A THEORETICAL MODEL FOR DOPPLER SPREAD OF BACKSCATTERED SOUND FROM A COMPOSITE ROUGHNESS SEA SURFACE

by

H. Schwarze
SACLANT ASW Research Centre
La Spezia, Italy

ABSTRACT

For calculating the Doppler spread of acoustical reverberation from the sea surface, good results are achieved by applying a composite-roughness model. The small wavelets primarily responsible for the backscattering are carried by long waves with higher amplitudes. Small roughness results are applied to conceptual "facets" on the long waves. As coupling parameters between short- and long-wave scattering, the rms-slope, the rms-vertical and horizontal velocities of the facets and their cross-correlation coefficients are used. They are calculated from the covariance function of the sea surface as a function of the facet length. For a Pierson-Moscowitz spectrum, some computer results are given.

INTRODUCTION

The general problem of scattering of acoustic waves from the rough sea surface is not completely solved at present. For special cases, solutions have been obtained, for instance when the amplitudes of the sea surface waves are much smaller [Ref. 1] or much larger [Ref. 2] than the length of the incident acoustic wave.

For an arbitrary acoustic wavelength and surface roughness, approximate solutions are obtained by using a composite-roughness sea-surface model and applying the results for the small-scale roughness sea surface to it [Refs. 3, 4, 5]. This means that the small wavelets, causing the resonant or "Bragg" scattering, are carried by plane facets, whose movements depend on the large-scale roughness of the sea.

This paper presents a few results of the frequency-spreading of a backscattered acoustic wave, using the facet model.

1. THEORY

The function that describes this frequency spreading is the spectral density function of the backscattered sound pressure. To calculate this function a source S illuminating a surface area A with a plane wave of frequency f_0 and pressure p_0 is assumed. The distance r between sources and surface area A is considered to be much larger than $\sqrt{A}$. The spectral density is normalized by the distance r, the area A, and the sound pressure f_0. This normalized function is called the doppler density $\varphi(f)$.

For the small-scale roughness sea surface, where the standard deviation of the wave height is much smaller than the wavelength of the acoustic incident wave, this doppler density is calculated by the well-known methods previously cited. The result is:

$$\varphi(f) = (4\pi)^2 \, k_{oz}^4 \, X_3(-2\, k_{ox}, 0, \Delta f) \qquad \text{[Eq. 1]}$$

where $k_0 = \dfrac{f_0}{c} = \dfrac{1}{\lambda_0}$ is the wave parameter of the acoustic wave

and $k_{ox} = k_0 \cos \varphi_0$, $k_{oz} = -k_0 \sin \varphi_0$, $\Delta f = f - f_0$ are the components of the incident wave parameter vector in the x and z directions respectively. The source is assumed to lie in the x-z plane, z looking downward.

The function X_3 is the three-dimensional wave parameter frequency spectrum of the sea-surface elevation. It is assumed that h(x,y,t) is a stationary Gaussian process, therefore X_3 describes the process completely.

For typical sonar frequencies the dispersion relation can be considered valid. Then Eq. 1 can be written in terms of the two-dimensional frequency direction spectrum $F_2(f, \varphi)$ and a mixing function $W(f, \varphi)$ that describes the ratio of incoming and outgoing surface waves at a given direction φ, the result being:

$$\varphi(f) = \frac{2g^2 k_{oz}^4}{f^{*3}} F_2(f^*, 0) \left[W(f^*, \pi)\cdot \delta(\Delta f - f^*) + W(f^*, 0)\delta(\Delta f + f^*)\right] \quad \text{[Eq. 2]}$$

where $f^* = \sqrt{\frac{g}{\pi} k_{ox}}$ is the Bragg frequency.

It should be noted that the function $F_2(f, \varphi)$ is not sufficient to describe the frequency-spreading process, as it contains no information about the travelling direction of the waves. Often, the product $F_2 \cdot W$ is denoted as F_2, but this leads to confusion because the application of the Wiener-Khintchine theorem to this function does not result in a real covariance function of the sea surface.

To obtain an approximate solution for an arbitrary roughness of the sea surface, the small-scale roughness result of Eq. 2 is applied to a composite-roughness sea-surface model. This model basically considers the short surface waves carried by long waves of large amplitude. The long waves are locally approximated by plane facets with randomly distributed slopes and velocities.

To apply this model to the small-scale results, the statistical properties of the facets have to be formulated in terms of the statistical properties of the sea surface and the facet diameter L. The following simplifying assumptions are made: If the incident acoustic wave vector lies in the x-z plane, the contribution of the facet slope and velocity in the y direction to the doppler spread and the backscattering strength are considered to be small compared to the velocities in the x and z directions and are therefore neglected. Then the facet movement is a three-dimensional stochastic process that is assumed to be Gaussian. Therefore the three-dimensional covariant matrix $\sigma_{\underline{u}}$ of the slope-velocity-vector $\underline{u} = (\epsilon, u_x, u_z)$ is sufficient to describe the process. The matrix $\sigma_{\underline{u}}$ is a function of the sea surface statistics and the facet length L, in fact the facet has the effect of a low-pass filter on the sea-surface spectrum. For certain cases, the involved integrals can be solved completely, otherwise rather simple computer calculations are performed.

With the knowledge of the facet statistics a general expression for the doppler density can be written down. The starting equation is

$$\varphi(f) = \iiint_{-\infty}^{\infty} (4\pi)^2 \; k_{oz}^4(\underline{u}) \; X_3\left[-2\, k_{ox}(\underline{u}),\, 0,\, \Delta f(\underline{u})\right] N(\underline{u}) \, d\underline{u}. \quad \text{[Eq. 3]}$$

The facet movements change the backscattered frequency stochastically. The total doppler density is given by an integral over these frequency shifts $\underline{u}$ multiplied by the probability $N(\underline{u})$ of their occurrence. This integral can be solved, in closed form, the result being:

$$\varphi(f) = \frac{-\pi^3}{4g^2 k_{oz}} \left[f_1 \cdot F_2(f_1, 0) \cdot W(f_1, \pi) \cdot I_1(\sigma_{\underline{u}}) + \right.$$

$$\left. + f_2 \cdot F_2(f_2, 0) \cdot W(f_2, 0)\, I_2(\sigma_{\underline{u}}) \right] \qquad \text{[Eq. 4]}$$

where

$$f_1 = \sqrt{\frac{g}{\pi} \frac{f-f^*}{c} \cos \gamma_0}\ , \qquad f_2 = \sqrt{\frac{g}{\pi} \frac{f+f^*}{c} \cos \gamma_0}\ .$$

The function $I_{\frac{1}{2}}(\sigma_{\underline{u}})$ contains the information of the facet statistics; it is a somewhat lengthy expression but contains no integral and can therefore be evaluated quite simply on the computer.

To evaluate Eq. 4, the facet length L has to be chosen. This choice is directed by two influences. Firstly, the facet length should be greater than the wavelength of the incident acoustic wave to reduce the finite aperture error as much as possible. Secondly, the sea-surface roughness $\sigma_h(L)$ on the facet should be much smaller than the acoustic wavelength to fulfill the Rayleigh condition.

To meet both requirements, the following procedure for choosing the facet length L is performed:
Choose L such that:

$$\lambda_0 \cdot N = L \qquad \text{[Eq. 5a]}$$

and

$$\sigma_h(L) = \frac{\lambda_0}{N}\ . \qquad \text{[Eq. 5b]}$$

The number N is a quality number that shows how far L is away from the border where the conditions for the validity of the composite sea-surface model are no longer valid. If N is less than, lets say, about two, this indicates that the facet model will yield no correct results.

Combining Eq. 5a and 5b gives

$$L \cdot \sigma_h(L) = \lambda_0^2\ , \qquad \text{[Eq. 6]}$$

which is used in an iterative computer procedure that calculates the facet length L.

2. RESULTS

One of the main difficulties in applying the general formulae is to find an appropriate sea-surface spectrum that leads to reasonable results for both the facet statistics and the doppler density calculations. The commonly used Pierson-Moscowitz spectrum with a cosine-square directivity was used for the calculation of the facet statistics. For the doppler density calculation this spectrum was not useful in all cases because it assumes a directivity for high surface frequencies where an omnidirectional Philipps spectrum would be more appropriate. Therefore a modified spectrum due to Scott [Ref. 6] was used in the doppler density calculation. The modification was done by introducing a cut-off frequency where the spectrum becomes very fast omnidirectional. The correct choice of this frequency is an open question.

Figure 1 demonstrates the influence of the grazing angle γ on the result. The x-axis shows the doppler shift in hertz, the y-axis the doppler density (the incident frequency is 3.5 kHz). The angle γ is changed from 2° to 20°, the wind speed is 16 kn = 8 m/s and the main direction of the surface waves is away from the sound source. This causes an asymmetry in the spectrum that is explained as follows: The correlation between the facet slope and the facet vertical velocity is negative in this case. This means that a large angle belongs to a negative velocity. This velocity causes negative doppler shift. As the slope is greater for negative velocity than for the positive velocity, the backscattered energy is greater. The sea surface spectrum is assumed to be omnidirectional in the frequency range where Bragg-scattering occurs, that means at about 2.7 Hz or 20 cm wavelength. The asymmetry is due only to the facet movement.

If the orientation of the sea surface spectrum is 90° off the incident wave vector this effect does not occur, as can be seen from Fig. 2. It is instructive to compare these results with another, very simple approximation. This approximation calculates the doppler spread by assuming that the small-scale results are just spread out on the frequency scale according to a normal distribution whose standard deviation is calculated as a second moment of the sea-surface spectrum. The slope of the sea surface is not considered. This model is a limiting case of the facet theory if the facet length is put to zero.

Figure 3 shows the result. Compared with the previous figure, it is seen that for small grazing angles, 2° to 6°, the differences are up to 20 dB. Moreover, the asymmetry of the curves, which must be expected from physical reasons, is not incorporated in the model. These considerable differences show the necessity of a more detailed model for the doppler-spread calculation.

Figure 4 shows the influence of the wind speed for a fully developed sea. The grazing angle is 6°, the wind speed varies from 4 kn (2 m/s) to 64 kn (32 m/s) in geometrical progression. For low wind speeds the facet model is seen to be superfluous, as the doppler density consists virtually of two δ-functions due to the Bragg resonant scattering. For high wind speeds the backscattered energy is considerably higher, but it is spread over a higher frequency range, the backscattered energy at the Bragg frequency does not change.

Figures 5 and 6 demonstrate the influence of the incident acoustic frequency. Figure 5 shows the doppler density versus the normalized frequency $\frac{\Delta f}{f_0}$ for f_0 = 0.33, 1, 3.3, 10 and 33 kHz at a wind speed of 8 m/s. For the low frequency, f_0 = 300 Hz, the Rayleigh condition is fulfilled, thus the doppler spectral density consists of only two lines. With increasing frequency the facets become more and more important. If the wind speed is very high, the facets become important at even low frequencies, as is shown in Fig. 6 for a wind speed of 16 m/s.

Figures 7 and 8 show the influence of the mixing function $W(f, \varphi)$. In the beginning of this chapter it was mentioned that a somewhat modified sea-surface spectrum according to Scott has been used, by introducing a cut-off frequency f_c. The result for f_c = 5 Hz is shown in Fig. 7. If, instead, the cut-off frequency is put to infinity, the result is as shown in Fig. 8. It uses the unmodified directional Scott spectrum up to the highest frequencies, which is a doubtful assumption.

CONCLUSION

Using a composite-roughness sea-surface model, an approximate solution for the frequency spread of backscattered sound from an arbitrarily rough sea surface is derived. The model needs a description of the statistical properties of the sea surface roughness including the directivity and the mixing of incoming and outgoing waves in one direction and one frequency. In particular, the mixing of waves is so far only theoretical and needs an experimental verification. The experiments would include both the frequency dependence and the relationship to the directivity of the sea-surface spectrum.

As a next step it is planned to incorporate the theory into the sonar models used at SACLANTCEN. It is hoped that it will be possible to check the frequency-spread model with experimental data from our sonar experiments. Incorporation in the sonar models will enable us to predict the time/frequency-spread of acoustic waves and, if verified, is considered to be the first step towards the estimation of the scattering function for the underwater sound channel.

REFERENCES

1. MARSH, H.W. Sound reflection and scattering from the sea surface. Jnl Acoustical Society of America 35, 1963: 240-244.

2. PARKINS, B.E. Scattering from the time-varying surface of the ocean. Jnl Acoustical Society of America 42, 1967: 1262-1267.

3. BACHMANN, W. Verallgemeinerung und Anwendung der Rayleighschen Theorie der Schallstreuung [Generalization and application of Rayleigh sound-scattering theory]. Acustica 28, 1973: 223-228.

4. BACHMANN, W. A theoretical model for the backscattering strength of a composite-roughness sea surface. Jnl Acoustical Society of America 54, 1973: 712-716.

5. ESSEN, H.H. Wave-facet interaction model applied to acoustic scattering from a rough sea surface. Acustica 31, 1974: 107-113.

6. SCOTT, J.R. Some average wave lengths on short-crested seas. Quarterly Jnl Royal Meteorological Society 95, 1969: 621-634.

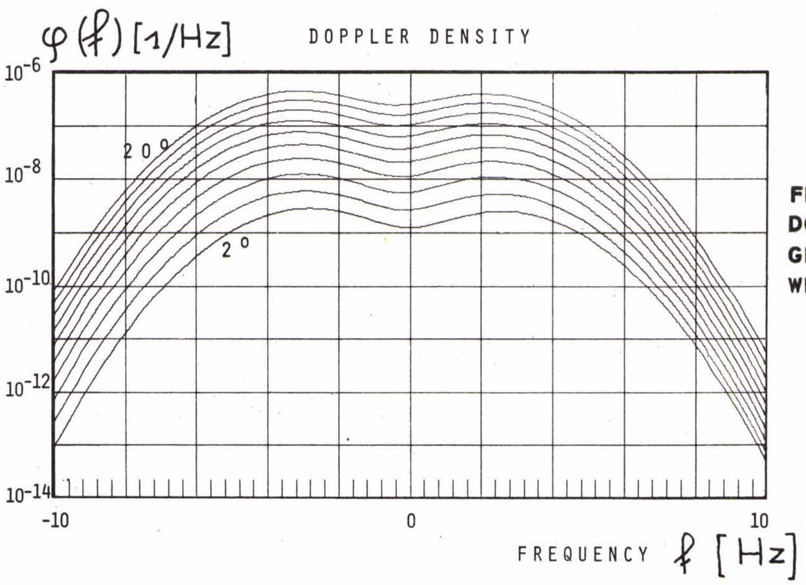


FIG. 1
DOPPLER DENSITY φ (f), INFLUENCE OF THE GRAZING ANGLE γ_0
WINDSPEED: 16 kn, DOWN WIND

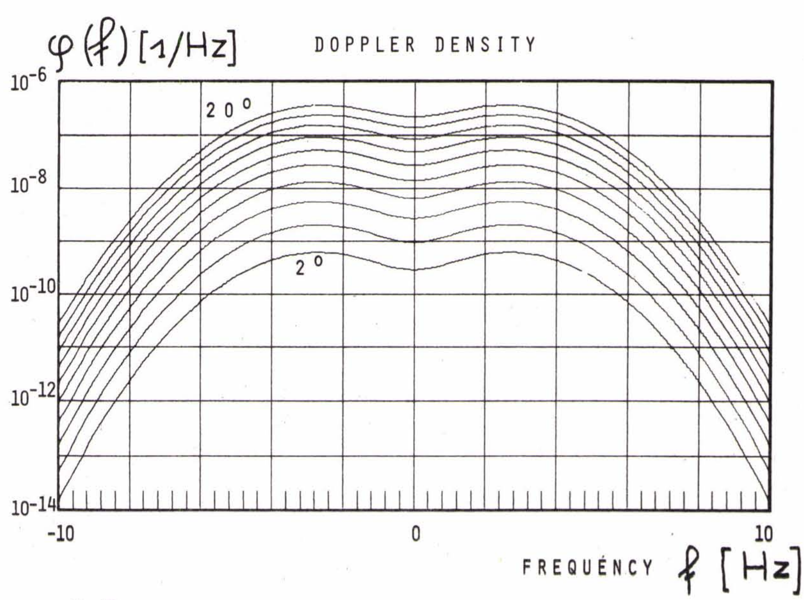


FIG. 2
DOPPLER DENSITY, INFLUENCE OF THE GRAZING ANGLE γ_0
WINDSPEED: 16 kn, CROSS WIND

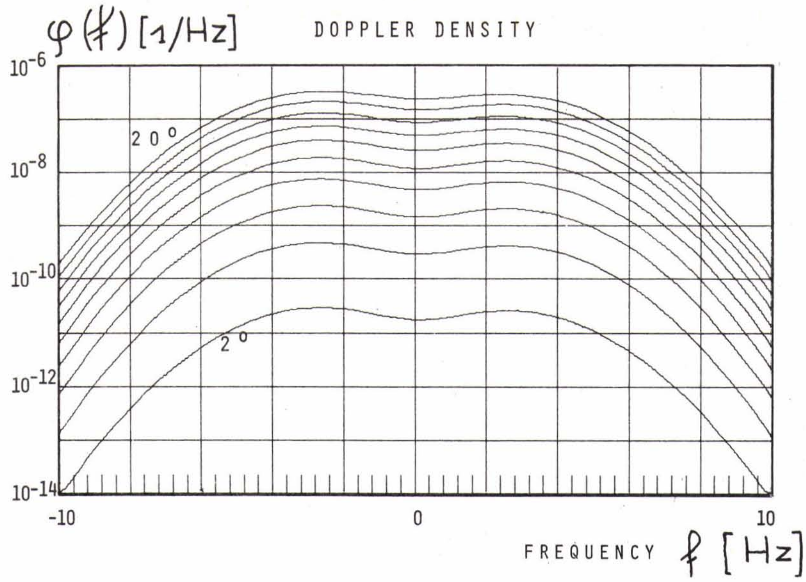


FIG. 3
DOPPLER DENSITY SIMPLIFIED MODEL, INFLUENCE OF THE GRAZING ANGLE γ_o
WINDSPEED: 16 kn, DOWN WIND

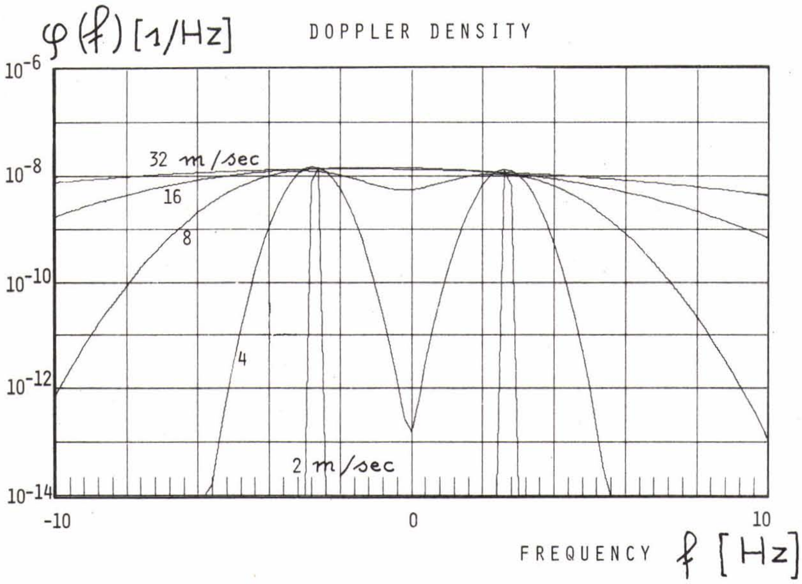


FIG. 4
DOPPLER DENSITY, INFLUENCE OF THE WINDSPEED, DOWN WIND
GRAZING ANGLE $\gamma_o = 6°$

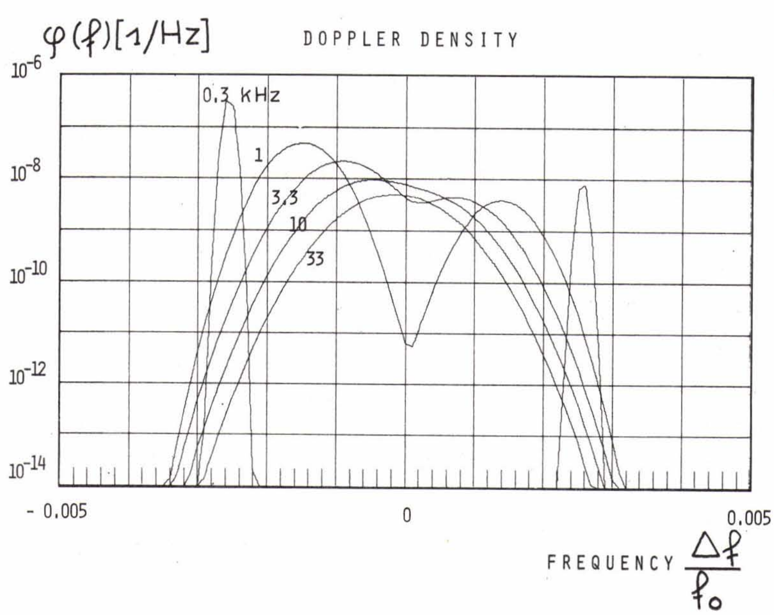
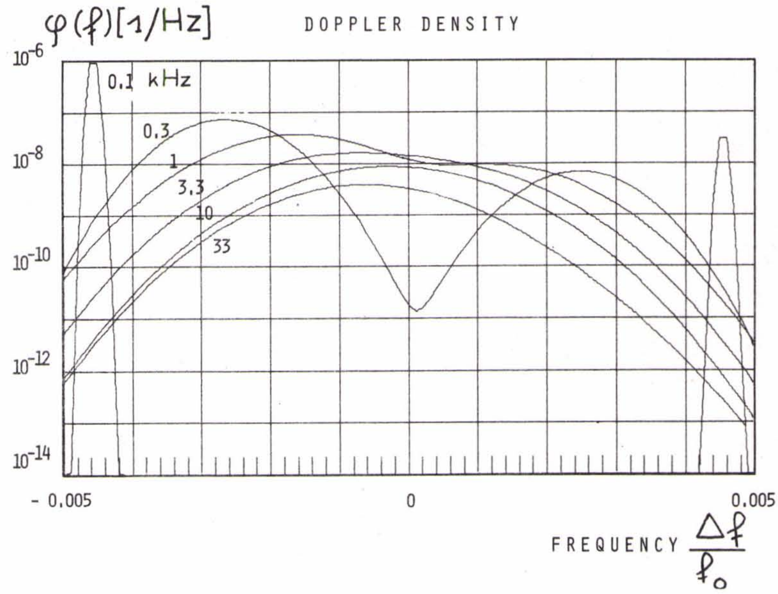


FIG. 5
DOPPLER DENSITY, INFLUENCE OF THE ACOUSTIC FREQUENCY
WINDSPEED: 16 kn, DOWN WIND
GRAZING ANGLE $\gamma_o = 6°$

FIG. 6
DOPPLER DENSITY, INFLUENCE OF THE ACOUSTIC FREQUENCY
WINDSPEED: 32 kn, DOWN WIND
GRAZING ANGLE $\gamma_0 = 6°$



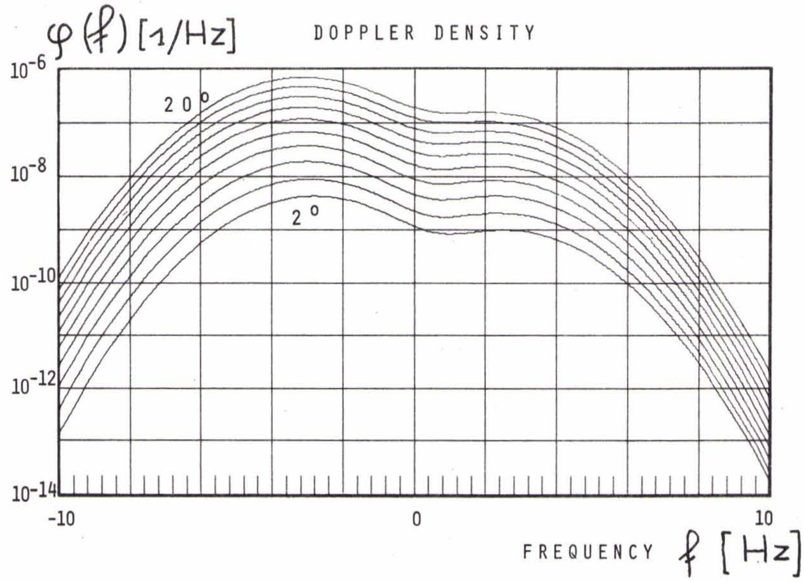


FIG. 7
DOPPLER DENSITY, INFLUENCE OF THE GRAZING ANGLE γ_0 FOR A MODIFIED SCOTT SPECTRUM
WINDSPEED: 16 kn, DOWN WIND
CUT OFF FREQUENCY $f_c = 5$ Hz

FIG. 8
DOPPLER DENSITY, INFLUENCE OF THE GRAZING ANGLE γ_0 FOR AN UNMODIFIED SCOTT SPECTRUM
WINDSPEED: 16 kn, DOWN WIND