AN INTRODUCTION TO ADAPTIVE ARRAY PROCESSING

by

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1. INTRODUCTION

Array processing systems which can respond to an unknown interference environment are currently of considerable interest. The fundamentals of such systems are by no means new - basically they depend on Weiner's filter theory - but application in practice has been limited both by technology and by the lack of robust algorithms rapid enough for real time operation. Rapid strides in the past decade or so in the twin fields of electronic components and computer technology have changed the situation considerably by offering the possibility of complicated signal processing in real time at economic costs. This has led to an increased interest in adaptive processing and although it is probably still finding its main application in the defence field the civilian applications are growing. As a consequence the literature on this subject has expanded rapidly but as this paper is intended for presentation rather than wide publication the number of references has been kept to a minimum. Ample references - and indeed good papers - can be found in the special issue of the IEEE Transactions[1] devoted to this subject.

An array comprises a set of sensors, the outputs of which are combined in some way to produce a desired effect, e.g. a set of beams 'looking' in various directions. The sensors may be of many forms, e.g. acoustic transducers for sonar, monopole aerials for h.f. reception, microwave horns in a radar system, the influence of the application on the processing required concerns the technology rather than the principles involved. Sensors may be distributed in space in various ways but the two most common are the linear array, normally a set of equally spaced sensors in a straight line and the circular array in which the sensors are arranged uniformly in a circular pattern. Although the distribution of sensors obviously affects the problem the effect in general is only of second order. The linear array, apart from being the most common in practice, is also the simplest to describe and understand and will be used in this paper as the basic model. The introduction to the theory will also be limited to 'band pass' systems, i.e. systems in which the signals can be described in terms of a carrier with a complex amplitude. In the theory the carrier will thus not be explicit and the calculations will be based on the complex numbers representing the amplitude and phase at any time.

In general wide-band systems can be dealt with by using an F.F.T. processor to provide a set of narrow-band systems to which the adaptive method is applied individually.

2. BASIC SYSTEM

The system which we are considering is shown in Fig. 1 and comprises a linear array of K equally spaced elements, the outputs of which are individually weighted before being summed. Basically we wish to adjust the weights in an adaptive manner such that the ratio of the output due to the signal (or noise) being received from a given direction compared to that due to the signals (or noise) arriving from other directions is optimised. We will define the output as

$$y = \sum_{i=1}^{K} w_i x_i \qquad \text{.......................} \quad (1)$$

where x_i are the individual element voltages and w_i the weights. The values of x_i will be real for a simple low pass system but for the more common bandpass systems the values will be complex since they represent the amplitude and phase of the received signals. In general the weights will be complex and hence also the output y. In practice we may be dealing with continuous or sampled values of the variables but since in the first part of the analysis we shall be using averages or expectation of the variable the method applies to either sampled or continuous systems.

For simplicity it is convenient to use vector notation and so we define the output as

$$y = W^{\dagger} X = X^{T} W^{*} \qquad \text{.......................} \quad (2)$$

where

$$W \triangleq \begin{vmatrix} w_1^{*} \\ w_2^{*} \\ | \\ | \\ | \\ w_K^{*} \end{vmatrix} \quad \text{and} \quad X \triangleq \begin{vmatrix} x_1 \\ x_2 \\ | \\ | \\ | \\ x_K \end{vmatrix}$$

T indicates the transpose
† indicates the complex transpose
* is a conjugate without transpose

The mean output power is given by

$$\begin{aligned} P &= E\left[|y|^2\right] = E\left[y \cdot y^{*}\right] \\ &= E\left[W^{\dagger} X \,.\, X^{\dagger} W\right] \\ &= W^{\dagger} R_{xx} W \qquad \text{.......................} \quad (3) \end{aligned}$$

where $R_{xx} \triangleq E\left[X \,.\, X^{\dagger}\right]$

R_{xx} is known as the Correlation Matrix. If the variables x_i have zero mean then R_{xx} can also be referred to as the Covariance Matrix.

3. STEERING VECTOR CONSTRAINT

If we know what is the desired output waveform y_D we may minimise the mean square error between y and y_D and in some systems this is possible. We shall be considering situations in which we do not know y_D and hence we need to constrain the system in some manner. One simple method is to constrain the gain in the wanted direction. This can be achieved by defining a steering vector C which is basically a vector representing the signals which would be present on the elements if a plane wave was arriving from the wanted direction θ viz:-

$$C^{\dagger} \triangleq \left[1,\ e^{-j\phi},\ e^{-j2\phi}\ \ldots\ldots\ldots\ e^{-j(K-1)\phi} \right] \qquad \ldots\ldots\ldots\ldots\ldots \ (4)$$

where $\phi = \dfrac{2\pi d}{\lambda} \sin\theta$

d = spacing between elements

λ = wavelength of plane wave signal

We wish to constrain the system such that the output from this signal is unity thus

$$W^{\dagger} C = 1 \qquad \ldots\ldots\ldots\ldots\ldots \ (5)$$

Since this constraint maintains the gain to the wanted signal constant we can now minimise the output subject to this constraint. To do this we use the method of Lagrange Multipliers and generate a cost function.

$$H(W) = P + \lambda\,(1 - W^{\dagger} \cdot C) \qquad \ldots\ldots\ldots\ldots\ldots \ (6)$$

Differentiating w.r.t. to W gives

$$\nabla H(W) = 2\, R_{xx} \cdot W - \lambda \cdot C \qquad \ldots\ldots\ldots\ldots\ldots \ (7)$$

Equating to zero and using the constraint to determine λ we get an expression for the optimal weight vector W_o.

i.e.
$$W_o = \frac{R_{xx}^{-1} \cdot C}{C^{\dagger} \cdot R_{xx}^{-1} \cdot C} \qquad \ldots\ldots\ldots\ldots\ldots \ (8)$$

Substitution of this optimal solution in the equation for the output power gives up the value for the optimal output power P_o.

i.e. $$P_o = \frac{1}{C^{\dagger} R_{xx}^{-1} C} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \text{(9)}$$

Hence $$W_o = P_o \cdot R_{xx}^{-1} \cdot C \qquad \ldots\ldots\ldots\ldots\ldots\ldots \text{(10)}$$

Fig. 2 shows the operation of a two element system in which the weights and inputs are assumed to be real. Although this is a gross over-simplification it does, however, illustrate a number of aspects of the behaviour of the more complicated systems. As will be seen from the diagram the constraint equation $W^{\dagger}C = 1$ defines a line (a surface in the multi-dimensional case) and the tip of the weight vector must lie on this line in order to maintain the gain as unity to the steering vector. The conventional weight vector lies along the direction of the steering vector as is shown. An interference vector can represent an unwanted signal arriving from a particular direction and the optimal solution is when the weight vector is in a direction orthogonal to that of the interference vector, i.e. when the output due to the inner product of these two vectors is zero. An illustrative example is shown in the figure.

This diagram can also be used to illustrate a problem that results from this simple method of constraint. It can be seen that unless the wanted signal is arriving from exactly the direction of the steering vector then in theory it will be nulled out. In fact, of course, uncorrelated noise is always present and as the length of the weighting vector increases the contribution from such noise in the output will increase. This then puts an effective limit on the magnitude of the weight vector. Fig. 3 shows the effective beam pattern of a simple constraint system for various signal/uncorrelated noise ratios. It can be seen that when the signal/background noise is high the beam is very narrow compared to the conventional beam formed by equal weighting of the element outputs.

4. THE REAL TIME PROBLEM

It can be shown that the solution derived in the last section is analogous to the matched filter approach for detecting a pulse in coloured noise. Thus if we are dealing with a stationary system and are not in a hurry the solution is fairly straightforward. The strategy would be to steer the system in as many directions as required and calculate the optimum 'weight vector' for each direction. This would enable the power received from each direction to be measured under the optimal conditions. Normally in practice neither the condition of stationarity, nor of ample time, hold and what we require is a system which can adapt to changing conditions and preferably do this very rapidly.

The most obvious approach is to use a recursive method and to update the weights as new data is processed. We have seen that our criterion is to minimise the expected mean square of the output (ℓ.m.s.) subject to the constraint. If we imagine a multi-dimensional space whose ordinates are the weight vector components then for a given environmental situation, i.e. a particular group of incoming signal interference/noise, a set of contours can be drawn for a given power output. These will form a 'bowl' and we are trying to seek the minimum (bottom) of this bowl subject to the constraint requirement. Fig. 4 illustrates this point using the very simple situation of a weight vector comprising two real components. One of the standard methods of searching for the minimum is to move along the direction of steepest descent. We require, however, also to satisfy our constraint and so we approach the solution in a series of steps.

Step I. We calculate the gradient vector from Equation (3) viz:-

$$\Delta = 2R_{xx} W \qquad \text{.....................} \quad (11)$$

Step II. Using a projection operator we decompose this vector into two components, one of which is parallel to the constraint vector and the other is orthogonal.

Step III. We subtract from our present weight vector W_k a proportion β of the component of the gradient vector which is orthogonal to the constraint vector and hence the new weight vector W_{k+1} will maintain the required constraint condition.

$$W_{k+1} = W_k - \beta\{I - \frac{CC^\dagger}{C^\dagger C}\} \cdot R_{xx} W_k \qquad \text{.....................} \quad (12)$$

Fig. 5 illustrates this process for the simple two weight example.

Unfortunately in a practical situation we do not know R_{xx} since this is the expectation of $XX^\dagger$. However, we can make an estimate from the data available, i.e.

$$\hat{R}_{xx}(k) = \frac{1}{k} \sum_{i=1}^{k} X_i X_i^\dagger \qquad \text{.....................} \quad (13)$$

and use this in equation (12).

What appears to be a more drastic approximation but in fact has advantages is to use the present value $X_k X_k^\dagger$ as an estimate for R_{xx}. Substitution in Equation (12) gives

$$W_{k+1} = W_k - \beta\,\{I - \frac{CC^\dagger}{C^\dagger C}\}\, X_k X_k^\dagger W_k$$

but $X_k^\dagger W_k$ is the current output y_k

$$\therefore \quad W_{k+1} = W_k - \beta\,\{I - \frac{CC^\dagger}{C^\dagger C}\}\, X_k y_k$$

This method is known as the stochastic steepest descent algorithm and results in fairly simple hardware for implementation,[2] an example of which is given in Fig. 6.

5. SAMPLE MATRIX INVERSION (S.M.I.)

The sample covariance matrix is defined for zero-mean data as

$$\hat{R}_{xx} = \frac{1}{M} \sum_{i=1}^{M} X_i X_i^{\dagger}$$

where M is the number data samples in the observation interval. The block may be formed in several ways, M can be equal to the total number of data samples available, or the data can be divided into blocks of M samples, or the sample matrix can be generated by a sliding average of M samples. The value of SMI in adaptive beamforming in ill-conditioned interference environments was discussed by Reed[3] et al who proposed the estimate for the optimum weight vector as

$$\hat{W}_o = \frac{\hat{R}_{xx}^{-1} C}{C^{\dagger} R_{xx}^{-1} C}$$

This is, of course, the same result as was obtained for the constrained solution discussed earlier. To determine R_{xx} requires something of the order of K^3 operations but the computational disadvantage of having to determine R_{xx}^{-1} may be compensated by the fact that convergence is normally faster for ill- conditioned environments. Another advantage is that if the problem is such that we are required to determine the angular distribution of the power received rather than concentrate on the signal from one direction then once R_{xx}^{-1} has been calculated the computation to obtain the appropriate optimum vector for each direction is relatively trivial. It should be pointed out that if the data is dealt with in blocks some method has to be used to prevent signal suppression, e.g. addition of white noise or limitation on the weight vector norm,as discussed earlier. A comparison of the SMI technique with the stochastic steepest descent is illustrated for a three-jammer environment in Fig. 7.

6. CONCLUSION

This paper has dealt with a relatively limited problem and only some of the possible solutions. We have not discussed the effect of truncation and quantization in the processing nor the need for robustness in dealing with, for example, variations of the sensitivities or positions. Suffice it to say that these introduce further complications but are capable of analysis and control. There is no panacea since the 'best' solution varies according to the environment in which it is to be applied. In simulation it is fairly easy to set up data for which a particular algorithm works well but just as easy to produce data for which it does not!

7. ACKNOWLEDGEMENT

The author would like to acknowledge the many sources from which information has been obtained and in particular the assistance of his colleague, Dr. J.E. Hudson.

8. REFERENCES

(1) Special Issue on Adaptive Arrays, IEEE Trans. Ant. and Prop. 1963.

(2) Frost, O.L., "An algorithm for linearly constrained adaptive array processing", Proc. IEEE, Dec. 1967, No. 12, pp 2143-2159.

(3) Reed, I.S., Mallett, J.D., Brennan, L.E. "Rapid convergence rate in adaptive arrays", IEEE Trans. Aerospace AES10 (1974) pp. 853-863.

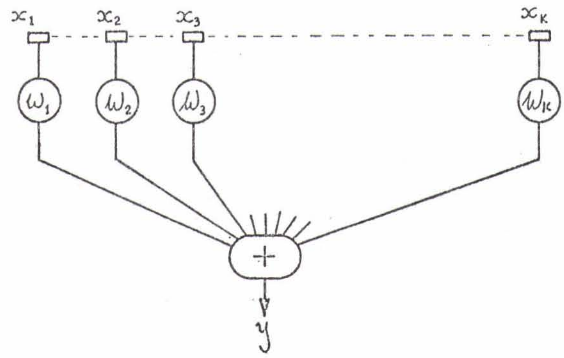
DISCUSSION

R. Seynaeve Can you comment on the accuracy required by the various algorithms?

J.W.R. Griffiths Depends on the problem. With the constraint technique, sensor inaccuracies in both position and gain can be tolerated. The number of bits depends on the noise "floor level".

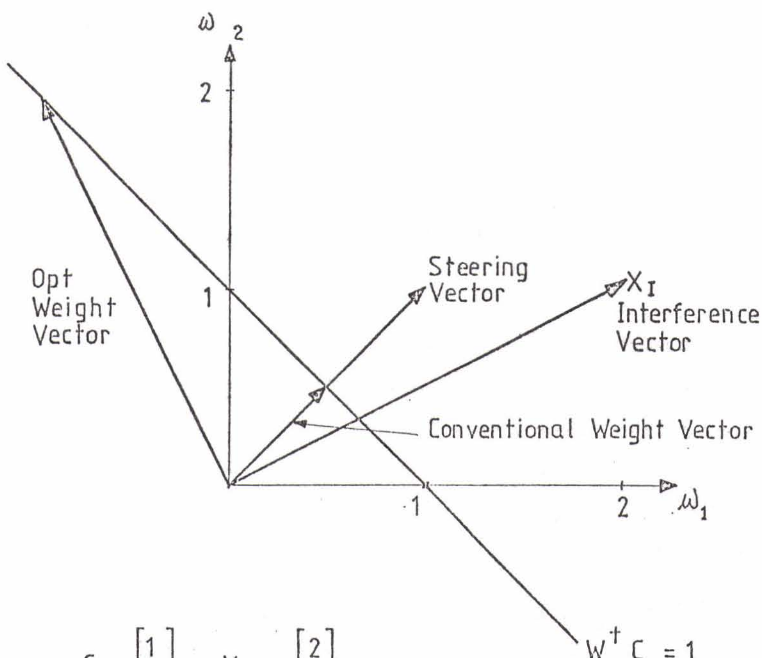
H.J. Alker In the U.K. are the real-time adaptive systems using steepest-descent or matrix-inversion techniques in the adaptive processing?

J.W.R. Griffiths Both are being considered.



Basic System.

FIG. 1



$$C = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad X_I = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$W_o = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \qquad W_o{}^t\, C = [-1, 2]\begin{bmatrix} 1 \\ 1 \end{bmatrix} = 1$$

$$W_o{}^t\, X_I = [-1, 2]\begin{bmatrix} 2 \\ 1 \end{bmatrix} = 0$$

Two element system with steering vector constraint.

FIG. 2

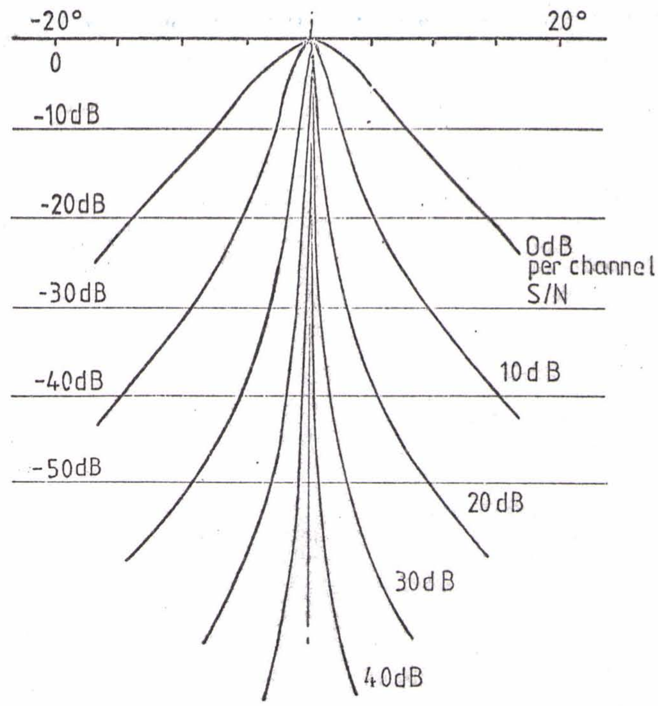


FIG. 3 Signal output power against angle. zero moment constraint, OdB signal.

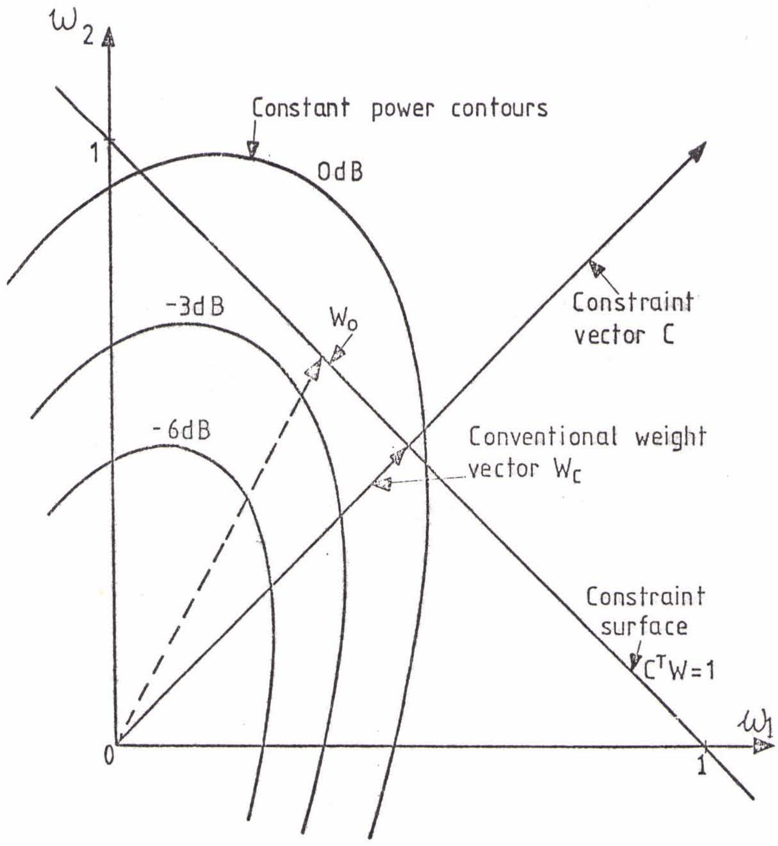
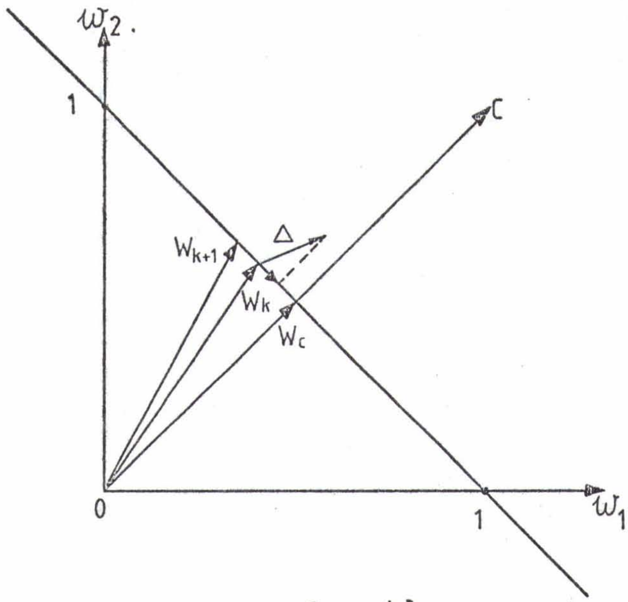


FIG. 4

Simple case showing equal power contours.



$$W_{k+1} = W_k - \beta\left[I - \frac{CC^{\dagger}}{C^{\dagger}C}\right]R_{xx}W_k$$

FIG. 5

Steepest descent algorithm .

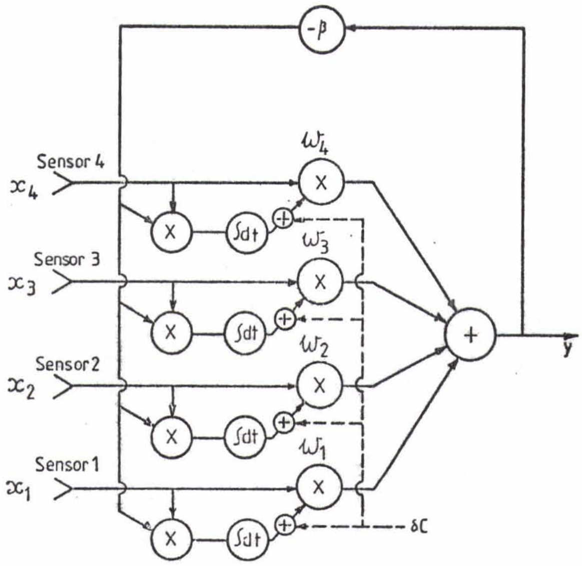
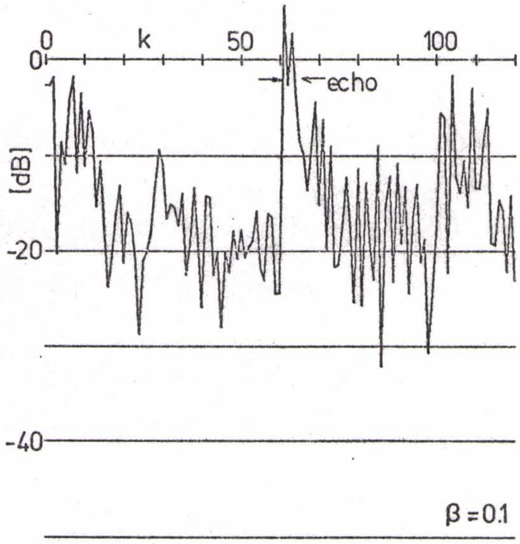
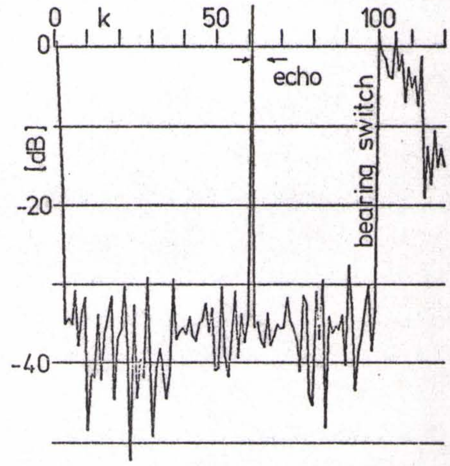


FIG. 6

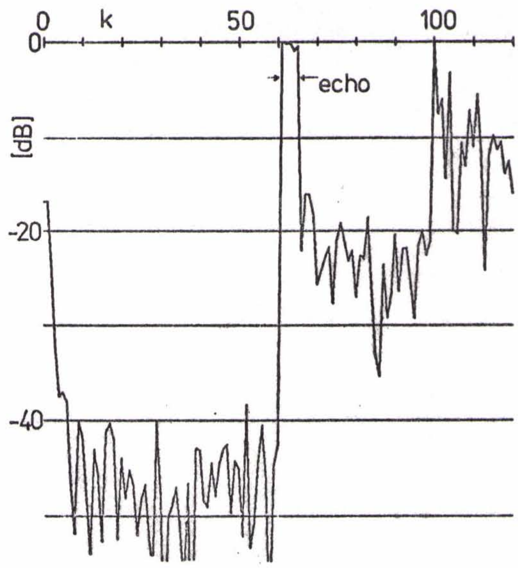
A Narrow-Band Array-Space Maximum Likelihood Circuit (After Frost.)



3 JAMMERS SIDELOBE CANCELLER
STOCHASTIC STEEPEST DESCENT.



3 JAMMERS: 0,-10, -20 dB NOISE:-40dB
N_e=5 SIDELOBE CANCELLER
SAMPLE MATRIX INVERSION



3 JAMMERS: 0,-10,-20 dB at-45°,-20°,+45°
N_e =5 NOISE 45 dB BROADSIDE CONSTR'NT.
SAMPLE MATRIX INVERSION.

FIG. 7