continuous and differentiable (but not convex) approximation of the ℓ_0 norm introduced in [6]. Since $\nabla J(\mathbf{c}_{i,mB})$ exists for every $\mathbf{c}_{i,mB}$ the minimization problem in (13) can be easily solved by a gradient descent method.

D. Network control

Since we have at our disposal a network of N sensors, a fundamental aspect to be taken into account is the optimal control of their trajectories. Since the main purpose of the network is to estimate the field $g(\mathbf{r},t)=\boldsymbol{\psi}(\mathbf{r})^T\mathbf{c}(t)$ assumed to be constant in time or slowly-varying, then the best set of trajectories will be the one that minimize the field estimation error. More formally, let $\mathbf{p}_{i,(m-1)B}$ be the position vector of the i^{th} sensor at time $k=(m-1)B$. We define as prediction error at time $k=mB$ the function:

$$\begin{aligned} l(\mathbf{r},mB) &= E\left\{\left(\boldsymbol{\psi}(\mathbf{r})^T\mathbf{c} - \boldsymbol{\psi}(\mathbf{r})^T\hat{\mathbf{c}}_{mB|(m-1)B}\right)^2\right\} \\ &= c + \boldsymbol{\psi}(\mathbf{r})^T\mathbf{C}_{i,mB|(m-1)B}\boldsymbol{\psi}(\mathbf{r}), \end{aligned} \tag{17}$$

where c collects all the constant terms w.r.t. $\mathbf{p}_{i,(m-1)B}$. An objective function for the trajectories can be defined as the spatial mean over a given region A of the prediction error:

$$s(\mathbf{p}_{i,(m-1)B}) = \int_A \boldsymbol{\psi}(\mathbf{r})^T\mathbf{C}_{i,mB|(m-1)B}\boldsymbol{\psi}(\mathbf{r})d\mathbf{r}. \tag{18}$$

Following [4], a control law for the i^{th} sensor can then be expressed as:

$$\mathbf{p}_{i,mB} = \mathbf{p}_{i,(m-1)B} - \gamma\nabla s(\mathbf{p}_{i,(m-1)B}), \tag{19}$$

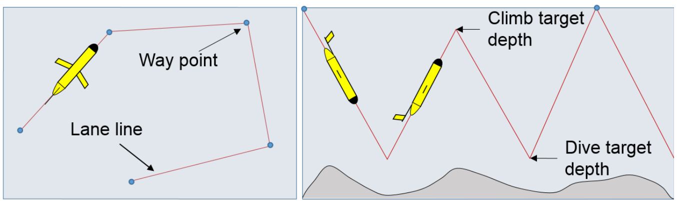
where γ is a positive constant gain and the gradient is taken with respect to the sensor position. The control law is applied at $k=(m-1)B$ and remains constant during the interval $[(m-1)B, mB]$ until the new CS measurement is available for the update step. The detailed calculation of the control law for a given set of base functions and the hypothesis under which the proposed control law is valid can be found in [4].

E. The kinematic model for underwater gliders

Generally, a glider moves through a 3D space following a saw tooth shape trajectory (see fig. 1) in the vertical plane. The trajectory is composed of a certain number of dive/climb cycles in the interval between two surfacing phases of the glider. Usually, the data, collected during each dive or climb cycle, are stored and are finally transmitted during the surfacing phase. The glider dynamic model considered in this work assumes a constant velocity without water current disturbances constrained to follow a yo-yo trajectory in the vertical plane with given climbing and diving target depths [7]. The glider, navigates in the vertical plane along a yo-yo segment with a given pitch angle. Since, in absence of currents, the gliders move with a constant speed v, the control law in eq. (19) has to be modified accordingly as:

$$\mathbf{p}_{i,mB} = \mathbf{p}_{i,(m-1)B} - v\nabla s(\mathbf{p}_{i,(m-1)B}) \Big/ \left\|\nabla s(\mathbf{p}_{i,(m-1)B})\right\|_2. \tag{20}$$

The control law eq. (20) is applied taking into account the kinematic of the vehicle as well as the practices involved in its operational use. Moreover, the guidance and navigation of the vehicle is based on a waypoint system. In this work, equation eq. (21) is used at each surfacing to decide the direction of the next way point to reach.



a) b)

Figure 1 - Glider mission plan: a) way points and lane lines in the horizontal plane, and b) yo-yo trajectory in the vertical plane.

III. DISTRIBUTED CONSENSUS ALGORITHM

In this section, the centralized (but unfeasible) estimation algorithm is modified to handle with the physical operational constraints of a glider networks and to allow a decentralized (local) implementation of the estimation algorithm and the control law. The resulting network architecture has a switching topology [8] and is based on the consensus paradigm in which the information is diffused among the agents through a fully connected subnet of surface relay nodes (RN), which act as information gateways.

A. Network protocol

The proposed sensor network is based on the following protocol:

1. Communications of the agents with the RN are asynchronous.
2. Agents can communicate with a RN at random instants,
3. Agents cannot communicate among each other,
4. Each agent sequentially estimates the coefficient vector and the relative error covariance matrix,
5. Agents transmit their local estimates to the RN when a communication link is established,
6. RN distributes its local estimates to the connected agents and update them by combining the local agent's estimates, when available, through the average consensus algorithm [9], or propagates the previous estimate if no agents are connected,
7. Agents connected to the RN update their local estimates as well by applying consensus with the RN estimates.
8. The RN sub-network is fully connected and each node updates its estimates using consensus.

The protocol allows the global information to "intermittently flow" into the network through relay nodes with a collaborative behavior among the agents who emerge above the sea surface to start the communication. Realistic simulations show that all local agent estimates and relay node estimates statistically converge close to the true global coefficient vector (i.e. the network reaches a consensus).

IV. SIMULATION RESULTS

The proposed estimation algorithm is tested by simulating a network of underwater glider vehicles carrying on board a sensor for measuring environmental parameters. The simulated network has one RN, reachable by a satellite link.

The true spatial field is modeled in 3D coordinates (the coordinates are normalized between 0 and 1 for convenience) as the weighted sum of K=4 Gaussian RBFs with different mean position vector and the same covariance matrix, $\mathbf{V}$=0.05 $\mathbf{I}_3$ ($\mathbf{I}_3$ being the 3×3 identity matrix). The true non-zero coefficients are constant in time and equal to $\mathbf{c}=[3,6,9,14]^T$.

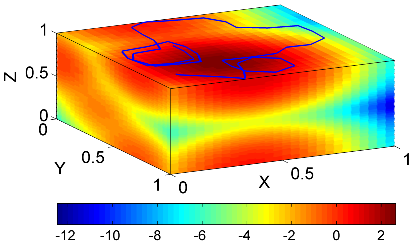


Figure 2 – True sparse field 3D view (intensity in log scale). The blue line represents the trajectory of an agent projected on the horizontal plane at z=1.

The field is reconstructed on a 27×27×27 3D regular grid representing a spatial domain of 30×30 Km along the x and y directions and 100m along the vertical direction (see fig. 2).

The sparse modeling of the field is obtained using a dictionary of 64 Gaussian RBFs with mean position vectors distributed on a regular 4×4×4 sub-grid of the reconstruction grid and with the same known covariance matrix $\mathbf{V}$. The mean positions of the true field RBFs are distributed on four points of this sub-grid. The true field coefficient vector, $\mathbf{c}$, is then a 64-dimensional sparse vector with four components different from zero and with support depending on the position of the mean of each true field RBF. The process noise covariance matrix set to $\mathbf{Q}_k=0.003\mathbf{I}_{64}$. The measurement equation of each agent is (8) with measurement noise variance equal to $\rho_k^2=0.001$. The network includes N=15 glider agents. The agent speed is 0.6 m/s, the pitch angle is $\phi=26^o$, the sampling rate is T=6 s and the CS block length is B=5. The time delay between adjacent surfacing/transmission phases of an agent is modeled as a uniform random variable $\Delta\sim U(\eta-\delta,\eta+\delta)$, to take into account random fluctuations due to unknown environmental conditions affecting the agent navigation. The average delay η is typically between 1-3 hours while in all the simulations δ is set to 15 min. The duration is T_D = 72h.

Initially, the agents are placed uniformly at random in the considered domain and the initial value of the coefficient estimates is supposed to be a Gaussian random variable with zero mean and 0.5 standard deviation. Performance statistics have been evaluated by carrying out 100 Monte Carlo runs. The performances are evaluated in terms of the steady state field root mean square error (RMSE) as a function of the ratio $\gamma=T_D/\ \eta$ (the total number of connections in the observation period). The RMSE curve is compared against the ideal case of $\gamma\rightarrow\infty$ and $\delta\rightarrow 0$, *i.e.* the centralized solution as in Section II-A. The network has been simulated for different values of the mean transmission delay parameter η={0.8h,1.2h, 1.6h,2h,2,4h,3h}. Fig. 3 depicts the estimation of the field at the end of the observation period showing the good match with true field in Fig. 2.

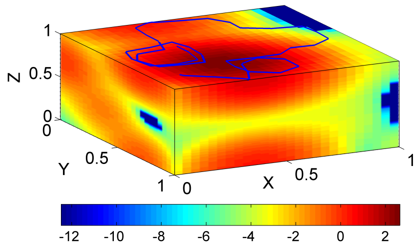


Figure 3 – Estimated field 3D view (natural logarithmic scale).

Fig. 4 provides the field RMSE versus time for γ=45 (η=1.6h), for both the RN and the sensors. After a transitory phase of about 48h, the sensors (in blue) achieve a consensus and the RMSE converge to the same value on average. The RMSE of the RN follows the same dynamic with 2σ confidence levels converging as well to a steady state value.

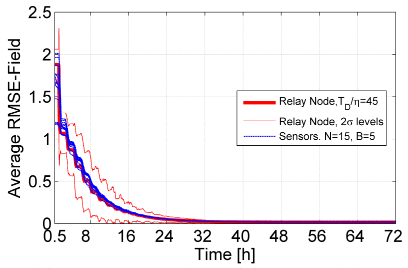


Figure 4 – Field RMSE versus time of the RN and the sensors for γ=45.

Fig. 5 shows the steady state phase of the graph in Fig. 4 confirming the convergence of the sensors and the RN RMSE. The RN average error slightly improves with respect to the mean of RMSE of sensors.

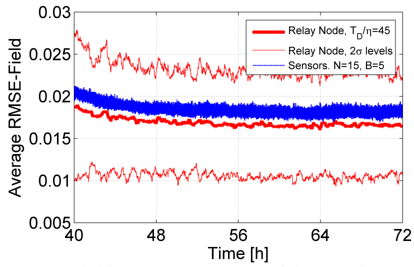


Figure 5 – Field RMSE. Zoom in of the steady state phase.

Figure 6 finally depicts the estimate of a non-zero coefficient versus time for the sensors and the RN (γ=45). The sensors estimates are initially very different producing oscillations in the RN estimate. The sensor and the RN estimates gradually converge to the true coefficient achieving consensus on average after about 48h.

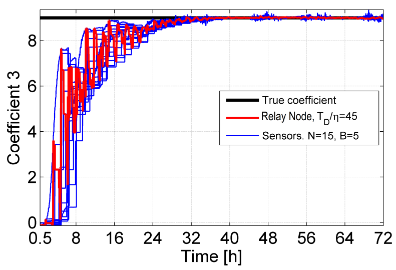


Figure 6 – Example of a non-zero coefficient estimate versus time.

V. Conclusions

In this paper, an autonomous dynamic sensor network is proposed and tested on a simulated but comprehensive scenario. The task of the sensor network is to sequentially estimate a 3D sparse scalar field from local field measurements. To this purpose, each agent locally estimates the field by using a SAKF scheme that refines the solution imposing SL0-norm sparsity constraint. The local sampling scheme is based on a CS sampling device (a random demodulator is used in this work) to reduce the sampling rate and retain the field information. The CS samples are directly used in the local SAKF, on-line, without the need of reconstructing the measurement sequence at the original sampling rate. In order to achieve a global estimate from the local ones, the glider network is integrated with a sub network of RNs that asynchronously diffuse information among the agents by fusing local estimates through an average consensus algorithm. The agents are optimally controlled to collect measurements in those areas where the global field uncertainty is higher. The system has been tested on simulated sparse fields modeled by a dictionary of Gaussian RBFs. The test here reported includes a constant sparse field in 3D. In this case and for typical fleet operational parameters and surveyed area size the average performance achieved in terms of relative error at steady state is within 2%.

Future work can be directed in several ways. For example, the on-line estimation of base functions unknown parameters, such as the mean and the covariance of Gaussian RBFs, is of particular importance in real applications. Further developments include the investigation of the effects of the water current affecting the agent navigation. Off grid effects as well is a topic of interest for future work.

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Title

Dynamic underwater sensor network for sparse field estimation

Abstract

A coordinated dynamic network of autonomous underwater gliders to estimate 3D environmental sparse fields is proposed and tested. Field spatial sparsity is exploited in the estimation algorithm. Moreover field measurements are acquired by compressive sensing devices. Integration with a network of surface relay nodes and asynchronous consensus are used to distribute local information and achieve the global field estimate. Tests on simulated data demonstrate the feasibility of the approach with relative error performance within 10%.

Keywords

Dynamic sensor network, underwater glider, random demodulator, SAKF, consensus protocols

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