SOME HIGH EFFICIENCY DIGITAL SIGNAL PROCESSING TECHNIQUES FOR HIGH-SPEED SIGNAL PROCESSING SYSTEMS

by

Vito Cappellini
Istituto di Elettronica, Facoltà di Ingegneria-Università
Florence, Italy

ABSTRACT Some high efficiency digital signal processing techniques, based on digital filtering, are presented for real-time general-purpose high-speed signal processing systems. At first high efficiency one-dimensional and two-dimensional digital filters are described, both of non-recursive FIR type (using Cappellini window) and of recursive IIR type (using transformations). Further a special technique, introduced by the author, for high-speed digital acquisition and processing with maximum efficiency, that is using the minimum number of sampled signal data for the required processing, is described. This technique using a single digital filter and a frequency shift procedure of the sampled signal spectrum, is particularly useful for performing a band-pass analysis or a spectral estimation with a tree-structure organization. Hardware and software implementations of the above technique are presented with typical application examples.

INTRODUCTION

Digital filtering is a very flexible and efficient technique for processing signals and images, in particular for underwater research. This technique can be useful in real-time general-purpose high-speed signal processing systems , both for reducing the amount of data and to process the important data according to any frequency characteristic which is required.

The reduction of the amount of data can be obtained through low-pass or band-pass digital filtering and subsequent "decimation" or, if the frequency behaviour of the signal is essentially required, through suitable spectral estimation algorithms.

In the following some high efficiency one-dimensional (1-D) and two-dimensional (2-D) digital filters are described. Hence a special technique is presented for high-speed digital acquisition and processing with maximum efficiency, that is using the minimum number of sampled data for the required processing. Hardware and software implementations are described with typical examples of results obtained in processing real signals having different frequency ranges.

1. SOME HIGH EFFICIENCY 1-D AND 2-D DIGITAL FILTERS

1-D digital filtering can be defined in general way by the following relation [1]

$$g(n) = \sum_{k=0}^{N-1} a(k)f(n-k) - \sum_{k=1}^{M-1} b(k)g(n-k) \tag{1}$$

where: $f(n)$, $g(n)$ are, respectively, the input and output data (samples of the input and output signal); $a(k)$ and $b(k)$ are the coefficients defining the digital filter (its frequency response); N and M are two integers.

If all $b(k)$ coefficients are equal to zero, the digital filter is called of finite impulse response (FIR) and the implementation structure is in general of non-recursive or trasversal type. If at least one $b(k)$ coefficient is different from zero, the digital filter is called of infinite impulse response (IIR) and the implementation structure is in general of recursive type [1].

2-D digital filtering can be defined in analogous way by the following general relation

$$g(n_1,n_2) = \sum_{k_1=0}^{N_1-1} \sum_{k_2=0}^{N_2-1} a(k_1,k_2)f(n_1-k_1,n_2-k_2) - \sum_{m_1=0}^{M_1-1} \sum_{\substack{m_2=0 \\ m_1+m_2\neq 0}}^{M_2-1} b(m_1,m_2)g(n_1-m_1,n_2-m_2) \tag{2}$$

where: $f(n_1,n_2)$ and $g(n_1,n_2)$ are, respectively, the input and output data (samples of the original image and processed image); $a(k_1,k_2)$ and $b(m_1,m_2)$ are the coefficients defining the digital filter (its frequency response); N_1,N_2,M_1 and M_2 are suitable integer numbers [1].

Again, if all $b(m_1,m_2)$ coefficients are equal to zero, the 2-D digital filter is called of finite impulse response (FIR) and the implementation structure is in general of non-recursive or transversal type. If at least one $b(m_1,m_2)$ coefficient is different from zero, the 2-D digital filter is called of infinite impulse response (IIR) and the implementation structure is in general of recursive type.

Many methods have been proposed and defined to design and implement FIR and IIR 1-D and 2-D digital filters [1] . Few methods are briefly recalled in the following, having good efficiency and relatively simple design and implementation properties.

For FIR digital filters (1-D and 2-D) a useful method is based on the use of suitable "window functions" to define the coefficients $a(k)$ or $a(k_1,k_2)$. Indeed the problem of designing a digital filter in the frequency domain is connected to the evaluation of the coefficients in such a way that the obtained frequency response satisfies the required characteristics. In the window method we start from an impulse response, which is the inverse Fourier Transform of the desired filter frequency response: this impulse response, in sampled form $h(k)$ or $h(k_1,k_2)$, is to be truncated for the practical implementation of the digital filter introducing the minimum error in the frequency response. To this purpose the values $h(k)$ or $h(k_1,k_2)$ are multiplied by the samples $w(k)$ or $w(k_1,k_2)$ of a suitable window function, having zero value in the region out of the truncation and a very high concentration in the frequency domain [1] . The obtained frequency response of the digital filter is therefore the convolution between $H(\omega)$ or $H(\omega_1,\omega_2)$, Discrete Fourier Transform (DFT) of $h(k)$ or $h(k_1,k_2)$, and $W(\omega)$ or $W(\omega_1,\omega_2)$, DFT of $w(k)$ or $w(k_1,k_2)$.

Many window functions are known for the design of FIR digital filters. If one window function is known for 1-D case, the 2-D extension is easily obtained, as shown by Huang [2] ,(with circular symmetry properties), by

$$w(x,y) = w\sqrt{(x^2 + y^2)} \tag{3}$$

Two window functions of particular interest are: the Lanczos-extension window (by Cappellini) having 1-D expression

$$w_{LC}(t) = \left(\frac{\sin(\pi t/\tau)}{\pi t/\tau} \right)^m \tag{4}$$

being zero for $|t| > \tau$ with m a positive number controlling the shape and the Weber-type window, having the property of giving a minimum value of the uncertainty product suitably modified. The expression of this last window is somewhat complicate: a practical approximation to it has been however defined by Cappellini [1] through a polynomial representation (up to the third order).

For IIR digital -filters (1-D and 2-D) the design must consider also the stability problem, due to the feedback action carried out by the coefficients $b(k)$ or $b(k_1,k_2)$. For 1-D case, techniques of transformation from the continuous domain (as the bilinear transformation) can be used. For 2-D case the design is more complicate and involved, because in the 2-D case it is not possible in general to factorize the z-transform $H(z_1,z_2)$, due to the lack of an appropriate factorization theorem of algebra: some approaches are however available making use of transformation from 1-D to 2-D or division of $H(z_1,z_2)$ in two or four quadrants [1] . An efficient method of this type was defined by Bernabo', Cappellini and Emiliani [1] .

2. MULTIPLE FAST PROCESSING BY MEANS OF DIGITAL FILTERING AND DECIMATION PROCEDURE

Digital filtering is very useful to perform multiple processing in efficient way, with the possibility to make any frequency operation (e.g. low--pass filtering for frequency band-limiting, high-pass or band-pass filtering for extracting some frequency band of interest...).

At first, considering a single sampled signal, it is possible with digital filtering of low-pass or band-pass type to extraxt only the frequency band of interest: a reduction of the sample number can be hence perfomed to the minimum value required by the sampling theorem [1] .

Multiple fast processing can be performed by using digital filtering as above and sample number reduction; two more interesting situations are: multiple filtering (as band-pass analysis) on a single sampled signal, a required filtering (low-pass, band-pass, high-pass) on several sampled signals.

A system of the second type, processing multiple signals (channels 1...n), with data compression (reduction of the sample number) and adaptive operation is shown - in block-diagram - in Fig. 1. Several sampled signals or data channels are digitally pre-filtered (low-pass or band-pass) in such a way to reduce the data to the minimum value required by the sampling theorem. The filtered data are sent to data compressors (constituted in the simplest form by sample number reduction or decimation, or by prediction--interpolation algorithms, variable length encoding, transformations in other forms [3]): the compressed data are finally sent to output buffers, where the remaining important data are eventually reorganized at constant time intervals to achieve bandwidth compression. The adaptivity is obtained by varying the bandwidth of the digital filters (in general at constant time intervals) by means of a "control circuit" which continuously measures the state of fullness of the output buffers, avoiding the undesirable situations of "overflow" and "underflow" [3] .

A general block-diagram of a system of the first type, performing multiple processing on a single signal, is shown in Fig. 2: the input unit to perform sampling and analog-to-digital (A-D) conversion and the output unit to make digital-to-analog conversion (D-A) and signal reconstruction are also enclosed. The programmer controls the arithmetic unit to perform the different digital filtering operations (of FIR type, using only input data) exchanging data with the memory (containing both the coefficients of digital filtering and eventually output data before to go to final reconstruction block).A multiple processing case of particular interest is represented by the band-pass analysis to be performed on a single input signal. Two methods on this line are described in the following.

A method uses a single digital filter to perform proportional bandwidth band-pass analysis. To illustrate the method, consider the Fig. 3, in which the frequency response of a band-pass digital filter is represented with reference to the maximum frequency of the input signal (ω_M): the band of the digital filter is set between ω/ω_M =1/8 and ω/ω_M = 1/4. In a first filtering the band of the signal between ω/ω_M= 1/8 and ω/ω_M = 1/4 is obtained; hence a time expansion of a factor 2 is applied on the input signal (as shown in Fig. 4) which corresponds to a frequency compression of the sampled signal spectrum of a factor 1/2: by using the same digital filter the band of the signal between ω/ω_M = 1/4 and ω/ω_M = 1/2 is obtained. In a third filtering a time expansion of a factor 4 is applied on the input signal (as shown in Fig. 4) and by using the same digital filter the band of the signal between $\omega/\omega_M = 1/2$ and $\omega/\omega_M = 1$ is obtained. To perform an analysis on a greater number of proportional bands, it is sufficient to set the band-pass digital filter at a lower frequency band with reference to the maximum frequency of the signal to be analysed.

Another method particularly efficient and fast uses a single low-pass digital filter and a frequency shift operation on the sampled signal spectrum, by multiplying the input data by an exponential factor. Indeed if an angular frequency shift ω_i is performed on the sampled signal spectrum $F(j\omega)$ to produce a spectrum $F(j\omega - j\omega_i)$, then this corresponds to a new sampled signal of the form (with T the sampling interval)

$$f_i(n) = f(n)e^{jn\omega_i T} \tag{5}$$

If ω_M is the signal maximum angular frequency, then angular frequency shifts $\omega_i = i(\omega_M/M)$, $i = 1,\dots, M-1$, are required to be operated on the signal so as to have M-band analysis by using a single low-pass filter of angular cut-off frequency $\omega_c = \omega_M/M$. It is clear that all the original spectrum frequency bands will be shifted to the low- frequency aband $(0,\omega_M/M)$. In this case the arithmetic unit (of Fig. 2) has to perform complex multiplications on $f_i(n)$ using the coefficients of the single low-pass digital filter [1].

On this line a special method has been proposed by Cappellini [1] [4], using the values $\omega_i = \omega_s/2$ with $\omega_s = 2\pi/T = 2\pi f_s$ as the angular sampling frequency. It is easy now to verify that the relation (5) becomes very simple as

$$f_i(n) = f(n)(-1)^n \tag{6}$$

which implies only an alternative sign change. If moreover $\omega_i = 2\omega_M$ and $\omega_c = \omega_M/2$, the following situation is obtained. Low-pass filtering of the original signal $f(n)$ produces the sampled signal $f_1(n)$ which corresponds to the original spectrum band $(0,\omega_M/2)$, while low-pass filtering of $f(n)(-1)^n$ produces the sampled signal $f_2(n)$ corresponding to the original spectrum band $(\omega_M/2,\omega_M)$, shifted and inverted within the frequency band $(0,\omega_M/2)$. This procedure can be applied again to the signals $f_1(n)$ and $f_2(n)$ by considering only one of two consecutive samples (i.e. involving sample reduction or decimation) so that the preceding relation (6) can be still valid if an angular frequency shift equal to $\omega_M/2 = \omega_s/4$ is used. Then four bands can be obtained. Therefore this method can separate, in a tree structure; the signal spectrum in a rapidly increasing number of bands. In general if r successive operations are performed, then 2^r bands are obtained.

The band-pass analysis resulting from the above procedure presents the following advantages with respect to the normal band-pass filtering:

(1) the band-pass analysis can be obtained by using a unique digital filter without changing its coefficients (depending only, as in FIR filters, on the number of processed samples and on the ratio ω_s/ω_c);
(2) high and constant efficiency of digital filtering;
(3) the speed of analysis is increased because at each step the minimum number of samples connected to the sampling theorem is processed and also because the analysis can be stopped in those parts of the frequency spectrum having zero value or which are of no interest;

(4) due to the sample reduction procedure a compressed representation of each analysed passband is obtained with fewer samples than normally needed, making faster subsequent digital processing operations;
(5) the practical implementation can be relatively simple because the coefficient memory contains only one coefficient set and the operation $f(n)(-1)^n$ is very simple, as shown in the block diagram of Fig. 5.

By performing the r.m.s. evaluation of the different band outputs (in a given time interval T_m) a short-time spectral estimation can be easily obtained through the above technique.

An hardware implementation of the above technique was realized, including the r.m.s. evaluation [1] [5] : an example of processing an audio signal is shown in Fig. 6, reporting the spectral estimation up to a resolution of 16 bands (r = 4).

A software implementation of the same technique was also implemented: an example of processing an audio signal is shown in Fig. 7 (a) with spectral estimation having a resolution up to 32 bands (r = 5). As comparison in Fig. 7 (b) the spectral estimation through the standard Fast Fourier Transform (FFT) is also shown : the good agreement of the two methods in giving the desired result is clearly appearing. The advantages of the presented technique in comparison with FFT are connected to the above considerations (from (1) to (5)) and to the great flexibility in obtaining spectral estimations in the desired time interval. These advantages are particularly relevant when both a band-pass analysis (with output samples corresponding to the different bands) and a spectral estimation are required, because spectral estimation is requiring simple and fast computation (r.m.s. evaluation).

CONCLUSIONS

The described digital processing techniques, using in different ways digital filtering operations, are interesting for high-speed digital acquisition and processing of signals and images (several presented techniques can be easily extended to image processing), in particular for underwater research.

A reduction of the amount of data to the minimum value required by the sampling theorem can be obtained through low-pass or band-pass digital filtering and subsequent decimation. Multiple efficient processing can be performed, particularly convenient - as obtained through the special technique described - when both band-pass analysis and spectral estimation are required, as confirmed by the presented examples of signal processing by means of hardware and software implementations.

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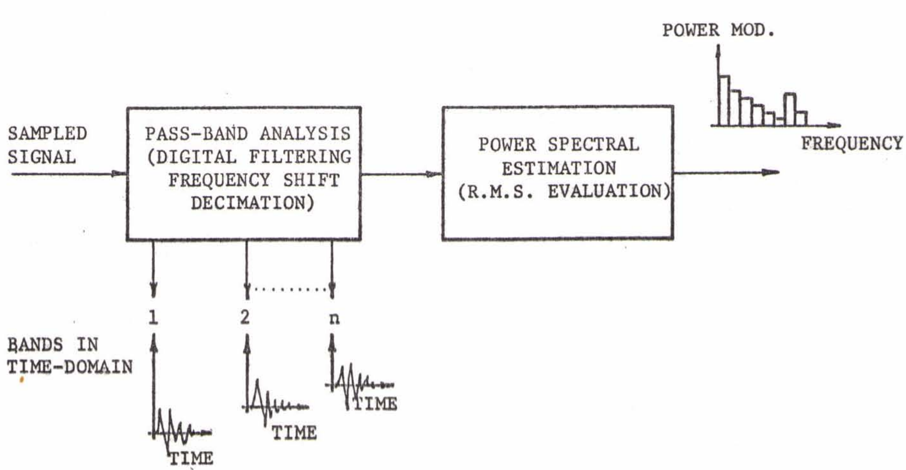
DISCUSSION

T.E. Curtis Is this decimation technique equivalent to a Radix N DFT? Does it require a comparable number of operations (i.e. multiply/accumulate cycle)?

V. Cappellini The decimation technique is near to a Radix N DFT. The overall procedure presented — consisting of a low-pass digital filter, frequency shift of the sampled signal spectrum, and decimation — is quite different, because here the first processing result is a time-domain sampled signal for each pass band from which, if required, a spectral estimation can be obtained (in particular, power spectral estimation through r.m.s. evaluation of each band output).

The number of operations involved can be comparable, if the number of coefficients of the FIR low-pass digital filter is suitably selected. (If the low-pass digital filter is selected with high efficiency — low ripple in band, high cutoff frequency, and low fluctuation out of band — the number of operations here can be higher but a more efficient band separation can be obtained.)

The described technique is of particular interest when a time-domain pass-band analysis in compressed form (that is, each signal in a band is represented through the minimum number of samples) is initially required and also a power spectral estimation is to be performed.



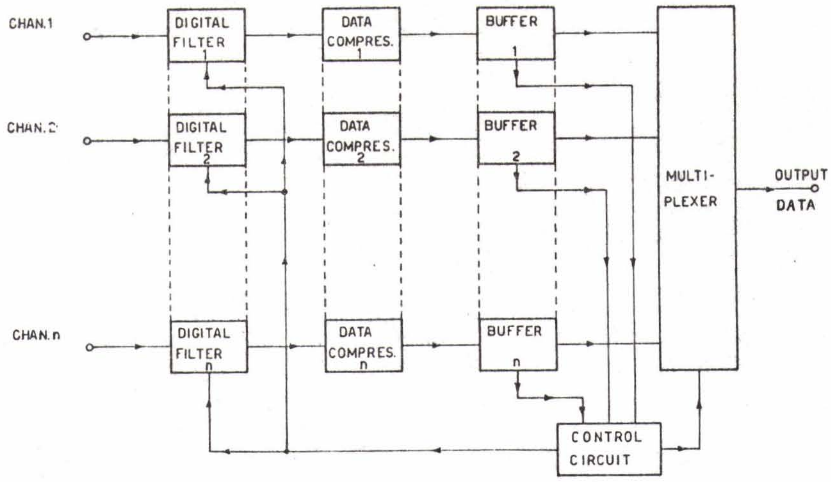


FIG. 1 BLOCK DIAGRAM OF A MULTIPLE PROCESSING SYSTEM WITH DIGITAL FILTERING AND DATA COMPRESSION OPERATIONS UNDER ADAPTIVE CONTROL BY A CONTROL CIRCUIT.

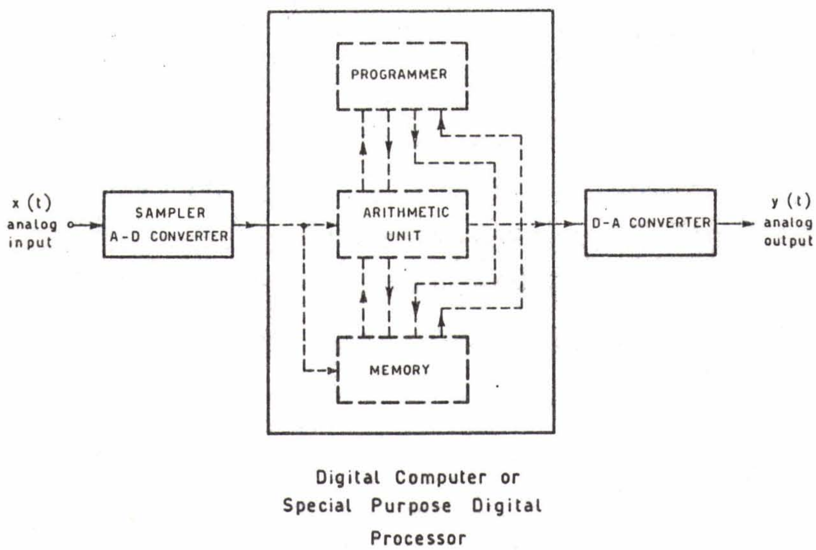


FIG. 2 BLOCK DIAGRAM OF SYSTEM PERFORMING MULTIPLE PROCESSING ON A SINGLE SIGNAL WITH INPUT-OUTPUT UNITS.

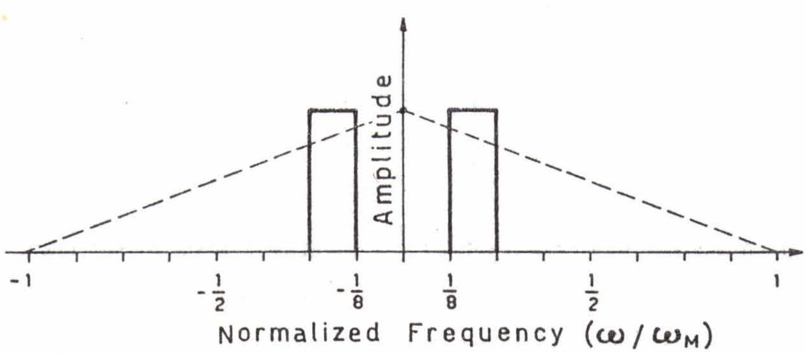


FIG. 3 FREQUENCY RESPONSE OF A BAND-PASS DIGITAL FILTER FOR PROPORTIONAL BANDWIDTH BAND-PASS ANALYSIS IN RELATION TO THE MAXIMUM FREQUENCY OF THE SAMPLED SIGNAL SPECTRUM ω_M.

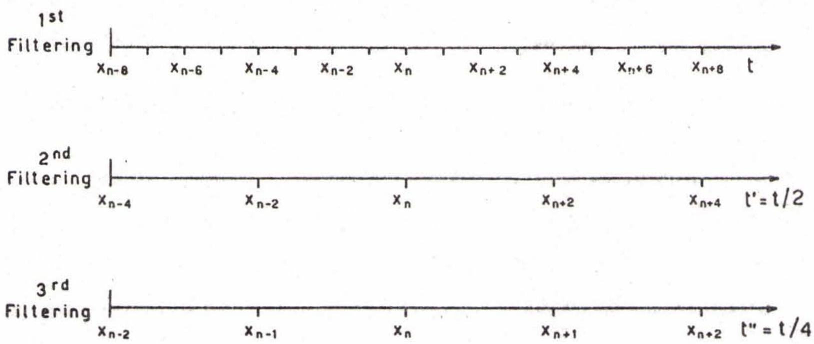


FIG. 4 SIGNAL SAMPLE POSITION IN TIME FOR PROPORTIONAL BANDWIDTH BAND-PASS ANALYSIS (TIME EXPANSION IN 2ND AND 3RD FILTERING)

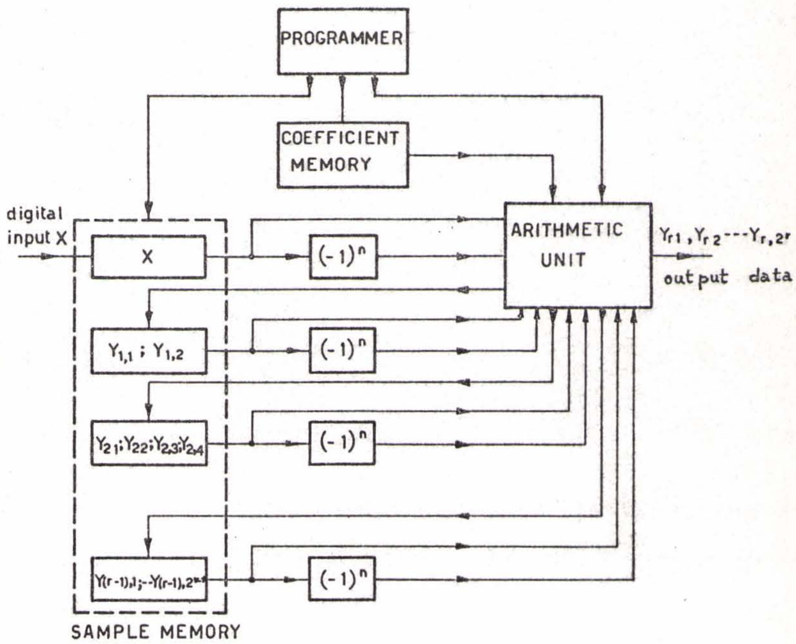


FIG. 5 BLOCK DIAGRAM OF HARDWARE IMPLEMENTATION OF THE DIGITAL PROCESSORS PERFORMING BAND-PASS ANALYSIS (CONSTANT BANDWIDTH) BY USING A SINGLE LOW-PASS DIGITAL FILTER AND A FREQUENCY SHIFT PROCEDURE OF THE SAMPLED SIGNAL SPECTRUM WITH DECIMATION

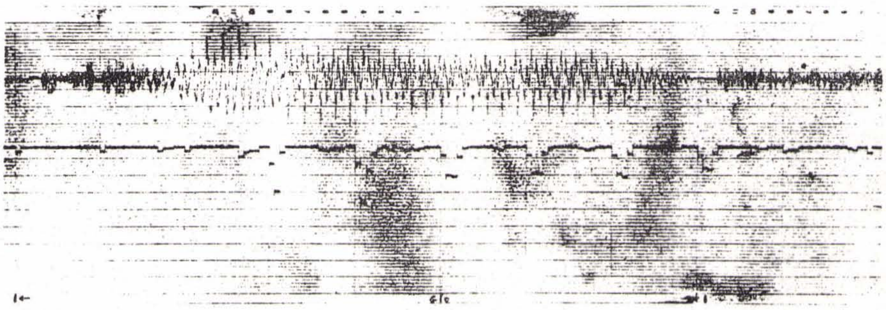


FIG. 6 EXAMPLE OF ON-LINE BAND-PASS ANALYSIS WITH SPECTRAL ESTIMATION ON AUDIO SIGNAL ("GIO") BY MEANS OF THE HARDWARE PROCESSOR: AT TOP - INPUT ANALOG SIGNAL; AT BOTTOM - SUCCESSIVE SPECTRAL ESTIMATIONS REPRESENTED BY 16 r.m.s. VALUES, ONE FOR EACH BAND, IN THE FREQUENCY RANGE 0-4 kHz.

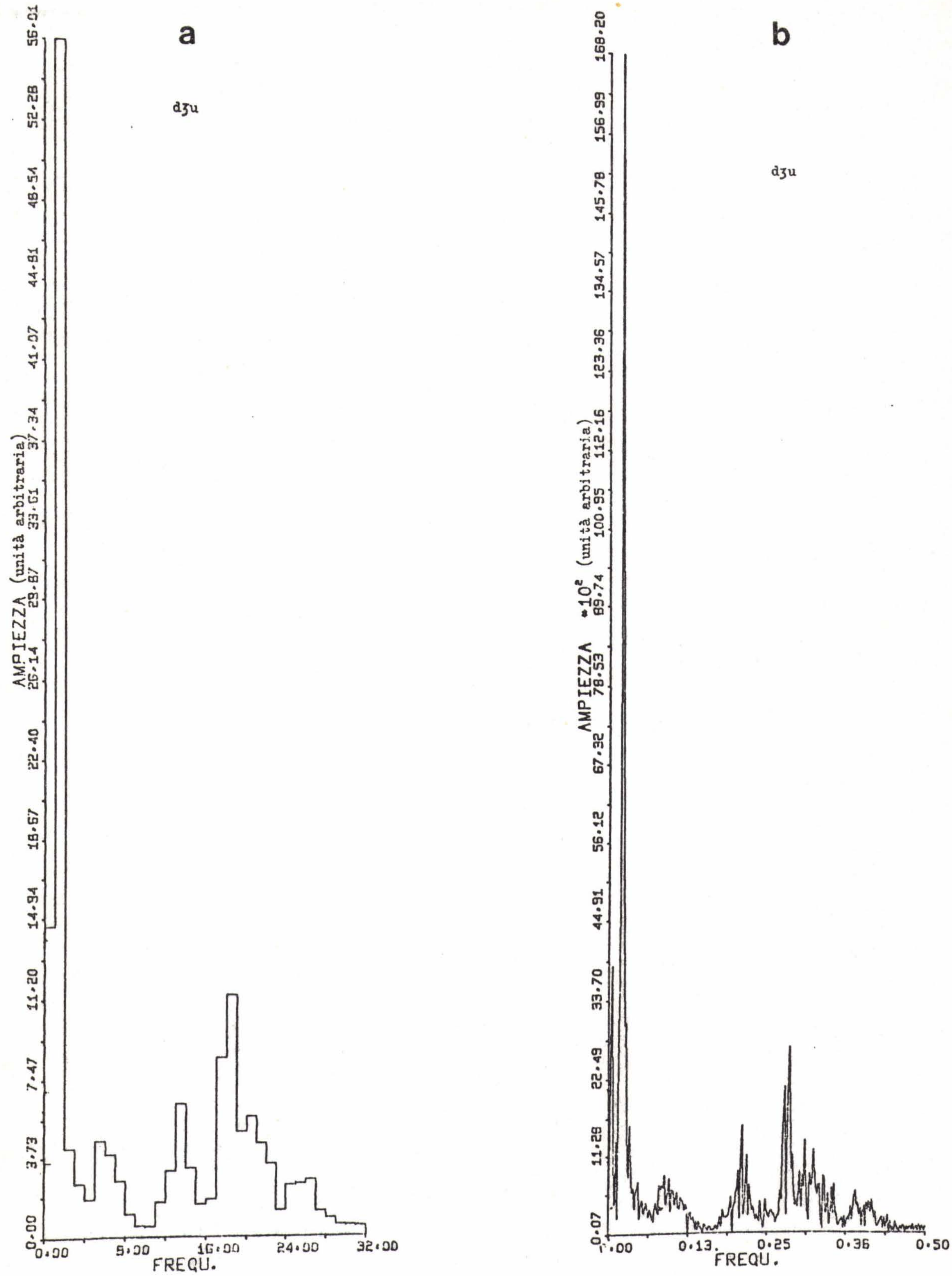


FIG. 7 EXAMPLE OF SOFTWARE IMPLEMENTATION OF THE BAND-PASS ANALYSIS AND SPECTRAL ESTIMATION METHOD AS IN FIG. 5: (a) SPECTRAL ESTIMATION (REPRESENTED BY r.m.s. VALUES) WITH 32 BANDS OF THE AUDIO SIGNAL ("DZU"); (b) SPECTRAL ESTIMATION OF THE SAME AUDIO SIGNAL BY MEANS OF THE STANDARD FFT WITH 512 POINTS.