GEOMETRICAL PROPERTIES OF UNDERWATER SOUND PROPAGATION

by

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ABSTRACT

During the past several years, the U.S. Naval Ordnance Laboratory has made a study of the geometrical properties of propagation of sound in the ocean. Measurements have been made at sea under controlled conditions by use of a deep-draft, stable receiving platform known as SPAR. Positions of the source ship and the receiving platform were precisely known by the use of electromagnetic positioning equipment. Reference positions thus obtained were compared with positions determined by acoustic means. Propagation paths investigated have included surface channel, near-surface refracted paths, bottom reflected paths, and combinations of bottom and surface reflected paths. Corrections for refraction were made by a laboratory ray trace program. This program, which uses a linear segmented velocity profile, has provisions for double precision calculations of the ray path, corrections for a sloping bottom and corrections for the curvature of the earth. The program has been successfully run both on an IBM-7090 computer and a CDC-6400 computer.

INTRODUCTION

The work I will describe has to do with a study of the geometry of underwater sound propagation as contrasted to intensity studies. Of course, the two are closely rleated. This study has been based largely on full-scale, at-sea measurements in which a stable, sea-going platform known as SPAR has been used. Since refraction is fundamental to this problem, ray tracing has been an important part of this work.

First, I will describe the ray trace program, particularly characteristics and features which are associated with the geometrical parameters. Next, I will describe briefly the experimental setup used for the at-sea measurements, and finally I will give some typical examples and results of the work.

LABORATORY RAY TRACE EQUIPMENT AND PROGRAM

The ray tracing program to be described here was originally coded in FORTRAN II by the U.S. Naval Oceanographic Office [Ref. 1] following the general pattern established by the Canadian Pacific Naval Laboratory [Ref. 2]. In 1964 the program was converted by NOL to run in FORTRAN IV on the IBM-7090 and later that year a plotting option was added. In 1967, a subroutine was added to correct the sound velocity profile for curvature of the earth, and at the same time, double precision computations were incorporated. Just recently the program has been adapted to and is being run on the CDC-6400. The program can also be run on the UNIVAC 1108.

The basic equations used in the ray trace program are (of course) based on Snell's law:

$$V_0 \cos\theta_1 = V_1 \cos\theta_0 \quad ,$$

where V_0 and θ_0 are the initial velocity and inclination of the ray to the horizontal and V_1 and θ_1 are taken at some other point on the ray. The parameter* θ is taken as positive for downward angles and negative for upward angles. A linear

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* θ and D/E are used interchangeably in this paper.

segmented sound velocity profile is used. Thus, the ray traverses an arc of a circle in each region of constant velocity gradient.

Curvature of the earth corrections are based on the formulations given by Watson [Ref. 3] at the Admiralty Research Laboratory in the U.K. This correction takes the form of a transformation of the sound velocity profile as follows:

$$z' = r_0 \ln [(r_0 - z)/r_0]$$

and

$$c' = c \exp (z'/r_0) \quad ,$$

where z and c are the original depth, velocity coordinates, z' and c' are the transformed coordinates, and r_0 is the radius of the earth at the sea surface. The program also takes into account the variation of the radius of the earth with latitude.

GENERAL PROGRAM CHARACTERISTICS AND SPECIAL FEATURES

The program has the following features:

a. A maximum of 40 depth-velocity coordinates (straight-line segments) may be used to represent the sound velocity profile.

b. As the ray progresses in the horizontal range, the sound velocity profile may be changed as many as twenty times.

c. Corrections for the curvature of the earth are optional. If corrections for curvature are chosen, the latitude associated with the sound velocity profile is inserted. If corrections for curvature are not chosen, a flat earth is assumed.

d. The independent operating parameter is the initial angle θ_0 at the source. Both upward and downward angles between ± 90 degrees, exclusive, are accepted.

e. Individual rays are terminated when they have reached some pre-set value of the horizontal range.

f. Reflections from both the ocean surface and the bottom, either flat or sloping, are permitted. A maximum of 100 regions of different bottom slopes are allowed for one ray field. Bottom slopes are represented by straight line segments.

g. The program includes an optional feature in which a tape is generated from which the ray field is plotted on a CalComp Plotter. Up to 1000 points are plotted which represents one individual ray. Also, sound velocity profiles are plotted with 40 points per profile permitted. Typical CalComp plots are depicted in Figs. 1, 2 and 3. Figure 1 shows both a profile relating to a flat earth and the same profile corrected for the curvature of the earth. Figure 2 shows a typical ray plot for a flat bottom and no corrections for earth curvature. The ray field, shown in Figure 3, relates to a uniform sloping bottom and includes corrections for earth curvature. (These are all plotted automatically by CalComp.)

i. Computation in either single or double precision are optional.

RAY PATH GEOMETRY

Before showing a sample printout, it is instructive to depict the ray path geometry and to define certain quantities. The geometry associated with the acoustic ray is shown in Fig. 4.

A sample printout is shown in Fig. 5. Note that when the ray becomes horizontal or changes direction (upgoing to downgoing, or vice versa) because of reflection at a boundary or because of refraction, all entries except the D/E angle are repeated.

SAMPLE RAY TRACE RESULTS

The next series of figures are sample runs which show the effect of a slight change in the sound velocity profile on ray path geometry for both bottom reflected and convergence zone propagation paths. The change in profile is just that induced by curved earth corrections. Thus, the results to be presented can be looked upon as the effect of curved earth corrections or the effect of an uncertainty in the velocity profile from any cause. (I will tie these and some experimental results together later.)

Figure 6, the first figure of this series, shows (on the left) a typical sound velocity profile for the Pacific Ocean. The profile shown relates to the original or uncorrected profile. The profile corrected for earth curvature is too little different to be resolved with the plotting scale used here. Instead, since the corrected profile is characterized by a slight increase in the deep-water sound velocity gradient (between channel axis and bottom), gradient values are shown instead. The gradient denoted by "g" relates to the uncorrected or reference profile and the gradient denoted by "g'" relates to the corrected or altered profile. Note that the difference in the gradients is only about one-fourth ft/s per 1000 ft.

An explanation of the plot on the right is as follows: Two sets of ray tracing runs were made -- one for the original profile and one for the corrected profile. From each set, the range at which the ray reached the surface for the same initial D/E angle was noted. Differences in range were taken and plotted as a function of range. This is the plot shown. The short ranges are associated with large depression angles and the long ranges, bottom bounce leading into convergence zone paths, are associated with shallow depression angles. Thus, a very slight change in profile causes a considerable change in the range at which the ray returns to the surface. For the bottom reflected path, the more severe gradient (g'), or the the correction for earth curvature, causes the range to be greater. This effect is highly range dependent, becoming as large as 1500 yds at about 55 kyd. In contrast, for convergence zone propagation, the more severe gradient, or the correction for earth curvature, causes the range to be less by about 500 yards.

The absolute difference in velocity at 18 000 ft is about +5 ft/s relative to the profile shown. If the entire profile is displaced by 5 ft/s, the difference in range is less than 15 yd for all bottom bounce ranges and 35 yd at the convergence zone.

Figures 7, 8 and 9 show the nature of this effect for profiles typical of the Caribbean, the Norwegian, and the Mediterranean Seas, respectively. In all cases the change in the deep-water gradient as affected by corrections for curved earth was from one-fourth to one-third ft/s per 1000 ft and has comparable effects on the ray geometry as for the Pacific profile. However, the effect within the convergence zone itself varies widely from area to area.

The next figure, presented in similar fashion, shows the effect of a small change in the sound velocity profile on the travel time. This has to do, of course, with our ability to measure ranges acoustically from travel times. As before, both bottom reflected and convergence zone propagation paths are included. In this case, however, the difference in range is plotted as a function of range using the same travel time as the independent analytical parameter. Figure 10 shows these results for the Pacific profile. (This profile is the same as shown previously for the Pacific Ocean.) Note that with travel time as a constraint, the change in range caused by a change in profile is considerably less than that for the data just shown. In contrast to the previous case, for both bottom reflected and convergence zone propagation, the more severe gradient, or corrections for earth curvature, causes an increase in range. In this case, however, a fixed displacement in the sound velocity profile of 5 ft/s caused a greater difference in the ranges than the effect of a change in the gradient — a difference of about 50 yd throughout the range. Since the other areas gave similar results, they are not repeated here. I will say more later concerning the significance of the sound velocity gradient as it relates to ray tracing accuracies.

AT-SEA MEASUREMENTS AND RESULTS

In order to provide some overall check on our knowledge of the geometrical properties of underwater sound propagation, the U.S. Naval Ordnance Laboratory has been engaged in an at-sea measurements program for this purpose. Ray tracing, of course, is an important part of this problem.

Measurements have been made under controlled conditions by use of a deep-draft, stable receiving platform known as SPAR [Ref. 4] (Seagoing Platform for Acoustic Research). An oceanographic vessel has served as a source platform. The experimental arrangement is depicted in Fig. 11.

Receiving arrays, rigidly fixed to the hull of SPAR, consist of both horizontal and vertical lines comprising 14 hydrophones, thus providing for the measurement of both azimuth and depression-elevation angles. The accuracy in reading either the azimuth or D/E angle is believed to be better than 0.1 degree. LORAN equipment, both A and C, is used for the geographical position of SPAR. A single-dimension RAYDIST system provides a reference as "true" range between SPAR and the source ship. A radio link is used for the measurement of acoustic travel times and for communication between operating units. Thus, reference or "true" positions were compared with positions determined by acoustic means. Propagation paths investigated have included surface channel, near-surface refracted paths, bottom reflected paths, and combinations of bottom and surface-reflected paths. Corrections for refraction were made by the laboratory ray trace program just described. On-site oceanographic data included bathythermographs, sea surface temperature measurements, precision depth recorded data, and direct-reading velocimeter data (throughout the water column) when available. Some typical experimental results relating to various propagation paths follow.

NEAR-SURFACE PROPAGATION PATHS

The purpose of this experiment was to determine the accuracy of acoustic ranges from travel time measurements for near-surface propagation paths. This accuracy, of course, is directly related to the accuracy of the assignment of the mean horizontal velocity — a sound velocity and ray trace problem.

The propagation paths referred to are shown in Fig. 12. For this type of propagation, the ray travels by repeated upward refractions and successive surface reflections. A simplified sketch of the experimental setup is shown in Fig. 13. The reference or true range was measured by a single-dimension RAYDIST. The error in the reference range was estimated to be 15 ft, independent of range. (This error is comprised of a RAYDIST interpolation error and the uncertainty in the position of the underwater projector.) The error associated with the measurement of the acoustic travel time was estimated to be about 1 ms, the source of which was in the reading of the timing record. Thus, the experimental error expressed in terms of the mean horizontal velocity was estimated to be about 3 ft/s for a source-receiver separation of 6 miles and was progressively less at longer ranges. (This quantity is also referred to as the true or reference MHV.)

In determining the MHV from sound velocity data, referred to as the acoustic MHV, sound velocity data were obtained from readings of temperature, salinity and depth. Water temperatures were read on station from standard 900-foot bathythermograms, salinity values were taken from tables published by the U.S. Naval Oceanographic Data Center for that area and season. Figure 14 gives estimates of the error in determining these input quantities. Using Wilson's tables to convert to sound velocity, the total error in the determination of sound velocities, from which the profile was constructed, was estimated to be about 3 ft/s and random in nature.

Acoustic mean horizontal velocities were obtained by use of the ray trace program described previously. For channel propagation, mean horizontal velocities are plotted as a function of channel depth in Fig. 15. Experimental results, expressed as the difference between the acoustic MHV and the reference MHV are presented in Fig. 16. The dashed curve gives the estimated error in the measurements themselves. Channel depths varied from 60 to 190 ft over the six days of measurements. The results show no significant difference between the acoustic MHV and the reference MHV based on an independent electromagnetic distance measurement. The small random differences observed, about 4 or 5 ft/s (0.1%) could be attributed to measurements errors alone. In terms of range, Fig. 17, the difference between the acoustic range and the reference range for a source-receiver separation of 12 miles, was about 15 yd, random in nature and with a mean error of substantially zero.

BOTTOM-REFLECTED PROPAGATION PATH

The purpose of this at-sea measurement was to determine the accuracy of predicting the horizontal range at which bottom-reflected sound reaches the surface from a knowledge of bottom slope, the depression angle at the source, and the sound velocity profile. Results of this experiment are shown in Fig. 18. Single-dimension RAYDIST ranges were used as the reference or true range. In this case, the acoustic range is obtained from the product of the measured travel time and the mean slant velocity derived from ray tracing. The ray tracing employed here included double precision computations and corrections for earth curvature. Note, however, the rather large error in the acoustic range -- about 1000 yd in 30 000 yd. Although not conclusive, the results suggest an error in determining the deep-water sound velocity gradient. A change in the deep-water gradient of about 2 ft/s per 1000 ft (0.016 to 0.018, for example) will remove this error. As indicated this profile was constructed from on-site bathythermograph data down to 900 ft and historical data below 900 ft. In both cases, sound velocity values were derived from temperature data using Wilson's equations [Ref. 5]. For this bottom bounce propagation path, the difference between the reference range and a range based on travel times and mean slant velocities obtained from ray tracing was less than about 20 yd out to 38 kyd. This is within the experimental error.

Wilson's equations relate sound velocity to temperature, salinity, and pressure (not depth). These results suggest that the measured pressure effect or the conversion from pressure to depth may be slightly in error. The relationship between depth and pressure is

$$p = \bar{\rho} g h \quad ,$$

where p is the pressure, $\bar{\rho}$ is the mean density of the water column from the surface to the depth considered, g is the acceleration of gravity, and h is the depth. Errors in assigning correct values to ρ and g can result in a cumulative type error in depth which causes and error in the sound velocity gradient.

The final example of at-sea measurements has to do with the measurement of bottom topography, both slope and roughness from refraction-corrected travel time differences between various multipaths [Ref. 6]. The geometry associated with this measurement technique is depicted in Fig. 19. For this work explosive charges were used for a sound source. The regional slope, α, is given by the equation shown at the bottom of the figure. Input quantities required are water depth at the receiver (SPAR), the horizontal distance between source and receiver (from RAYDIST), source and receiver depths, and certain time differences. The time differences denoted by the primes in the equation are time differences corrected for refraction effects. These corrections are made by the ray tracing program previously described by obtaining an effective mean slant velocity appropriate to any two propagation paths. By judicious choice of the experimental parameters, the extent of the bottom over which the slope is measured can be controlled. This method has the advantage of yielding bottom slope characteristics which result from sound energy reflected at the bottom at oblique angles of incidence. Thus, the reflection process is that which is associated with an effective bottom reflecting area (Fresnel zone configuration) comparable to that which exists for sonar applications.

SUMMARY AND RECOMMENDATIONS

As concerns the geometry of propagation, based on our work to date, the following comments and recommentations are made:

a. For precision ray tracing, curved earth corrections are necessary, particularly at ranges greater than about 20 kyd.

b. The principal deficiency is believed to be in the accuracy of the sound velocity profile and not in the ray trace program itself.

c. For near-surface propagation paths, the sound velocity is known sufficiently accurately.

d. The critical input quantity is the sound velocity gradient, not the absolute value of the sound velocity itself. That is, a fixed or systematic error (such as a calibration error) is not as serious as a depth dependent error.

e. It is recommended that equipment be designed and built which will directly read the sound velocity gradient as a function of depth. It would be advantageous to measure the sound velocity gradient directly, and as a function of depth.

f. Additional comparisons should be made between direct-reading velocimeter data and values derived from temperature, salinity, and depth.

g. Finally, I feel that the only meaningful evaluation of a ray trace program and the input data to this program is by full-scale, at-sea experiments.

REFERENCES

1. T.L. Goodin, "A FORTAN IV Ray Tracing Program", U.S. Naval Ordnance Laboratory, NOLTR 70-74, 1 May 1970.

2. Dosso, Lokken, Maunsell and Greenhouse, "Ray Tracing with an LGP-30: Multiple Velocity Profiles, Multiple Bottom Profiles, Bottom and Surface Reflections, Travel Time", Pacific Naval Laboratory, British Columbia, March 1960

3. A.G.D. Watson, "The Effect of the Earth's Sphericity in the Propagation of Sound in the Sea", Admiralty Research Laboratory, Teddington, Report ARL/N29/L, December 1958.

4. U.S. Naval Ordnance Laboratory, Brochure entitled "SPAR as a Research Vessel".

5. W.D. Wilson, "Equation for the Speed of Sound in Sea Water", J.Acous.Soc.Am., Vol.32, 1960, p.1357.

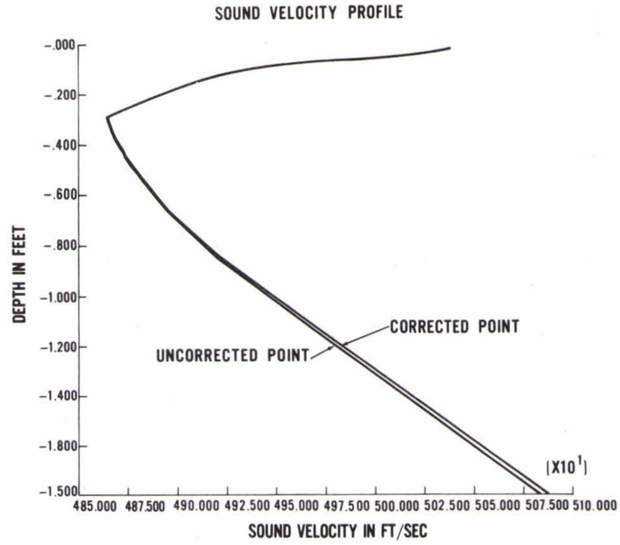
6. A.T. Jaques, "Determination of Bottom Slope and Roughness from Travel Time Differences", U.S. Naval Ordnance Laboratory Report NOLTR 67-100, 6 September 1967.

DISCUSSION

The author said that the expansion of the program to double-precision was not a necessary modification.

The question was asked whether the incorporation of the curved-earth correction by modifying the velocity profile was an expression of latitude-dependent gravity effects. Spofford answered that the modification was by a coordinate transformation, the effect of which could be incorporated in the velocity profile.

FIG. 1



A PLOT OF BOTH THE INPUT VELOCITY PROFILE, AND THE PROFILE CORRECTED FOR EARTH CURVATURE

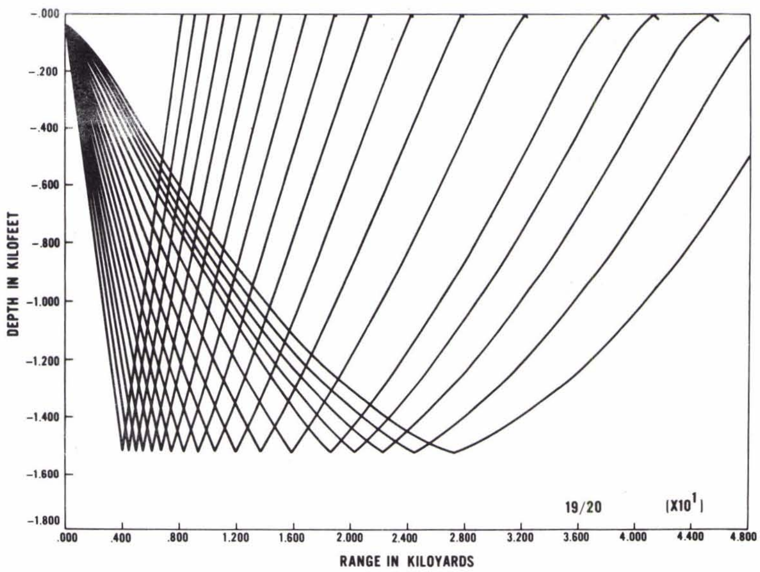
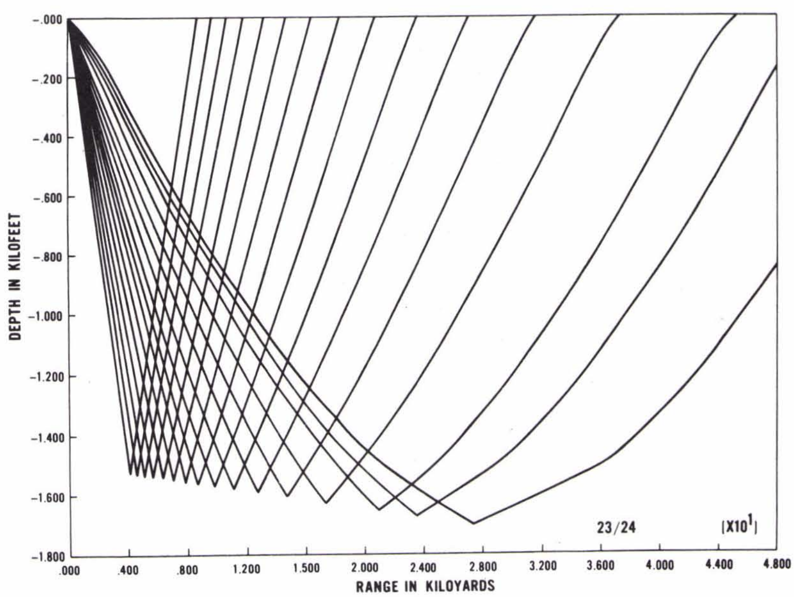


FIG. 2

A RAY FIELD PLOT WITH A FLAT BOTTOM AND NO EARTH CURVATURE CORRECTIONS

FIG. 3



A RAY FIELD PLOT WITH A SLOPING BOTTOM - CORRECTIONS FOR EARTH CURVATURE

RAY PATH GEOMETRY

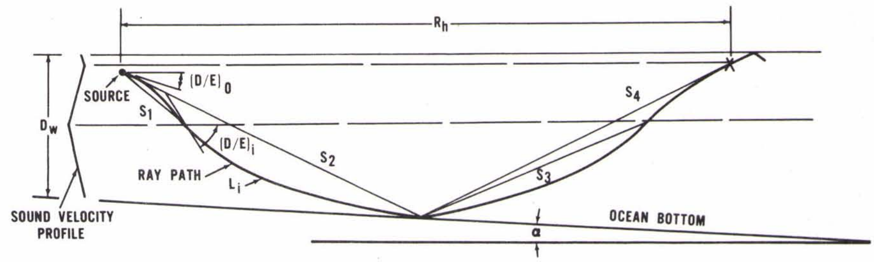


FIG. 4

D_w – WATER DEPTH AT SOURCE
R_h – HORIZONTAL RANGE
D/E – DEPRESSION-ELEVATION ANGLE
T – TRAVEL TIME ALONG THE RAY
L – PATH LENGTH ALONG THE RAY
R_s – SLANT RANGE
MHV – MEAN HORIZONTAL VELOCITY DEFINED BY R_h/T
MRV – MEAN RAY VELOCITY DEFINED BY L/T
MSV – MEAN SLANT VELOCITY DEFINED BY R_s/T
α – BOTTOM SLOPE

SAMPLE PRINTOUT

DOUBLE PRECISION COMPUTATIONS
CORRECTIONS FOR CURVATURE OF THE EARTH

WATER DEPTH AT SOURCE: 15,222 FT (2537 FMS) BOTTOM SLOPE: 0.025 (1.4 DEG.) $(D/E)_0$ = 50.50 DEG
SOURCE DEPTH; 100 FT

DEPTH (FT)	SOUND VELOCITY (FT/SEC)	D/E (DEG)	HORIZONTAL RANGE R_h (FT)	TIME T (SEC)	PATH LENGTH L (FT)	SLANT RANGE R_s (FT)	MHV (FT SEC)	MRV (FT SEC)	MSV (FT SEC)
100.00	5037.02	50.50	0.	0.	0.	0.	0.	0.	0.
150.00	5035.54	50.51	41.2	0.0129	64.8	64.8	3202.99	5036.28	5036.28
—									
—									
2400.14	4879.06	51.97	1835.6	0.5959	2942.9	2942.8	3080.50	4938.78	4938.62
—									
—									
12,003.44	4982.86	51.01	9428.1	3.0888	15,185.2	15,184.9	3052.37	4916.26	4916.16
—									
—									
15,299.26	5040.78	50.46	12,122.2	3.9381	19,442.0	19,441.3	3078.15	4936.85	4936.67
15,299.26	5040.78	−47.64	12,122.2	3.9381	19,442.0	19,441.3	3078.15	4936.85	4936.67
—									
—									
2400.14	4879.06	−49.29	23,567.3	7.4165	36.613.0	.. 23,567.3	3162.54	4936.73	3177.71

NOTE: 56 DEPTH ENTRIES

FIG. 5

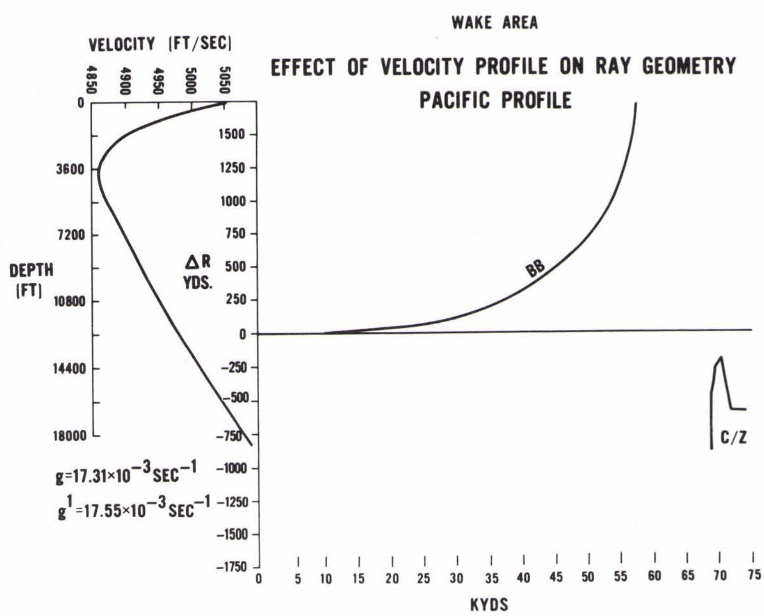
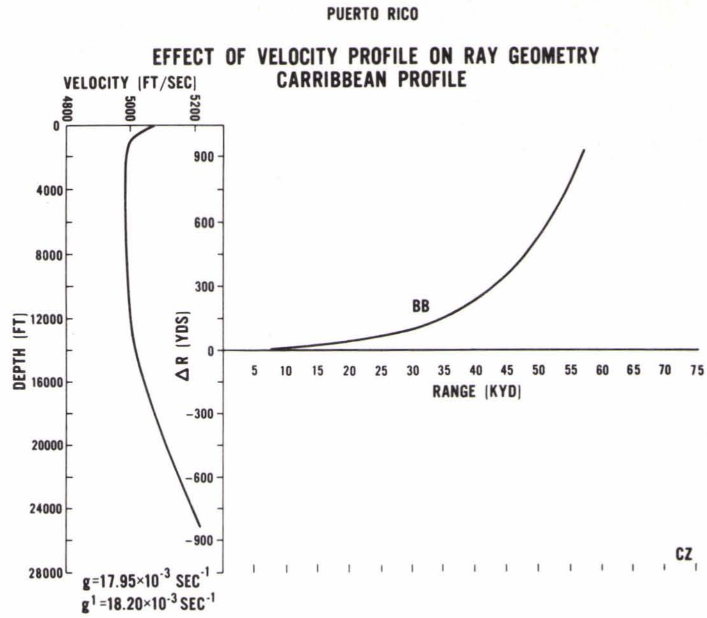


FIG. 6

FIG. 7



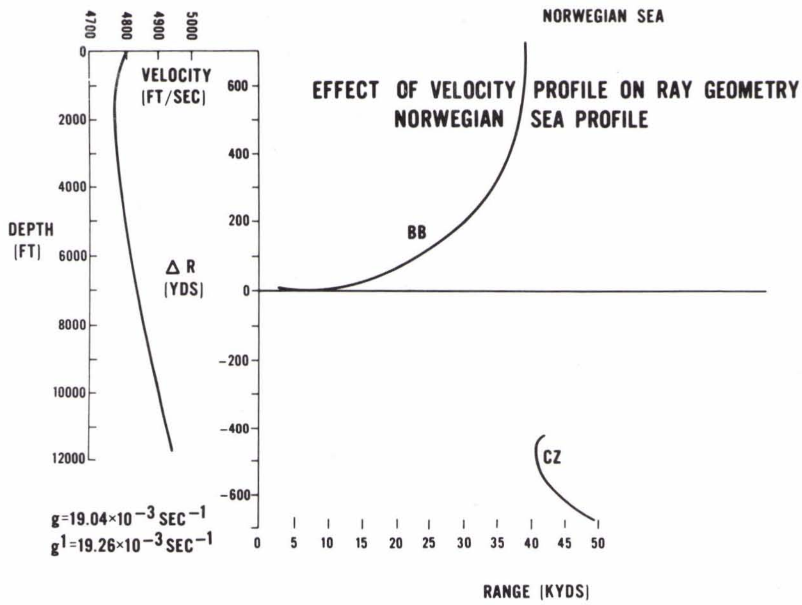


FIG. 8

FIG. 9

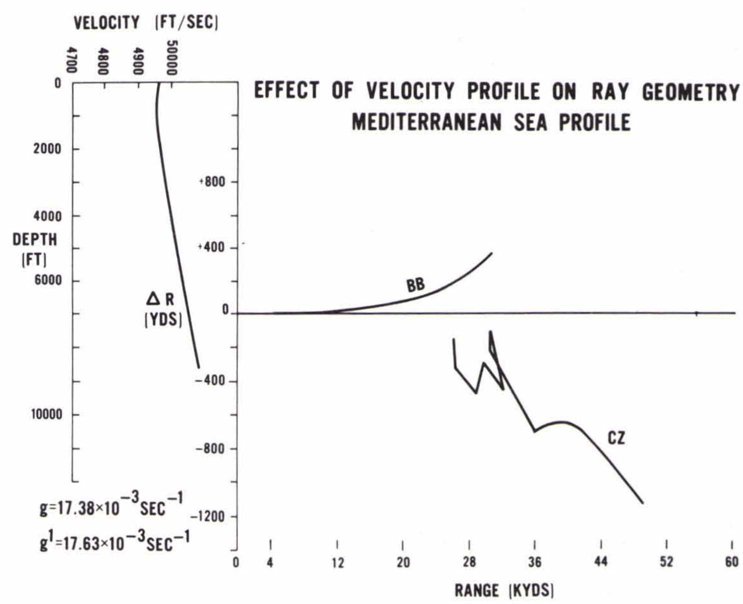
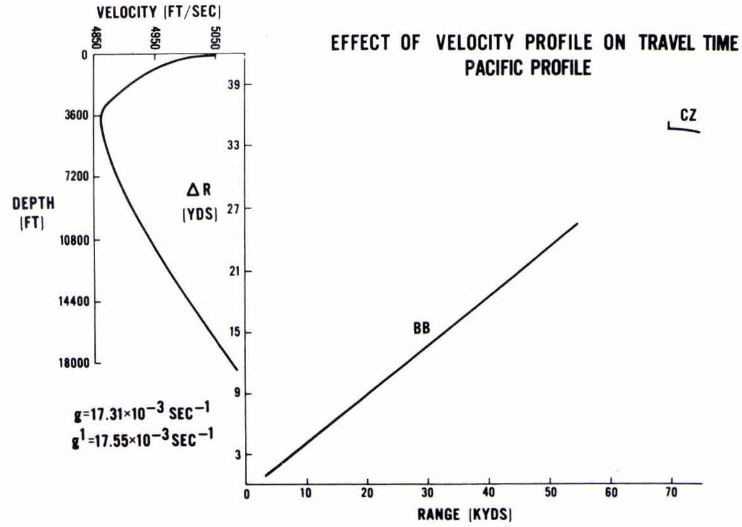
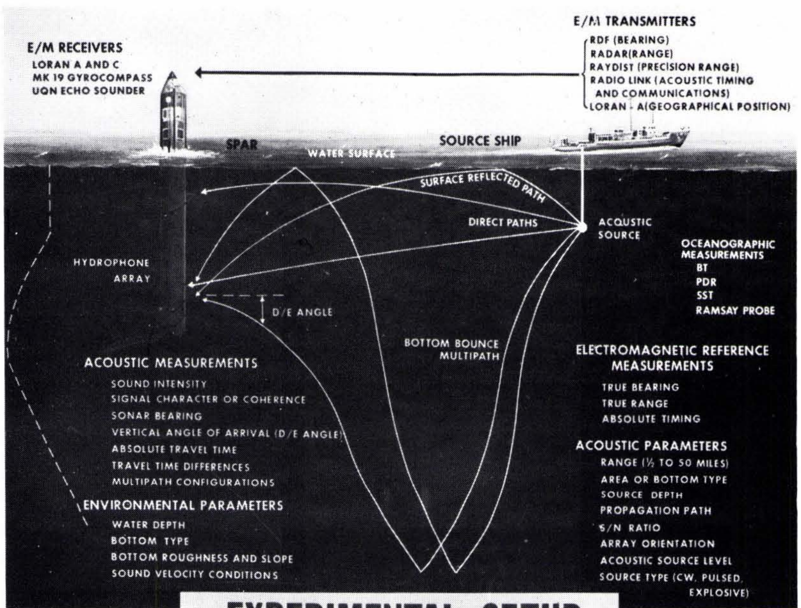


FIG. 10

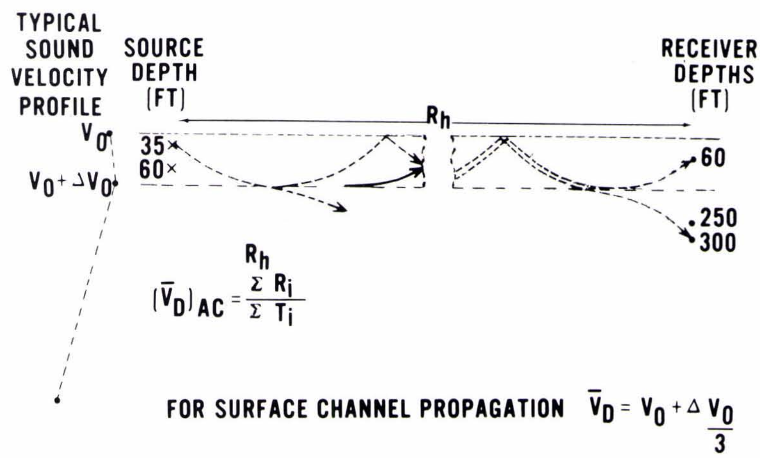




EXPERIMENTAL SETUP

FIG. 11

FIG. 12



NEAR-SURFACE ACOUSTIC PROPAGATION PATHS

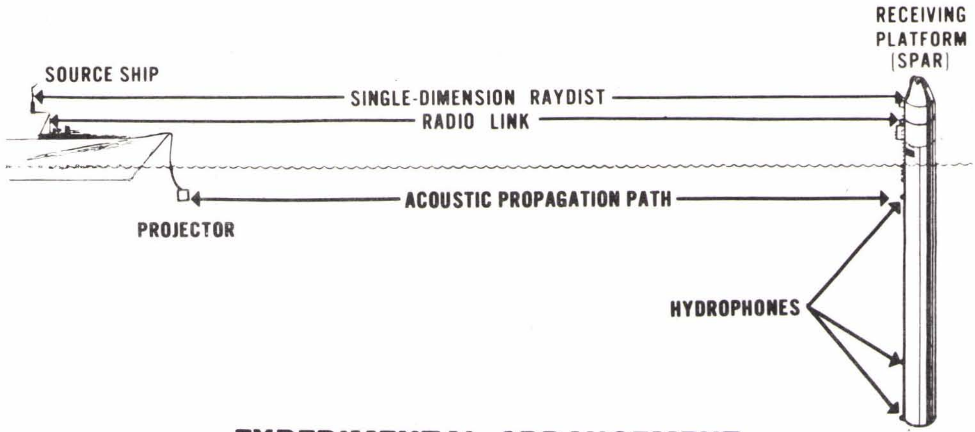


FIG. 13

EXPERIMENTAL ARRANGEMENT

ESTIMATE OF ERROR IN THE DETERMINATION
OF
THE SOUND VELOCITY

$$V = f\,(T, S, D)$$

WHERE T = TEMPERATURE

S = SALINITY

D = DEPTH

1) TEMPERATURE READING FROM BT: ± 1 FT/SEC (± 0.2 to 0.3°F)

2) SALINITY VALUES FROM TABLES: ± 2.5 FT/SEC (± 0.5 PPM)

3) DEPTH ERROR: 0.0 FT/SEC

TOTAL ERROR : ± 3 FT/SEC

FIG. 14

COMPUTED MEAN HORIZONTAL VELOCITY
CHANNEL PROPAGATION
(FROM RAY TRACE)

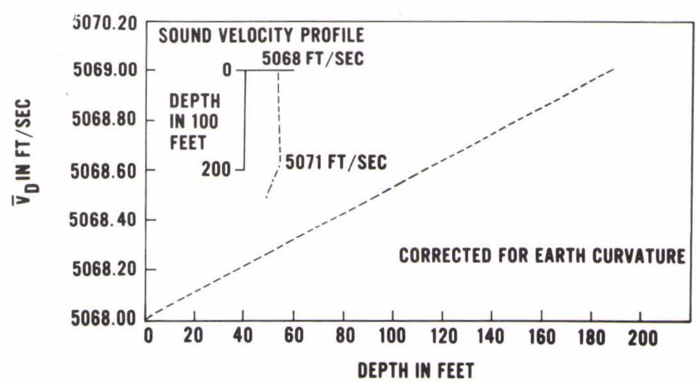


FIG. 15

COMPOSITE PLOT
SURFACE CHANNEL PROPAGATION

SOURCE DEPTH : 35 FEET
RECEIVER DEPTH: 60 FEET
CHANNEL DEPTH : 60 TO 190 FEET

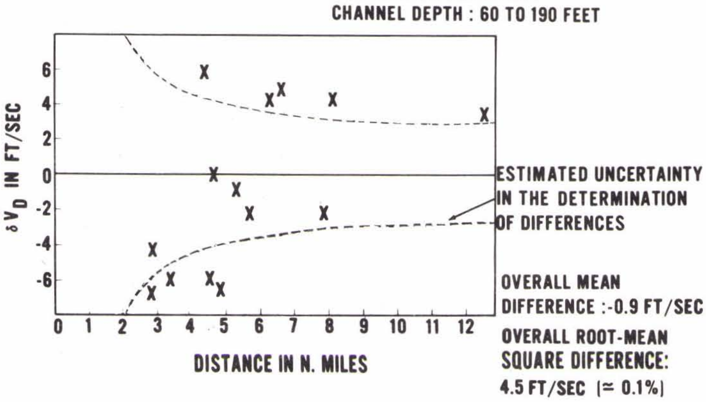
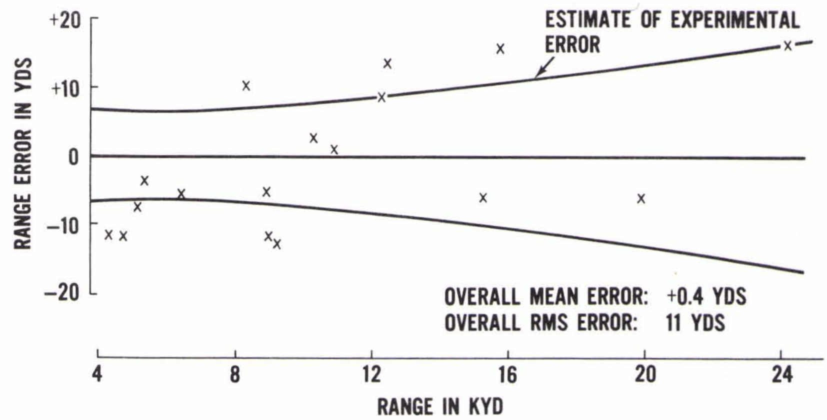


FIG. 16

ACTIVE RANGE ERROR
SURFACE CHANNEL PROPAGATION
REFERENCE RANGE: SINGLE-DIMENSION RAYDIST

FIG. 17



COMPARISON OF BOTTOM BOUNCE ACOUSTIC VS E/M RANGES

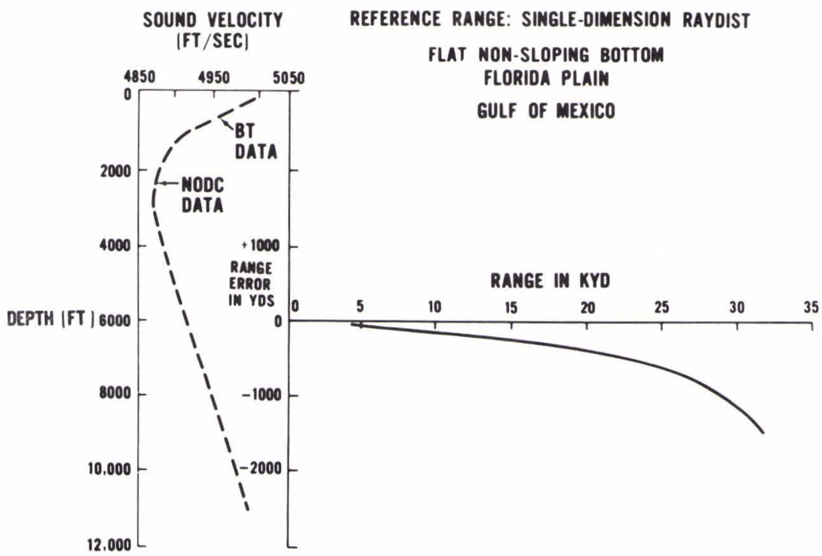
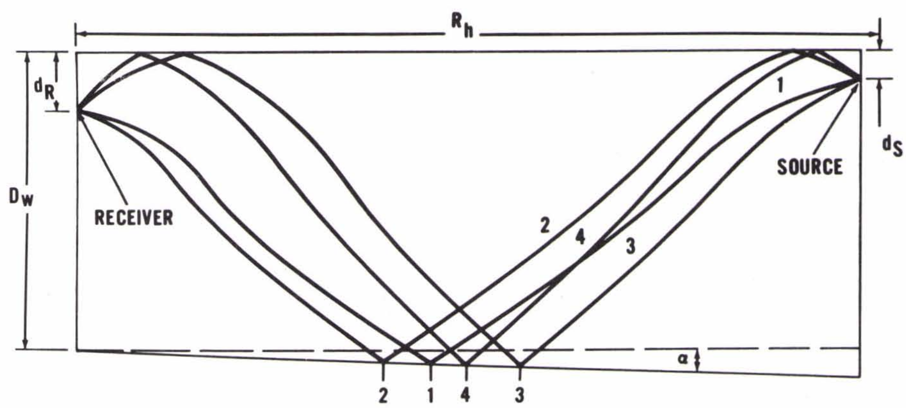


FIG. 18

BOTTOM SLOPE MEASUREMENTS FROM TRAVEL TIME DIFFERENCES CORRECTED FOR REFRACTION

FIG. 19



$$\text{TAN } \alpha = \frac{D_w}{R_h}\left\{\frac{d_S}{d_R}\left[\frac{(\Delta^1 T_{4-1} + \Delta^1 T_{3-2})}{(\Delta^1 T_{4-1} - \Delta^1 T_{3-2})}\right] - 1\right\}$$