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Multiple Sensor Bayesian Extended Target Tracking Fusion Approaches Using Random Matrices

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***Abstract*—The tracking of extended targets is attracting a growing literature thanks to the high resolution of several modern radar systems. A fully Bayesian solution has been proposed in the random matrix framework. In this paper, the fusion of detections acquired by multiple sensors is analyzed. Four different methods are proposed to track and to estimate jointly both the kinematic and extent parameters. All of them use the same multi-sensor kinematic vector measurement update. The first approach is based on a particle approximation of the extent state probability density function, whereas the other three are based on an inverse Wishart representation of the latter. Extensive simulations evaluate the performance of the different approaches. The best performance is obtained by the particle filter-based approach paid by an increased computational burden. Comparable performance are observed for the two updates based on multi-sensor generalization, while the worst performance is obtained by the updated based on fusion approximation.**

I. INTRODUCTION

High resolution radars are becoming very common and widespread. In such systems targets appear as a cloud of detections instead of a point. Standard target tracking algorithms rely upon the hypothesis of at most one detection per target, which is no longer valid in this case. Therefore, the development of new strategies for target tracking is required to deal with this issue. The rubric is *extended target tracking* (ETT) in the related literature.

Several approaches have been already proposed, see *e.g.* [3], [4], [18]. An overview of the state-of-art for group and extended target tracking techniques is given in [19]. A popular and computationally efficient framework for extended target tracking is provided by Koch in [14]. An approximate Bayesian solution to the target tracking problem is illustrated under the hypothesis of an elliptical spread of the target. Random matrices are exploited to model the ellipsoidal object extensions, which are treated as additional state variables to be estimated or tracked. The target kinematic states are modeled using a Gaussian distribution, whereas the ellipsoidal target extension is modeled using an inverse Wishart distribution. Random matrices are used to model extended targets under kinematic constraints in [15]. In [10], [17], [23], the integration of the random matrices into the probabilistic multi-hypothesis tracking and into the PHD and CPHD filters, respectively, is addressed for the multi-target tracking problem. Furthermore, a new approach is derived in [7] to overcome some of the weaknesses in [14]. Indeed, in [14] sensor inaccuracies are neglected and, if they are large in comparison to the target size, the lack of modeling may lead to an overestimation of the target size, see [6]. New measurement and time updates for [7] are proposed in [20] and [12], respectively. An extension of random matrices for non-ellipsoidal group and extended target tracking based on a combination of multiple ellipsoidal sub-objects, each represented by a random matrix, is discussed in [16]. An interesting application of the random matrix framework is reported in [8], [9] using radar data. The problem of the conversion between polar/bistatic and Cartesian coordinates into random matrices is addressed in [21], [22], respectively, and tested on real data.

The original single sensor random matrix model, presented in [14], modeled the dependence between the kinematic state and the extent state. Later work showed that by not explicitly modeling this dependence, more general models for both sensor noise and dynamic target motion could be used [7], [11]. Results using both simulated and experimental data showed that the single target updates and predictions provide for sufficient interdependence between the kinematic state and the extent state to compensate for lack of an explicit model of the dependence. Following the related literature, the explicit modeling of the dependence between the kinematic state and the extent state is neglected here. Thus, in this paper, the multiple sensor fusion problem is solved for the kinematic update in a closed form and four different methods are proposed to update the extent state and compared using extensive simulations.

The paper is organized as follows. The paper's contributions are explained in Sect. II. The four extensions to the multiple sensor case are illustrated in Sect. III. The extended target prediction is reviewed in Sect. IV. Numerical results are presented in Sect. V. Finally, conclusions are drawn in Sect. VI.

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II. Paper contributions

Consider a multistatic radar system that consists of S sensors that acquire data within the same surveillance area. The set of measurements acquired at time k by the s-th sensor is denoted

$$\mathbf{Z}_k^s = \left\{\mathbf{z}_k^{s,j}\right\}_{j=1}^{n_k^s} \tag{1}$$

The detections $\mathbf{z}_k^{s,j}$ are assumed independent and each sensor s acquires n_k^s measurements of the target at time k. The full set of detections at time k is the union of the sets of detections from the S sensors,

$$\mathbf{Z}_k^c = \bigcup_{s=1}^{S} \mathbf{Z}_k^s \tag{2}$$

Let $\mathbf{Z}^{c,k} = \{\mathbf{Z}_\ell^c\}_{\ell=0}^k$ denote all such measurement sets up to, and including, time k.

The multi-sensor measurement likelihood, conditioned on the extended target state (which consists of the target kinematic state x_k and X_k that describes the target's extent exploiting an elliptical model) and the number of detections n_k^s, is

$$\begin{aligned} p&\left(\mathbf{Z}_k^c \,\middle|\, \mathbf{x}_k, X_k, \{n_k^s\}_{s=1}^S\right) \\ &= \prod_{s=1}^{S}\prod_{j=1}^{n_k^s} \mathcal{N}\left(\mathbf{z}_k^{s,j}\,;\, H_k\mathbf{x}_k, \rho X_k + R_k^s\right) \end{aligned} \tag{3}$$

where ρ is a scaling factor, H_k is a measurement model at time k that selects the position components in the state vector, and R_k^s is the covariance matrix for the sensor s at time k.

Remark: *For brevity and notational simplicity, the measurement noise covariance's (R_k^s) possible dependence on the kinematic state is not explicitly indicated. However, all derivations in the paper are valid also in the case of kinematic state dependent covariance matrices, see [21].*

Given a prior distribution for the extended target state,

$$p(\mathbf{x}_k, X_k|\mathbf{Z}^{c,k-1}) \approx p(\mathbf{x}_k|\mathbf{Z}^{c,k-1})p(X_k|\mathbf{Z}^{c,k-1}), \tag{4}$$

the objective of the paper is to find the posterior

$$p(\mathbf{x}_k, X_k|\mathbf{Z}^{c,k}) \approx p(\mathbf{x}_k|\mathbf{Z}^{c,k})p(X_k|\mathbf{Z}^{c,k}). \tag{5}$$

These densities approximate the kinematic and extent states as independent, however, as noted in [7], the measurement update step "*provides for the interdependency between kinematics and extension estimation*".

The prior and posterior kinematic state probability density functions (pdfs) are approximated by Gaussian distributions,

$$p(\mathbf{x}_k|\mathbf{Z}^{c,k-1}) \approx \mathcal{N}\left(\mathbf{x}_k\,;\, m_{k|k-1}, P_{k|k-1}\right) \tag{6}$$
$$p(\mathbf{x}_k|\mathbf{Z}^{c,k}) \approx \mathcal{N}\left(\mathbf{x}_k\,;\, m_{k|k}, P_{k|k}\right) \tag{7}$$

For the extent state, two different approximations will be considered. The first is a particle approximation of the extent pdf,

$$p\left(X_k|\mathbf{Z}^{c,k-1}\right) \approx \sum_{i=1}^{N_p} \mathcal{W}_{k|k-1}^{(i)} \delta_{X_k}\left(X_k^{(i)}\right) \tag{8}$$
$$p\left(X_k|\mathbf{Z}^{c,k}\right) \approx \sum_{i=1}^{N_p} \mathcal{W}_{k|k}^{(i)} \delta_{X_k}\left(X_k^{(i)}\right) \tag{9}$$

Here the paper presents the following contribution,

- An update based on a Rao-Blackwellized particle representation of the extended target pdf.

The second extent state pdf approximation that is considered is an inverse Wishart pdf,

$$p(X_k|\mathbf{Z}^{c,k-1}) \approx \mathcal{IW}_d\left(X_k\,;\, v_{k|k-1}, V_{k|k-1}\right) \tag{10}$$
$$p(X_k|\mathbf{Z}^{c,k}) \approx \mathcal{IW}_d\left(X_k\,;\, v_{k|k}, V_{k|k}\right) \tag{11}$$

where the inverse Wishart pdf is defined as in Table I. Here the paper presents the following three contributions:

- An update based on approximate fusion.
- A multi-sensor generalization of the single sensor update from [7].
- A multi-sensor generalization of the single sensor update from [1].

The four different updates are compared in an extensive simulation study in which a network of radar sensor is simulated. Some notation that is used throughout the paper is listed in Table I.

III. Multi-sensor updates

This section presents the kinematic state update and the four different multi-sensor updates.

A. Gaussian kinematic state update

The kinematic state is updated as follows

$$m_{k|k}^{\mathrm{MS}} = m_{k|k-1} + K_{k|k-1}^c\left(\bar{\mathbf{z}}_{k|k-1}^c - H_k m_{k|k-1}\right) \tag{18}$$
$$P_{k|k}^{\mathrm{MS}} = P_{k|k-1} - K_{k|k-1}^c H_k P_{k|k-1} \tag{19}$$

where

$$\hat{Y}_{k|k-1}^c = \left(\sum_{s=1}^{S}\left(\hat{Y}_{k|k-1}^s\right)^{-1}\right)^{-1} \tag{20a}$$
$$S_{k|k-1}^c = H_k P_{k|k-1} H_k^{\mathrm{T}} + \hat{Y}_{k|k-1}^c \tag{20b}$$
$$K_{k|k-1}^c = P_{k|k-1} H_k^{\mathrm{T}}\left(S_{k|k-1}^c\right)^{-1} \tag{20c}$$
$$\bar{\mathbf{z}}_{k|k-1}^c = \hat{Y}_{k|k-1}^c\left\{\sum_{s=1}^{S}\left[\left(\hat{Y}_{k|k-1}^s\right)^{-1}\bar{\mathbf{z}}_k^s\right]\right\} \tag{20d}$$

TABLE I
NOTATIONS

- $\mathbb{R}^n$ is the set of real column vectors of length n, $\mathbb{S}^n_{++}$ is the set of symmetric positive definite $n \times n$ matrices, $\mathbb{S}^n_{+}$ is the set of symmetric positive semi-definite $n \times n$ matrices, and $\mathbb{N}$ is the set of non-negative integers.
- $\mathcal{N}(\mathbf{x}\,;\,\mathbf{m}, P)$ denotes a multi-variate Gaussian pdf over the vector $\mathbf{x} \in \mathbb{R}^{n_x}$ with mean vector $\mathbf{m} \in \mathbb{R}^{n_x}$, and covariance matrix $P \in \mathbb{S}^{n_x}_{+}$.
- $\mathcal{IW}_d(X\,;\,v, V)$ denotes an inverse Wishart pdf over the matrix $X \in \mathbb{S}^d_{++}$ with scalar degrees of freedom $v > 2d$ and parameter matrix $V \in \mathbb{S}^d_{++}$, see, e.g. [13, Definition 3.4.1]

$$\mathcal{IW}_d(X\,;\,v, V) = \frac{2^{-\frac{v-d-1}{2}} \det(V)^{\frac{v-d-1}{2}}}{\Gamma_d\left(\frac{v-d-1}{2}\right) \det(X)^{\frac{v}{2}}} \operatorname{etr}\left(-\frac{1}{2} X^{-1} V\right), \tag{12}$$

where $\operatorname{etr}(\cdot) = \exp(\operatorname{tr}(\cdot))$ is exponential of the matrix trace, $\det(\cdot)$ is the determinant operator, and $\Gamma_d(\cdot)$ is the multivariate gamma function. The multivariate gamma function $\Gamma_d(\cdot)$ can be expressed as a product of the ordinary gamma function $\Gamma(\cdot)$, see [13, Theorem 1.4.1].

- $\mathcal{W}_d(X\,;\,w, W)$ denotes a Wishart pdf over the matrix $X \in \mathbb{S}^d_{++}$ with scalar degrees of freedom $w \geq d$ and parameter matrix $W \in \mathbb{S}^d_{++}$, see, e.g. [13, Definition 3.2.1]

$$\mathcal{W}_d(X\,;\,w, W) = \frac{2^{-\frac{wd}{2}} \det(X)^{\frac{w-d-1}{2}}}{\Gamma_d\left(\frac{w}{2}\right) \det(W)^{\frac{w}{2}}} \operatorname{etr}\left(-\frac{1}{2} W^{-1} X\right). \tag{13}$$

- Expected prior extent [13]

$$\hat{X}_{k|k-1} = \frac{V_{k|k-1}}{v_{k|k-1} - 2d - 2} \tag{14}$$

- Expected added effect of extent and sensor noise, normalized by number of detections,

$$\hat{Y}^s_{k|k-1} = \frac{\rho \hat{X}_{k|k-1} + R^s_k}{n^s_k} \tag{15}$$

- Centroid measurement and measurement spread for sth sensor are

$$\bar{\mathbf{z}}^s_k = \frac{1}{n^s_k} \sum_{j=1}^{n^s_k} \mathbf{z}^{s,j}_k \;; \qquad Z^s_k = \sum_{j=1}^{n^s_k} \left(\mathbf{z}^{s,j}_k - \bar{\mathbf{z}}^s_k\right)\left(\mathbf{z}^{s,j}_k - \bar{\mathbf{z}}^s_k\right)^{\mathrm{T}} \tag{16}$$

Note: measurement spread is not equivalent to sample covariance.

- Cholesky factorization of positive definite matrix A is denoted

$$A = A^{\frac{\mathrm{T}}{2}} A^{\frac{1}{2}} \tag{17}$$

B. Particle approximation extent update

The Bayesian extent state posterior is usually impossible to evaluate in closed form. Different approaches to approximating it exist, e.g., Sequential Monte Carlo (SMC) methods [2], [5]. Assume that at time $k-1$, a set of weighted particles representing the posterior is available

$$\left\{\mathcal{W}^{(i)}_{k-1}, X^{(i)}_{k-1}\right\}_{i=1}^{N_p} \tag{21}$$

$$p\left(X_{k-1} | \mathbf{Z}^{c,k-1}\right) \approx \sum_{i=1}^{N_p} \mathcal{W}^{(i)}_{k-1} \delta_{X^{(i)}_{k-1}}(X_{k-1}), \tag{22}$$

where $\delta_{\mathbf{y}}(X)$ is a Dirac delta function centered in $\mathbf{y}$.

Similarly to how in the kinematic state update the extent state is approximated by its predicted expected value, here we approximate the kinematic state with its predicted expected value. In other words, we approximate $\mathbf{x}_k \approx m_{k|k-1}$, and the single sensor log-likelihood can be written as

$$\begin{aligned}&\log\, p\left(\mathbf{Z}^s_k \,|\mathbf{x}_k, X_k, n^s_k\right) \\ &\quad\approx -\frac{n^s_k}{2} \log\det(\rho X_k + R^s_k) - \frac{1}{2}\operatorname{tr}\left[(\rho X_k + R^s_k)^{-1} \Sigma^s_k\right]\end{aligned} \tag{23}$$

where

$$\Sigma^s_k = \sum_{j=1}^{n^s_k} \left(\mathbf{z}^{s,j}_k - H_k m_{k|k-1}\right)\left(\mathbf{z}^{s,j}_k - H_k m_{k|k-1}\right)^{\mathrm{T}} \tag{24}$$

$$= Z^s_k + n^s_k(\bar{\mathbf{z}}^s_k - H_k m_{k|k-1})(\bar{\mathbf{z}}^s_k - H_k m_{k|k-1})^{\mathrm{T}} \tag{25}$$

and all terms that are independent of the detections and the target state are omitted.

The logarithm of the multi-sensor likelihood (3) is the sum of the single sensor log-likelihoods (23),

$$\log\;\, p\left(\mathbf{Z}^c_k \middle| \mathbf{x}_k, X_k, \{n^s_k\}_{s=1}^S\right) = \sum_{s=1}^S \log p\left(\mathbf{Z}^s_k \,|\mathbf{x}_k, X_k, n^s_k\right) \tag{26}$$

$$\approx -\frac{1}{2}\sum_{s=1}^S n^s_k \log\det(\rho X_k + R^s_k) + \operatorname{tr}\left[(\rho X_k + R^s_k)^{-1}\Sigma^s_k\right] \tag{27}$$

$$= \log\;\, \mathcal{L}_k(\mathbf{Z}^c_k | X_k) \tag{28}$$

where $\mathcal{L}_k(\mathbf{Z}^c_k | X_k)$ is shorthand notation for the approximate multi-sensor likelihood as a function of the set of measurements $\mathbf{Z}^c_k$ and the extent state X_k. The particle filter (PF) proceeds to approximate the posterior at time k by a new set of weighted particles $\left\{\mathcal{W}^{(i)}_k, X^{(i)}_k\right\}_{i=1}^{N_p}$ as described in the sampling importance resampling (SIR) filter [2].

Particles are sampled from the importance sampling distribution that is represented by a Gaussian distribution along the ellipse orientation, minor axis, and major axis extracted from the particle $X^{(i)}_{k-1}$.

Then particle weights are updated based on the likelihood function of the observed data $\mathbf{Z}^c_k$ from all sensors . The likelihood $\mathcal{L}_k\left(\mathbf{Z}^c_k | X_k\right)$ is given in Eq. (28), and takes into account all information at time k. Lastly, a resampling strategy is adopted to avoid the particle degeneracy problem, *e.g.* see [2].

A target extent estimate is given by the *Minimum Mean Square Error* (MMSE) estimator, which is optimal in terms of MSE. Afterwards, the estimation at time k, *i.e.* $\hat{X}_{k|k}$, is used for the next prediction step.

C. Inverse Wishart extent updates

1) FFK generalization: The multi-sensor generalization of the FFK (here called FFK, after the authors initials) single sensor update [7] is

$$v^{\mathrm{MSFFK}}_{k|k} = v_{k|k-1} + \sum_{s=1}^S n^s_k \tag{29}$$

$$V^{\mathrm{MSFFK}}_{k|k} = V_{k|k-1} + \sum_{s=1}^S Z^s_{k|k-1} + N^c_{k|k-1} \tag{30}$$

where

$$Z^s_{k|k-1} = \hat{X}^{\frac{\mathrm{T}}{2}}_{k|k-1} \left(\hat{Y}^s_{k|k-1}\right)^{-\frac{\mathrm{T}}{2}} Z^s_k \left(\hat{Y}^s_{k|k-1}\right)^{-\frac{1}{2}} \hat{X}^{\frac{1}{2}}_{k|k-1} \tag{31}$$

$$N^c_{k|k-1} = \hat{X}^{\frac{\mathrm{T}}{2}}_{k|k-1} \left(S^c_{k|k-1}\right)^{-\frac{\mathrm{T}}{2}} \left(\bar{\mathbf{z}}^c_{k|k-1} - H_k m_{k|k-1}\right) \times \left(\bar{\mathbf{z}}^c_{k|k-1} - H_k m_{k|k-1}\right)^{\mathrm{T}} \left(S^c_{k|k-1}\right)^{-\frac{1}{2}} \hat{X}^{\frac{1}{2}}_{k|k-1} \tag{32}$$

2) ULL generalization: The multi-sensor generalization of the ULL (unbiased log-likelihood linearisation) single sensor update [1] is

$$v^{\mathrm{MSULL}}_{k|k} = v_{k|k-1} + \sum_{s=1}^{S} n^s_k \tag{33}$$

$$V^{\mathrm{MSULL}}_{k|k} = V_{k|k-1} + \sum_{s=1}^{S} M^s_{k|k-1} \tag{34}$$

where

$$M^s_{k|k-1} = n^s_k \hat{X}_{k|k-1} + n^s_k \rho \hat{X}_{k|k-1} (C^s_{k|k-1})^{-1} \times \left(\Psi^s_{k|k-1} - C^s_{k|k-1}\right) (C^s_{k|k-1})^{-1} \hat{X}_{k|k-1} \tag{35}$$

$$\Psi^s_{k|k-1} = \frac{1}{n^s_k} \sum_{j=1}^{n^s_k} \left(\mathbf{z}^{s,j}_k - H_k m_{k|k-1}\right) \left(\mathbf{z}^{s,j}_k - H_k m_{k|k-1}\right)^{\mathrm{T}} \tag{36}$$

$$C^s_{k|k-1} = H_k P_{k|k-1} H^{\mathrm{T}}_k + \rho \hat{X}_{k|k-1} + R^s_k \tag{37}$$

3) Fusion approximation: This update is based on fusion approximation (FA) and was presented in [22].

IV. Extended Target Prediction

For completeness, the extended target prediction is given here. The kinematic state transition density is

$$p(\mathbf{x}_{k+1}|\mathbf{x}_k) = \mathcal{N}\left(\mathbf{x}_{k+1} ; f_{k+1,k}(\mathbf{x}_k), Q_{k+1}\right) \tag{38}$$

where $f_{k+1,k}(\mathbf{x}_k)$ is a function that describes the motion of the target. The updated mean and covariance are given by the extended Kalman filter prediction

$$m_{k+1|k} = f_{k+1,k}(\mathbf{x}_k) \tag{39}$$

$$P_{k+1|k} = F_{k+1,k} P_{k|k} F^{\mathrm{T}}_{k+1,k} + Q_{k+1} \tag{40}$$

where

$$F_{k+1,k} = \nabla_{\mathbf{x}_k} f_{k+1,k}(\mathbf{x}_k)|_{\mathbf{x}_k = \hat{\mathbf{x}}_{k|k}} \tag{41}$$

is the Jacobian of $f_{k+1,k}(\mathbf{x}_k)$ with respect to the state $\mathbf{x}_k$, evaluated at the expected value $\hat{\mathbf{x}}_{k|k}$.

In the case the extent pdf is approximated by an inverse Wishart pdf, the extent state transition density is

$$p(X_{k+1}|X_k, \mathbf{x}_k) = \mathcal{W}_d\left(X_{k+1} ; \nu_{k+1,k}, \frac{\Phi(\mathbf{x}_k) X_k \Phi(\mathbf{x}_k)^{\mathrm{T}}}{\nu_{k+1,k}}\right) \tag{42}$$

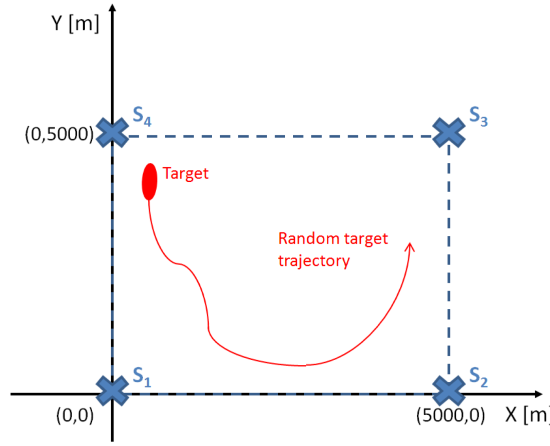


Fig. 1. Simulator setup with four sensors and random target trajectories.

where $\Phi(\cdot)$ is a rotation matrix and the Wishart pdf is defined as in Table I. Given the posterior extent X_k and the posterior kinematic state $\mathbf{x}_k$, the expected predicted extent is $\Phi(\mathbf{x}_k) X_k \Phi(\mathbf{x}_k)^{\mathrm{T}}$. The parameter $\nu_{k+1,k}$ governs the extent process noise: the larger $\nu_{k+1,k}$ is, the lower the process noise. Thus, the extent prediction corresponds to rotating the extent by an amount specified by the kinematic state, and increasing the variance. In [11] the predicted parameters $v_{k+1|k}$ and $V_{k+1|k}$ are computed using a series of density approximations that minimize the Kullback-Leibler divergence. Here we use a computationally efficient approximation and update the parameters as follows

$$v_{k+1|k} = 2d + 2 + e^{-T/\tau}\left(v_{k|k} - 2d - 2\right) \tag{43}$$

$$V_{k+1|k} = \left(v_{k+1|k} - 2d - 2\right) \frac{\Phi(\hat{\mathbf{x}}_{k|k}) V_{k|k} \Phi(\hat{\mathbf{x}}_{k|k})}{v_{k|k} - 2d - 2} \tag{44}$$

where τ is a time constant related to the agility with which the target may change its extent over time. Thus, in this approximate prediction the parameter τ replaces the Wishart transition density degrees of freedom $\nu_{k+1,k}$. However, note that the effect of the prediction is the same as if the prediction from [11] had been used: the extent expected value is rotated, and the covariance is increased.

V. Numerical Simulation Study

The presented Bayesian extended target tracking updates are compared using simulated data.

A. Setup

The simulator setup is shown in Figure 1. The target's initial positions are randomly sampled from inside the sensor network, and the target tracks are randomly generated. The simulated target follows both linear and curved trajectories. The number of detections per scan is variable and depends on the distance between the target and the sensor, as well as on the simulator's parameters, i.e., the azimuthal accuracy and the range resolution. The parameters are summarized in Table III. The goal of this simulator is to obtain average performance of

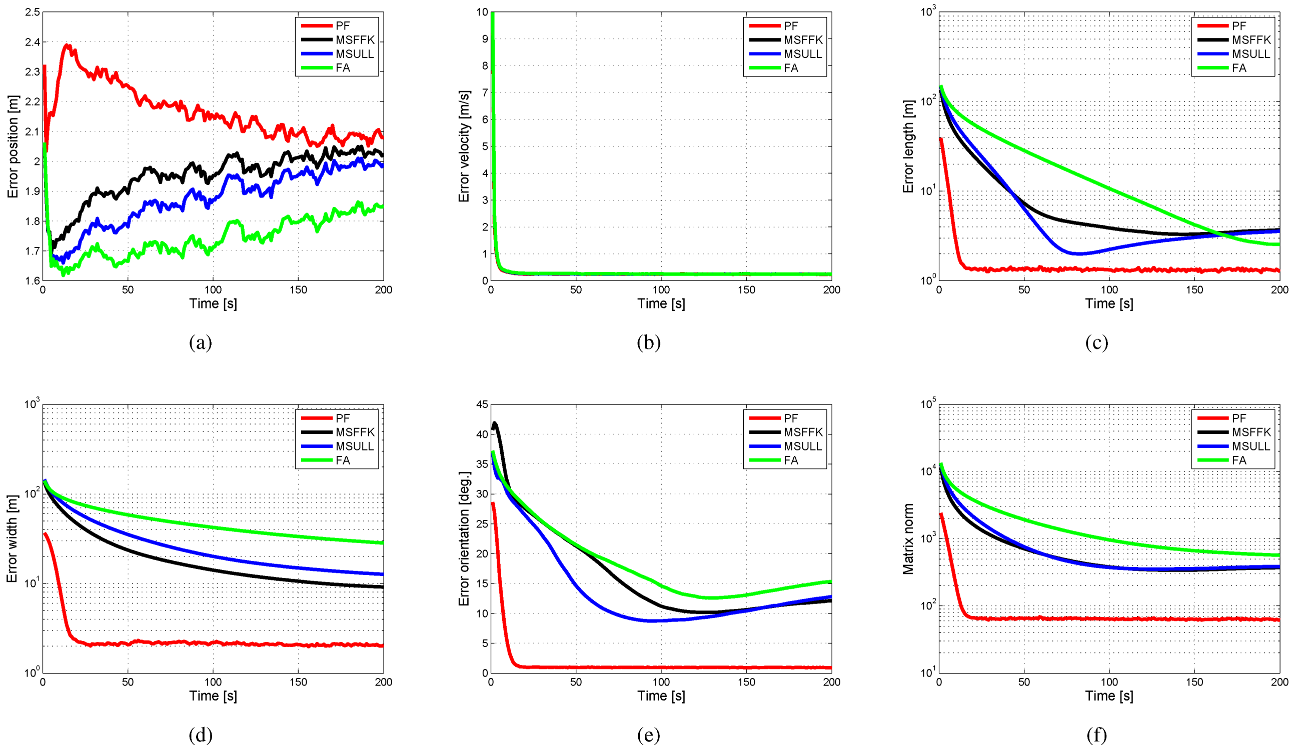


Fig. 2. Errors in (a) position, (b) velocity, (c) length, (d) width, (e) orientation, and the (f) Frobenius matrix error are depicted for the four compared approaches averaged on 10^3 Monte Carlo trials on Simulator 1.

TABLE II
ESTIMATION ERRORS AVERAGED ON 10^3 MONTE CARLO TRIALS

Algorithm	ϵ^{pos} [m]	ϵ^{vel} [m·s^{-1}]	ϵ^{wid} [m]	ϵ^{len} [m]	ϵ^{or} [deg.]	ϵ^{FE}
FA	**1.75**	0.328	49.1	21.3	18.1	1698.7
MSULL	1.88	0.324	30.0	10.9	14.0	969.3
MSFFK	1.95	0.324	22.2	10.3	16.3	830.1
PF	2.17	**0.317**	**3.2**	**2.0**	**1.72**	**110.9**

the different updates when Monte Carlo trials are performed. Table IV shows the sets of parameters used by the compared updates.

The random vector $\mathbf{x}_k = [\mathbf{p}_k,\ \mathbf{v}_k,\ \omega_k]^{\mathrm{T}} \in \mathbb{R}^5$ is the kinematical state, and describes the target's position $\mathbf{p}_k \in \mathbb{R}^2$, velocity $\mathbf{v}_k \in \mathbb{R}^2$ and turn-rate $\omega_k \in \mathbb{R}^1$. The motion model $\mathbf{f}(\cdot)$ and process noise covariance $\mathbf{Q}$ are

$$\mathbf{f}(\mathbf{x}_k) = \begin{bmatrix} 1 & 0 & \frac{\sin(\omega_k T_s)}{\omega_k} & \frac{-1+\cos(\omega_k T_s)}{\omega_k} & 0 \\ 0 & 1 & \frac{1-\cos(\omega_k T_s)}{\omega_k} & \frac{\sin(\omega_k T_s)}{\omega_k} & 0 \\ 0 & 0 & \cos(\omega_k T_s) & -\sin(\omega_k T_s) & 0 \\ 0 & 0 & \sin(\omega_k T_s) & \cos(\omega_k T_s) & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{x}_k, \tag{45a}$$

$$\mathbf{Q} = \mathbf{G}\mathrm{diag}\left(\left[\sigma_a^2\ ,\ \sigma_a^2\ ,\ \sigma_\omega^2\right]\right)\mathbf{G}^{\mathrm{T}}, \quad \mathbf{G} = \begin{bmatrix} \frac{T_s^2}{2}\mathbf{I}_2 & \mathbf{0}_{2\times1} \\ T_s\mathbf{I}_2 & \mathbf{0}_{2\times1} \\ \mathbf{0}_{1\times2} & T_s \end{bmatrix} \tag{45b}$$

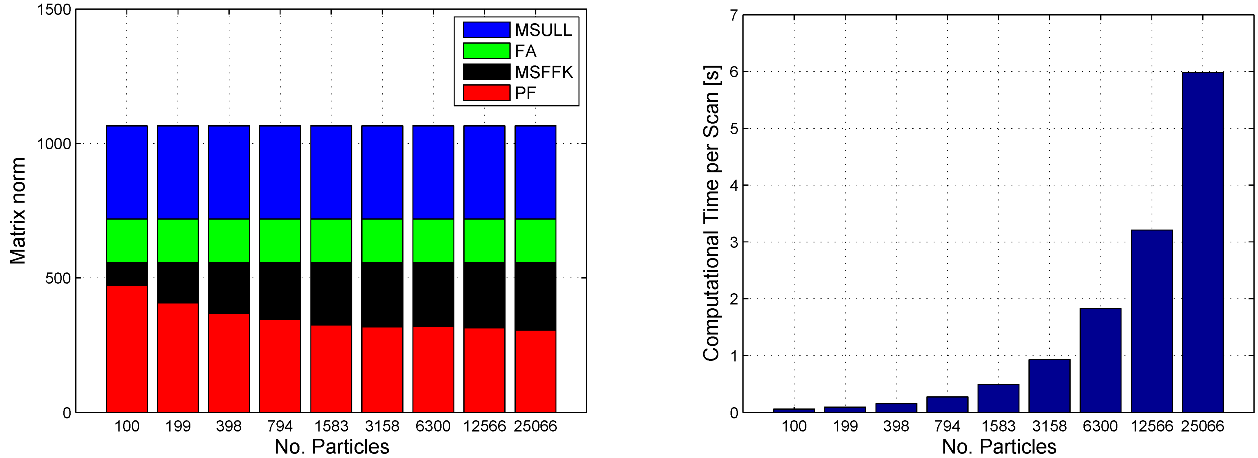
where T_s is the sampling time, σ_a is the acceleration standard deviation, and σ_ω is the turn-rate standard deviation. The rotation matrix is defined as

$$\Phi(\mathbf{x}_k) = \begin{bmatrix} \cos(\omega_k T_s) & -\sin(\omega_k T_s) \\ \sin(\omega_k T_s) & \cos(\omega_k T_s) \end{bmatrix} \tag{46}$$

The performance metrics used for the assessment are the root mean square errors in position (*i.e.* ϵ^{pos}), velocity (*i.e.* ϵ^{vel}), length (*i.e.* ϵ^{len}), width (*i.e.* ϵ^{wid}), and orientation (*i.e.* ϵ^{or}). Furthermore, the Frobenius matrix error (*i.e.* ϵ^{FE}) for the target extent X_k is adopted to have an overall performance index for the target's extent estimation.

B. Results

Figure 2 shows the results averaged over 10^3 Monte Carlo trials. The PF clearly has the smallest extent estimation errors and also converges faster, however, the price for the improved performance is an increase in the computational burden. How the computational time scales with the number of particles and the related improvements in the Frobenius matrix error metric are shown in Figure 3. Thus, a number of particles of 10^3 (used in this paper) can be advisable given the trade-off between accuracy and computational time. The other approaches require



(a) (b)

Fig. 3. The histogram representation of the Frobenius matrix error versus the number of particles is depicted in (a), whereas the computational time per scan versus the number of particles for the particle filter-based approach is shown in (b).

TABLE III
SIMULATOR PARAMETER SETTING

Parameter	Value	Specification
T_s	1 s	Sampling time
σ_{pos}	$0.1\ \mathrm{m}\cdot\mathrm{s}^{-2}$	Std. process noise
θ_{max}	1°	Maximum turn-rate
σ_ϕ	0.1°	Std. turn-rate noise
σ_h	1°	Std. heading noise
l_t	70	Target length
w_t	15	Target width
k_{max}	200	Number of frames
Δ_r	0.5 m	Range resolution
hbw	0.1°	Azimuthal accuracy
v_{max}	15	Maximum velocity target
v_{min}	0.5	Minimum velocity target
N_d	variable	Num. detects. time per sensor
S	4	Num. of sensors
σ_r	10 m	Std. noise range
σ_θ	2°	Std. noise azimuth

TABLE IV
TRACKING PARAMETER SETTINGS

Parameter	Value	Specification
T_s	1 s	Sampling time
σ_{pos}	$0.1\ \mathrm{m}\cdot\mathrm{s}^{-2}$	Std. process noise
τ	10	Time constant
ρ	1	Scaling factor
Np	10^3	Num. of particles
$\sigma^p_{ax_{min}}$	1 m	Std. prediction axis min.
$\sigma^p_{ax_{max}}$	2 m	Std. prediction axis max.
σ^p_θ	2°	Std. prediction orientation

about 0.005 s to process a single scan (negligible with respect to the particle filter-based computational burden depicted in Fig. 3(b)). One can note that the kinematics (i.e. position and velocity) are well estimated by all of the four updates. Very small differences can be seen for the position estimates before the convergence of the algorithms. However, these are limited to less than 1 m.

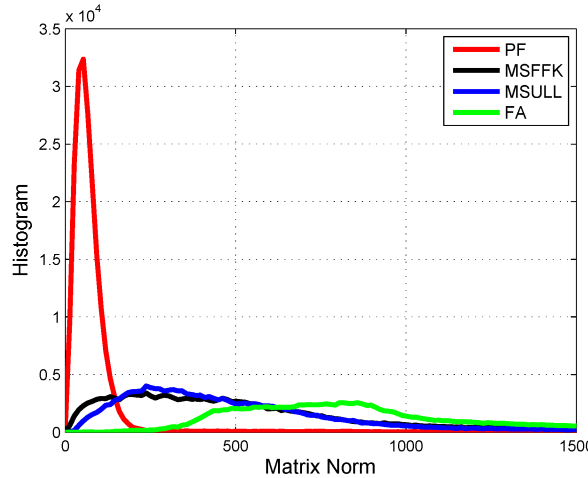


Fig. 4. Histogram of the Frobenius matrix errors for the four compared approaches averaged on 10^3 Monte Carlo trials.

Regarding the estimated extent, the errors for length, width and orientation follow the same trend as the Frobenius matrix error does. FA is the worst algorithm, whereas MSULL and MSFFK have comparable average performance. Table II summarizes the estimation errors (the best outcomes are highlighted in boldface text). Figure 4 provides a histogram representation of the Frobenius matrix errors for the four updates, confirming the above analysis and ranking of the updates.

VI. CONCLUSIONS

In this paper, a study of multi-sensor Bayesian extended target tracking was presented. The extended targets were modeled in the random matrix framework. Four different extent matrix measurement updates were proposed, and each uses the same multi-sensor kinematic vector measurement update. One of the updates is based on a particle approximation of the extent state pdf, the other three are based on an inverse Wishart representation of the extent state pdf. Extensive numerical results demonstrate that the best performance is obtained by the PF-based approach paid by an increased computational burden. Comparable performance is observed for the two updates based on multi-sensor generalization, and the worst performance is obtained by the updated based on fusion approximation.

In future research the theoretical differences and similarities between the four updates will be analyzed. Further simulation study will evaluate how the different updates scale with the number of sensors and the magnitude of the sensor noise.

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Title

Multiple sensor Bayesian extended target tracking fusion approaches using random matrices

Abstract

The tracking of extended targets is attracting a growing literature thanks to the high resolution of several modern radar systems. A fully Bayesian solution has been proposed in the random matrix framework. In this paper, the fusion of detections acquired by multiple sensors is analysed. Four different methods are proposed to track and to estimate jointly both the kinematic and extent parameters. All of them use the same multi-sensor kinematic vector measurement update. The first approach is based on a particle approximation of the extent state probability density function, whereas the other three are based on an inverse Wishart representation of the latter. Extensive simulations evaluate the performance of the different approaches. The best performance is obtained by the particle filter-based approach paid by an increased computational burden. Comparable performances are observed for the two updates based on multi-sensor generalization, while the worst performance is obtained by the updated based on fusion approximation.

Keywords

Kinematics, probability density function, target tracking, time measurement, covariance matrices, radar tracking, Bayes methods

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