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MULTIPATH PULSE SHAPE IN A REFRACTING ENVIRONMENT

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Abstract: *Multiple boundary interactions in shallow water propagation introduce a more or less Gaussian angle distribution at a distant receiver. When all paths are added the result is 'mode-stripping', i.e. the steep rays are stripped away leaving only ray angles near horizontal. Because the steeper, more highly attenuated, rays are also more delayed, the same phenomenon results in a calculable time smearing of an impulse source. It has been shown* [Harrison, C.H., J. Acoust. Soc. Am., **114**, 2744-2756, (2003)] *that in isovelocity water the pulse tail falls off exponentially with a time constant that is dependent only on the derivative of the reflection loss and the water depth and is consequently independent of range. It is therefore a useful geoacoustic inversion tool. Here we investigate the effects of refraction, following a more recent paper* [Harrison, C.H., and Nielsen, P.L., J. Acoust. Soc. Am., **121**, 1362-1373, (2007)]. *Although the leading edge of the pulse is altered, the tail (resulting from steep arrivals) still behaves as in isovelocity water. Results are confirmed by comparisons with C-SNAP.*

Keywords: *Pulse shape, multipath propagation, refraction.*

1. INTRODUCTION

The multipaths of shallow water propagation spoil the resolution of active sonars by introducing a spread in travel times. The broadening of pulse transmission is therefore a nuisance to sonar detection and underwater communications. On the other hand it has been shown [1,2] that the pulse shape contains easily extractable environmental information. So for both reasons the shape of the pulse and its dependence on environmental properties are of interest. One could investigate these effects with ray traces, but here the more general behaviour is established by studying the pulse shape analytically.

In a multipath shallow water environment acoustic travel times and boundary losses vary according to the trajectories of the rays. In the absence of additional constraints, such as source or receiver beam patterns, or target vertical directionality this results in a calculable spreading of the transmitted pulse shape. If the sonar system has a broad frequency band then individual eigenray arrivals may be seen inside this spread. If it has a narrow band then interference effects make the rays group into modal arrivals. In both cases the arrivals tend to increase their separation as time advances. The mathematical approach here is insensitive to these detailed arrivals because it treats the ray angles as a continuum. Nevertheless it takes account of both their changing amplitudes and their changing separations in such a way that the cumulative time integral of the pulse shape matches the more “steppy” cumulative integral of the true eigenray or mode arrival pulse shape. The pulse envelope calculated here corresponds physically to a depth average or locally range-averaged pulse shape. The depth average is particularly close for the tail of the pulse where rays interact with both seabed and sea surface.

Smith [1] and Harrison [2] investigated this behaviour for isovelocity water and found that for a two-way path the received pulse decayed exponentially (i.e. the roll-off was a fixed number of dBs per unit time) with a time constant that was independent of travel time or range but fixed by the angle dependence of the reflection loss. Subsequently Prior and Harrison [3] applied the findings to experimental data, and demonstrated that the derived reflection properties were consistent with the literature.

This paper extends the earlier analytical work on pulse shape firstly by including the critical angle’s truncation of the pulse (trivial for one-way path but not trivial for two-way path), and secondly by including a uniform vertical gradient refraction using the approach of Harrison [4]. In both cases one-way and two-way paths are considered. The former would be appropriate for direct blast measurements while the latter would be appropriate for the multipath echo from a point target or an echo repeater.

Finally some comparisons in three environments with the normal mode model C-SNAP [5] demonstrate the relationship between the closed-form pulse shapes and the sequence of resolvable eigenray arrivals.

2. EIGENRAYS

Generally, representing a propagation formula as the integral of a quantity $Q(\nu)$ over a parameter ν, and knowing the conversion from ν to t one obtains a pulse shape I_{1way}

$$I_{1way}\, dt = \left\{ Q(\nu(t)) \left| \frac{d\nu(t)}{dt} \right| \right\} dt \tag{1}$$

The modulus sign is required for the case where t is not a single-valued function of v, for instance v could be angle. Integrating I_{1way} in t would, of course, give the same result as integrating $Q(v)$ in v. The pulse shape for the two-way path is given by the convolution of the one-way pulse shape I_{1way} with itself

$$I_{2way}\, dT = \int_0^T I_{1way}(t)\, I_{1way}(T-t)\, dt\, dT \tag{2}$$

The function I_{1way} may be discontinuous or split into several regions in each of which there is one continuous function. For a one-way path this is straightforward, but for the two-way path the convolution will contain several cross term contributions with various integration limits.

In the isovelocity one-way case at range r, water depth H, angle θ, and power reflection coefficient $\exp(-\alpha\theta)$, we have [4]

$$I_{1way}\, d\theta = \frac{2}{rH} \exp\left(-\frac{\alpha r \theta^2}{2H} \right) d\theta \tag{3}$$

but travel time t is related to horizontal range through $ct = r\sec\theta$ which leads to the pulse shape, in terms of τ, the time after first arrival

$$I_{1way}\, d\tau = \frac{2}{rH} \frac{\exp\{-\alpha\tau / t_H\}}{\sqrt{2 t_o \tau}} d\tau \tag{4}$$

where $t_o \equiv r/c$ and $t_H = H/c$. The truncation of this function at a time corresponding to the critical angle results in the two-way path having two distinct analytical forms. For $0 < T < \tau_c$ it is

$$I_{2way}\, dT = \frac{2\pi}{r^2 H^2 t_o} \exp\{-\alpha T / t_H\}\, dT \tag{5}$$

and for $\tau_c < T < 2\tau_c$ it is

$$I_{2way}\, dT = \frac{4}{r^2 H^2 t_o} \operatorname{asin}((2\tau_c / T) - 1)\, \exp\{-\alpha T / t_H\}\, dT \tag{6}$$

3. PULSE SHAPE WITH REFRACTION

Some approximate propagation formulas taking account of uniform sound speed gradient refraction were derived in the context of reverberation [4]. Intensity is written in terms of the reciprocal of the cycle distance u (instead of angle), but there are two distinct angle ranges, to be treated separately. In the first, rays interact with only one boundary; in the second they interact with both, and the relationship between angle and cycle distance is different in the two. Since the rays are always arcs of circles one can easily calculate exact cycle distances and cycle times analytically. It is shown by Harrison and Nielsen [6] that the contribution to the pulse shape in the first regime (following Eq.(1)) can be written in terms of properties on the low sound speed side of the duct (subscript 'L') as

$$\begin{aligned} I_1\, dt &= \frac{4}{r} \exp(-R_L r u) \frac{du}{dt} dt \\ &= \frac{4}{r}\, u \big/ \left(t - (r/c_L) \operatorname{sech}(tc'/(2ru)) \right) dt\, \exp\{-\alpha_L c' r / 2c_L\} \end{aligned} \tag{7}$$

whose functional form in t can be seen by interpolation since formulas relating t, u, and θ_L are available. An example of this is seen in the first rise at the left of Fig. 1.

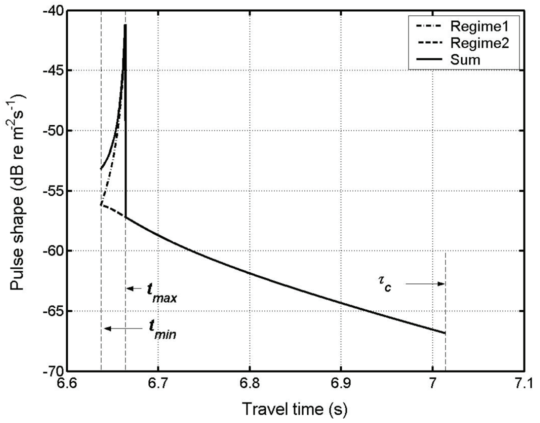

Fig. 1. Contributions to the pulse shape from refraction regime 1 (dash-dot), regime 2 (dashed), and their sum during the period in which they overlap (solid).

An approximate contribution to the pulse shape can be written as an explicit function of t (and $t_L = r/c_L$)

$$I_1\,dt = \frac{dt}{(t_L - t)^{3/2}}\left\{\frac{t_L^{3/2}\,c'}{\sqrt{6}\,r^2}\exp[-(\alpha_L c'/2c_L)r]\right\} \tag{8}$$

The equivalent pulse shape for the two-boundary-interacting component (denoting the properties at the high sound speed side by subscript 'H') is

$$\begin{aligned} I_2\,dt &= \frac{4}{r}\exp\{-(R_L + R_H)ru\}\frac{du}{dt}dt \\ &= \exp\{-2rHu^2(\alpha_L a_L + \alpha_H a_H)\}\frac{du}{dt}dt \\ &\quad\times\left\{\frac{4}{r}\exp\{-c'r(\alpha_L/c_L - \alpha_H/c_H)/4\}\right\} \end{aligned} \tag{9}$$

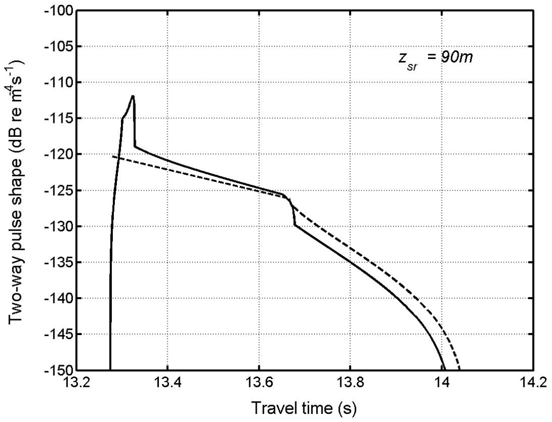

Fig. 2. Two-way pulse shape (solid) for parameters as in Fig. 1 except that z_{sr} (depth of the shallowest of source and receiver) takes the value 90m. Isovelocity equivalent for average sound speed superimposed (dashed).

Again this can be evaluated by interpolation since formulas are available for du/dt, and the result is seen on the right of Fig. 1. This can be written approximately as an explicit function of t as

$$I_2\, dt = \frac{t}{\sqrt{t^2 - t_0^{\,2}}} \exp(-a_1 t - a_0)\ \frac{1}{t_0} dt\, f_0 \tag{10}$$

where a_0, a_1, f_0, t_0 are constants. The effect of a two-way path (calculated by convolution) is shown in Fig. 2.

4. RULES OF THUMB

It was shown [6] that the dimensions and shape of the typical sail shape of the pulse (Fig. 1) can be quantified by two rules of thumb. First, the characteristic time spread of the initial rise T_o; second, the amplitude ratio of the rise F_o.

$$T_o = t_{\max} - t_{\min} = \frac{r}{6 c_L^{\,3}} (c_H^{\,2} - c_{sr}^{\,2}) \approx \frac{r z_{sr} c'}{3 c_L^{\,2}} \approx \frac{1}{3}\, t_L \frac{\delta c_{sr}}{c_L} \tag{11}$$

$$F_o = \left(\frac{c_H^{\,2} - c_L^{\,2}}{c_{sr}^{\,2} - c_L^{\,2}} \right)^{3/2} = \left(\frac{H (c_H + c_L)}{(H - z_{sr})(c_{sr} + c_L)} \right)^{3/2} \approx \left(\frac{H}{(H - z_{sr})} \right)^{3/2} = \left(\frac{H}{h_{sr}} \right)^{3/2} \tag{12}$$

Both depend on the angular spread of regime 1 which depends on c_{sr} = MAX(c_s, c_r), the greater of the speeds at source and receiver.

5. COMPARISON WITH A WAVE MODEL

Figure 3 shows the closed-form solutions superimposed on the depth average of the C-SNAP isovelocity solution. The dashed line is the linear reflection loss approximation using Eq. (4). However this approximation is not necessary and we can substitute R as in the first line of Eqs. (7) or (9) to give the solid line which agrees with C-SNAP.

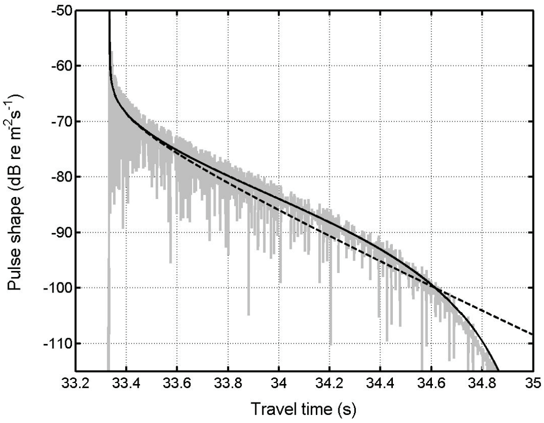

Fig. 3. Depth averaged pulse shape computed by C-SNAP for isovelocity water overlaying a half-space seabed defined by parameters in [6] (grey). Superimposed are the analytical solutions with Rayleigh reflection coefficient (black solid), and linear approximation (dashed).

The uniform gradient refraction example in Fig. 4 shows the analytical solution with the given R_L superimposed on the depth-averaged C-SNAP curve. Although agreement is good for the tail of the pulse it can be improved for the early portion by including depth averaged focussing effects (dashed line).

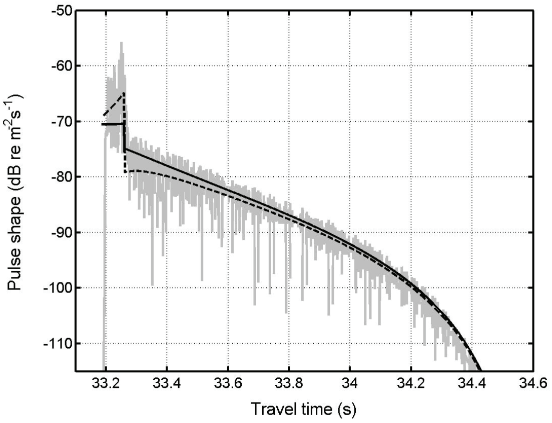

Fig. 4. Depth averaged pulse shape computed by C-SNAP for uniform sound speed gradient overlaying a half-space seabed defined by parameters in [6] (grey). Superimposed are the analytical solutions (also depth averaged) with Rayleigh reflection coefficient, excluding (black solid), and including focusing effects (dashed).

Using the cycle distance and travel time formulas piece-wise one can construct the components of Eq. (1), and therefore a pulse shape, numerically. Figure 5 shows an example for a three layer summer profile. Again agreement is good for the tail, but the early part is improved by including focussing effects.

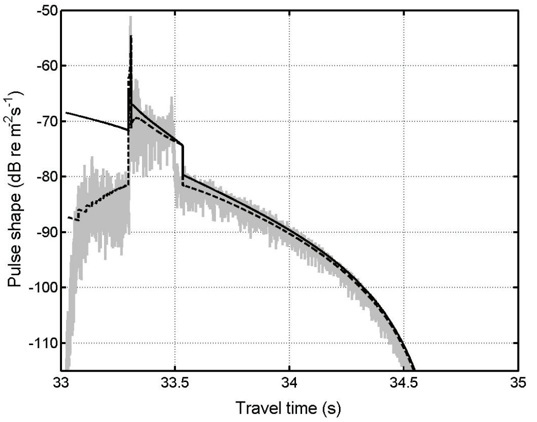

Fig 5 Depth averaged pulse shape computed by C-SNAP for a summer sound speed profile overlaying a half-space seabed defined by parameters in [6] (grey). Superimposed are the analytical solutions (also depth averaged) with Rayleigh reflection coefficient, excluding (black solid), and including focusing effects (dashed).

6. CONCLUSIONS

Formulas for the envelope of a multipath pulse in a shallow water environment with a uniform sound speed gradient were taken from [6]. The refracted arrivals that interact only with the low sound speed boundary form a characteristic quadrilateral, or sail shape, near the leading edge of the pulse. In contrast, the steeper rays that interact with both boundaries form a long tail that decays more or less exponentially (linearly in dB) with a rate dependent only on the reflection loss and the water depth. Furthermore this rate is independent of range or refraction. Some rules of thumb enable one to estimate the duration of the first refracted arrival and the intensity ratio of its maximum and minimum.

Two approaches were taken to calculate the complete pulse shape. One was to write an exact expression in terms of mixed variables (time, cycle distance, and derivatives) which require numerical interpolation to reveal pulse shape. The other was to make approximations to find explicit functions of time. Two-way pulse shapes were obtained by numerical convolution rather than attempting analytical convolution, as was possible in the isovelocity case.

To establish correspondence between these pulse shapes and the expected eigenray or modal arrivals, three comparisons were made against the normal mode model C-SNAP, transforming its output into the time domain. The formulas for the three cases, isovelocity, uniform sound speed gradient, and summer profile, show very good agreement with a depth average or a time smoothing. Also the interpolation approach can easily handle the Rayleigh reflection loss, instead of its linear approximation.

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