SOME STATISTICAL CHARACTERISTICS OF OCEAN MOTIONS FROM CURRENT MEASUREMENTS

by

J.A. Gonella*
Museum National d'Histoire Naturelle
Paris, France

ABSTRACT

A rotary-component method for analysing vector series in two dimensions was developed for the study of the coupling between wind-stress and surface current [Gonella, 1972]. This method provides a good tool for instrumental intercomparison and for pointing out statistical characteristics of ocean motions from current data. In particular, rotary-waves seem to be an important characteristic of ocean motions.

The rotary-component method developed for analysing the coupling between wind and surface current [Gonella, 1972] is used here for pointing out some statistical characteristics of ocean motions in open sea. Before, a short review of this method and its application to some currentmeter intercomparison will be given.

ANALYSIS OF VECTOR SERIES

Data collected with a currentmeter provide a vector series:

$$u(t) = |u(t)|e^{i\theta(t)} = u_1(t) + i\,u_2(t) \qquad i = \sqrt{-1}$$

with $t = m\,\Delta t \leq d$ where d is the length of the record, m an integer (≥ 0), Δt the sampling interval and, u_1 and u_2 the scalar components of u along horizontal rectangular axes.

*The author was visiting the Woods Hole Oceanographic Institution from Sept. 70 to Oct. 71.

From the Fourier transform u_ω for each angular velocity ω we can write:

$$u(t) = \sum_{n=-N}^{n=+N} u_\omega e^{i\omega t} \quad \text{with} \quad \omega = 2Mn/d$$

summed over integral values of n from -N to N ($N = d/2\Delta t$; $\sigma_N = \pi/\Delta t$, Nyquist angular frequency). Because $u(t)$ is a vector, u_ω will be different from the complex conjugate at $-\omega$: $u_\omega \neq u^*_{-\omega}$. Thus at the angular frequency $\sigma = |\omega| > 0$, the hodograph will be an ellipse:

$$M_\sigma = u_{+\sigma} e^{+i\sigma t} + u_{-\sigma} e^{-i\sigma t}$$

which foci coordinates (c, φ) are defined by the equation:

$$u_+ u_- = \frac{c^2}{4} e^{i2\varphi}$$ where symbol σ is removed in the index.

The area is: $\pi \quad |u_-|^2 - |u_+|^2$.

For statistical analysis, we can take into account several records or pieces of same length in order to have more degrees of freedom. With records of the same vectorial parameter, it is easy now to define at a given angular frequency σ the following characteristics invariant under coordinate rotation:

- the clockwise kinetic energy or clockwise spectrum $S_- = \frac{1}{2}\langle u_- u_-^* \rangle$, the symbol $\langle\ \rangle$ represents the average over all the pieces;
- the anticlockwise spectrum: $S_+ = \frac{1}{2}\langle u_+ u_+^* \rangle$;
- the total spectrum: $S_t = S_- + S_-$;
- the rotary coefficient: $C_{R_\sigma} = \dfrac{S_- - S_+}{S_+ + S_+}$, its sign is related to the polarisation of the ellipse (positive for clockwise motion) and its magnitude is varying as the complementary to one of the square of the ellipse exentricity.

The ellipse coefficient given by the following equation

$$E_\sigma = |E_\sigma| \, e^{i2\Phi_\sigma} = \frac{\langle u_+ u_- \rangle}{2(S_+ S_-)^{1/2}}$$

is a complex quantity similar to a coherence; we can assign it a confidence limit using the distribution properties of coherence (Amos and Koopmans, 1963). The magnitude ($|E_\sigma| \leq 1$) defines the stability of the ellipse orientation Φ_σ which is not invariant under coordinate rotations.

With simultaneous records of two different vectorial parameters u and V, the coherence is defined at each angular velocity ω by:

$$C_\omega = |C_\omega| \, e^{i\Phi_\omega} = \frac{\langle u_\omega v_\omega^* \rangle}{2(S_{u_\omega} S_{v_\omega})^{1/2}} .$$

The coherence will be one for a linear process and Φ_ω is the phase between u_ω and v_ω: Φ_ω is invariant under coordinate rotation. If there is some asymmetric effect on the angular velocity in the process generating u and v, we can expect different order of magnitude for $C_{+\sigma}$ and $C_{-\sigma}$.

CURRENTMETER INTERCOMPARISON

Current data are collected at site D (39°N, 70°W) in Atlantic Ocean [Fig. 1] by the Woods Hole Oceanographic Institution from moored buoys. On moorings 377, 378 and 379 [Fig. 2], two types of currentmeters were used for measuring current in the mixed layer (8 - 20 m): the Geodyne 850 and the VACM (Vector Averaging Current-Meter) which has the same sensors as the Geodyne but a different sampling system (J. McCullough, 1972). The 95% significant coherence band [Fig. 3] is decreasing in the following order:

- two VACM on the same mooring (3783 and 3785): data are coherent in all the band ($-\sigma_N + \sigma_N$) when σ_N is the Aquist angular frequency.

- two VACM on moorings 2 km apart (3783 - 3772)
- a VACM (3783) and a Geodyne (3784) on the same mooring
- two Geodynes on moorings 2 km apart.

It is obvious that the VACM gives a measurement of better quality than the Geodyne which is contaminated by noise coming from surface waves. Total spectra for VACM and Geodyne are given by Fig. 4; the level of the noise appears clearly for the Geodyne from 1/4 to 2 cycles per hour.

The phase relationship between all the currentmeters used in this array gives a variance of 6° at the angular velocity close to $-f$ (f = Coriolis parameter) but reaches 11° for the mean value over 36 hours (about twice the inertial period). Because inertial oscillations are locally in phase, we can consider that the value of 6° represents the accuracy on the direction given by the currentmeter. Nevertheless, for two given currentmeters the maximum difference in the phase was 20°; so before an experiment, it would be good to make some intercomparison in order to get the compass relative error for these two particular meters.

SPECTRAL CHARACTERISTICS OF OCEAN MOTION

In the linear approximation, the motion equation can be written (Gonella, 1971) in Northern hemisphere:

$$\frac{\partial u}{\partial t} + ifu = \gamma$$

where γ is the forcing function; it might be related to the wind stress or the pressure gradient. If we assume that the forcing function is equal in the clockwise and in the anticlockwise

$$|\gamma_+| = |\gamma_-|$$

and because $i(f+\omega)\,u_{\omega} = \gamma_{\omega}$, the theoretical rotary coefficient will be

$$C_{R_\sigma} = \frac{|u_-|^2 - |u_+|^2}{|u_-|^2 + |u_+|^2} = \frac{2\sigma f}{\sigma^2 + f^2} > 0 \ .$$

In the ocean motion in northern hemisphere, there is more energy in the clockwise than in the anticlockwise: ellipse at a given frequency will be polarized clockwise. The three spectra (total, clockwise and anticlockwise) are shown in Fig. 5 for the current and 2000 meters at site D. This method allows to point out the diurnal tide in the anticlockwise spectrum which is not contaminated by inertial peak ($\omega = -f$). The observed rotary coefficient at 1000 and 2000 m [Fig. 6] is in good agreement with the theoretical coefficient (dashed line) for $\sigma > f$, but not in the low frequency ($\sigma < f$) where anticlockwise forcing is larger than the clockwise one ($|\gamma_+| > |\gamma_-|$). In the ellipse orientation [Fig. 7] no particular direction is found in the internal wave field in the ocean at site D, except may be at the semi-diurnal frequency.

Now if we look at the coherence between 1000 and 2000 m data [Fig. 8], the distribution in the magnitude of this coefficient is asymmetric with respect to $\omega = 0$: mostly at diurnal and semi-diurnal angular velocities.

The high level of the coherence for both tides in the anticlockwise must be related to the fact that both cotidal lines are turning anticlockwise in Northern Atlantic. In other hand, the coherence between surface current 50 km apart [Fig. 9] has a significant level in the angular velocity ranging (0, -f) exactly in the same range where wind-stress and current are coherent [Fig. 10]; [Gonella, 1972]. Note the discontinuity in the phase for $\omega = -f$; it is explained by the change of the sign of $(f + \omega)$ in Eq. 1. This asymmetry has some similarities with free oscillations in a rotating circular basin [Lamb, 1952]; no rotary wave can be propagated with "cotidal line" turning clockwise with angular velocity ranging (-f, 0). In this range, no dissipation by wave propagation is possible and surface currents, forced by wind, will be coherent in a relatively small area (50 - 100 km) where the wind is coherent too. But in deep ocean, no external force as the wind stress appears and the pressure gradient associated to the rotary waves will have an anticlockwise component larger than the clockwise one in the frequency range: $0 < \sigma < f$. This asymmetry explains, in deep sea, the discontinuity in the observed rotary coefficient at $\sigma = f$ [Fig. 6] and the larger magnitude of the coherence in the anticlockwise [Fig. 8].

CONCLUSION

Our purpose was to show briefly how this rotary component method can be used for currentmeter intercomparison or for pointing out some statistical characteristics of ocean motion. For frequency smaller than f, it seems that rotary waves are an important feature of ocean motions at any depth in open sea.

ACKNOWLEDGEMENTS

This work has been supported by the Office of Naval Research under contract No. N000.14.66.C0.241 and by the Centre National pour l'Exploitation des Oceans under contracts No. 70/150 and 71/253.

Special thanks are due to F. Webster for arranging the author's stay in Woods Hole and for helpful advice in this work. The author acknowledges all the WHOI "Buoy Group" for the considerable amount of help and cooperation in data collecting, processing and programming.

REFERENCES

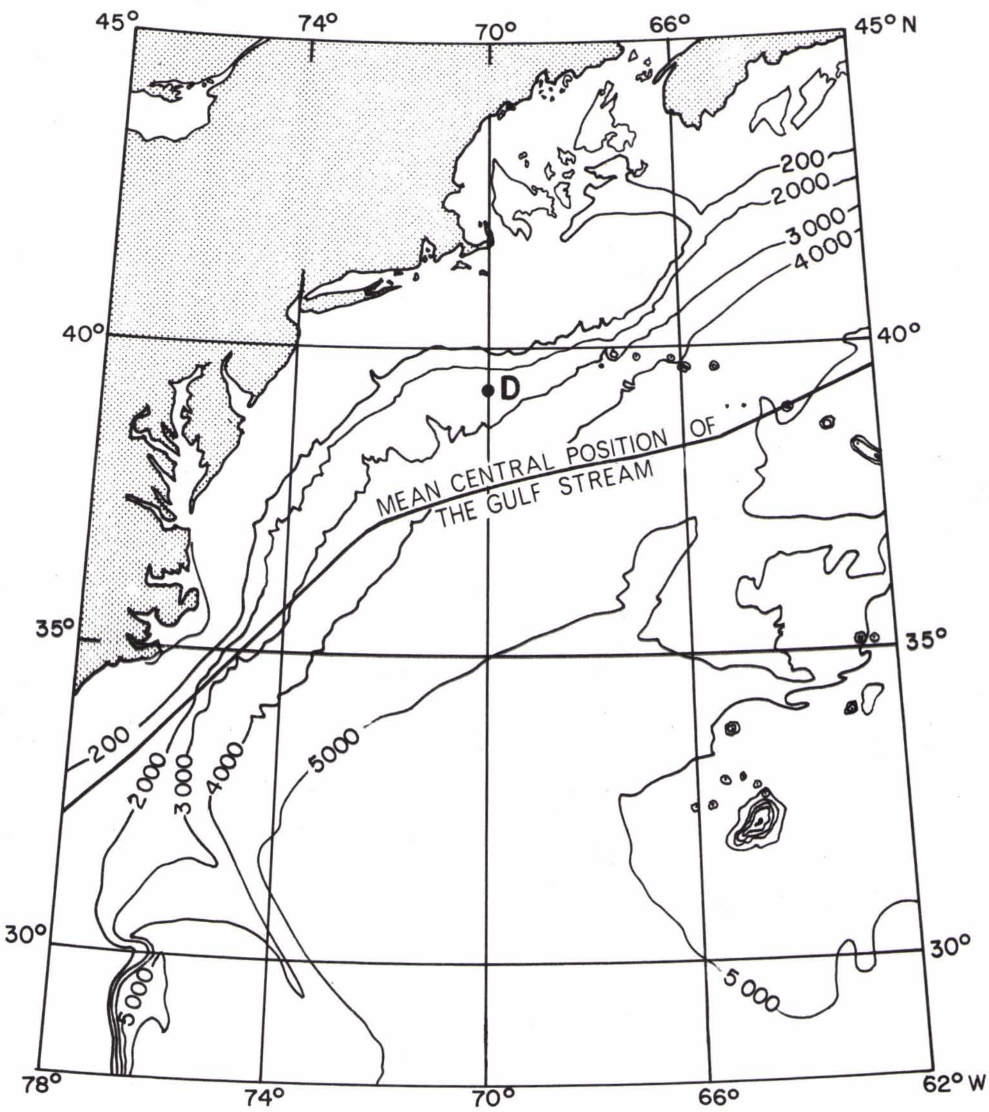
D.E. Amos and I.H. Koopmans, "Tables of the Distribution of the Coefficient of Coherence for Stationary Bivariate Gaussian Processes", Saudia Corporation, SCR 483, Monograph, 1963.

J. Gonella, "Sur la polarisation des courants marins par la rotation terrestre", C.R. Acad. Sci. Paris, 273, pp. 162-164, 1971.

J. Gonella, "A Rotary Component Method for Analysing the Coupling between Wind and Surface Current", Deep Sea Res., (Submitted), 1972.

J. McCullough, "The Vector Averaging Currentmeter", Woods Hole Oceanographic Institution, unpublished manuscript in press, 1972.

H. Lamb, "Hydrodynamics", University Press, Cambridge, pp. 320-324, 1932.



Site D { 39° N
70°W

FIG. 1

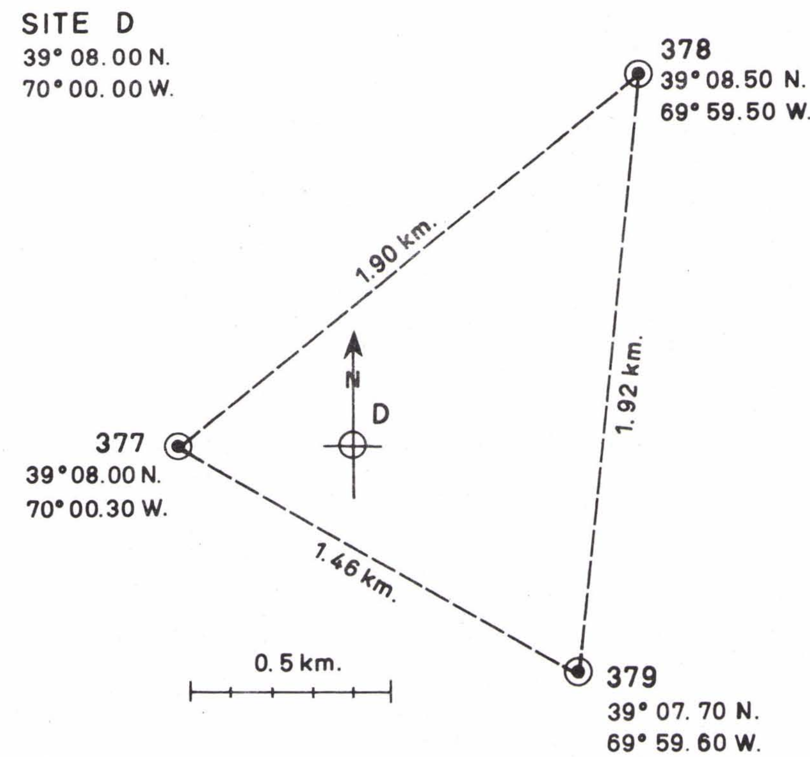


FIG. 2

SITE D

NEAR-SURFACE CURRENTMETER INTERCOMPARISON

VACM(*) & GEODYNE

95% SIGNIFICANT COHERENCE BAND

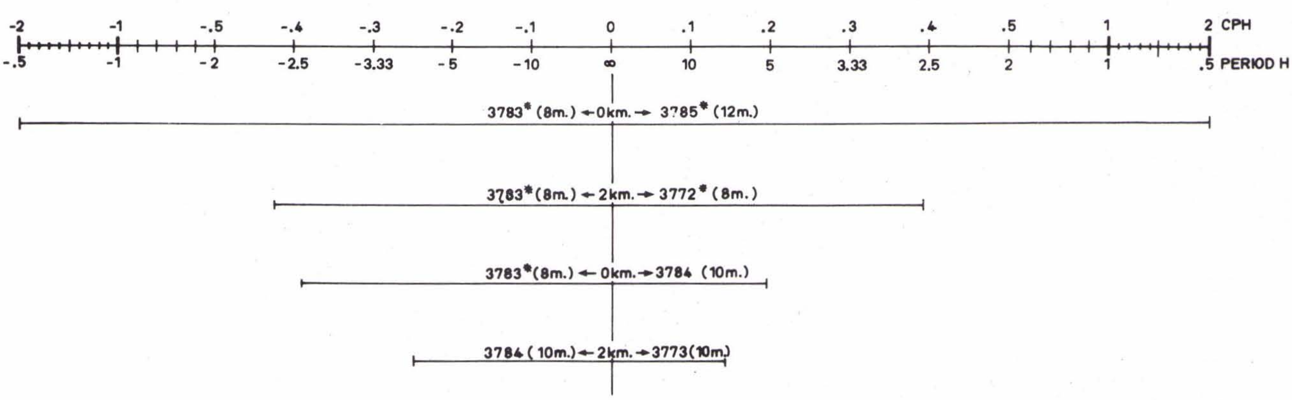


FIG. 3

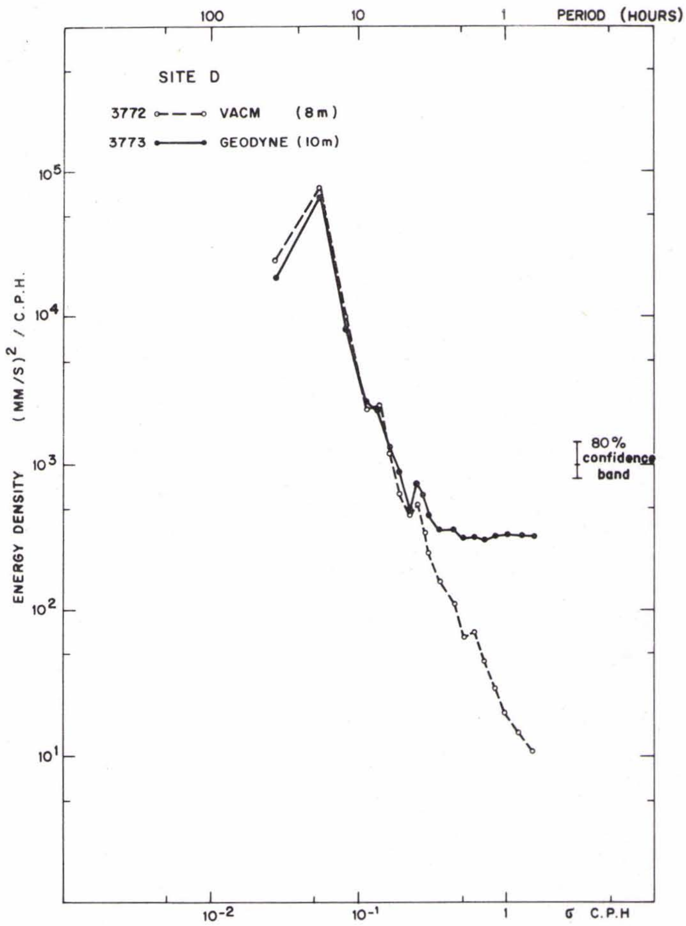

PERIOD (HOURS)
100
10
1
SITE D
3772 VACM (8 m)
3773 GEODYNE (10m)
ENERGY DENSITY (MM/S)^2 / C.P.H.
10^5
10^4
10^3
10^2
10^1
80% confidence band
10^{-2}
10^{-1}
1
σ C.P.H

FIG. 4

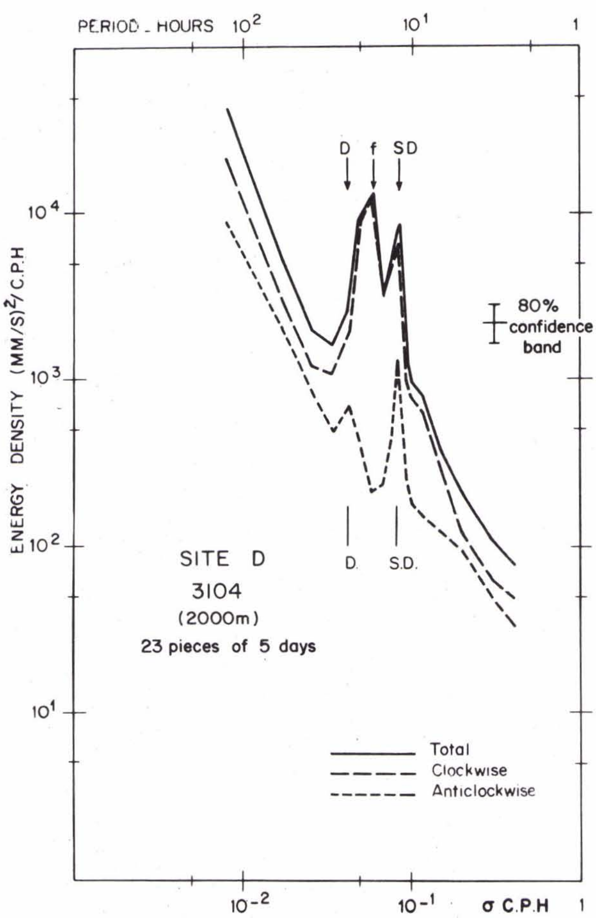
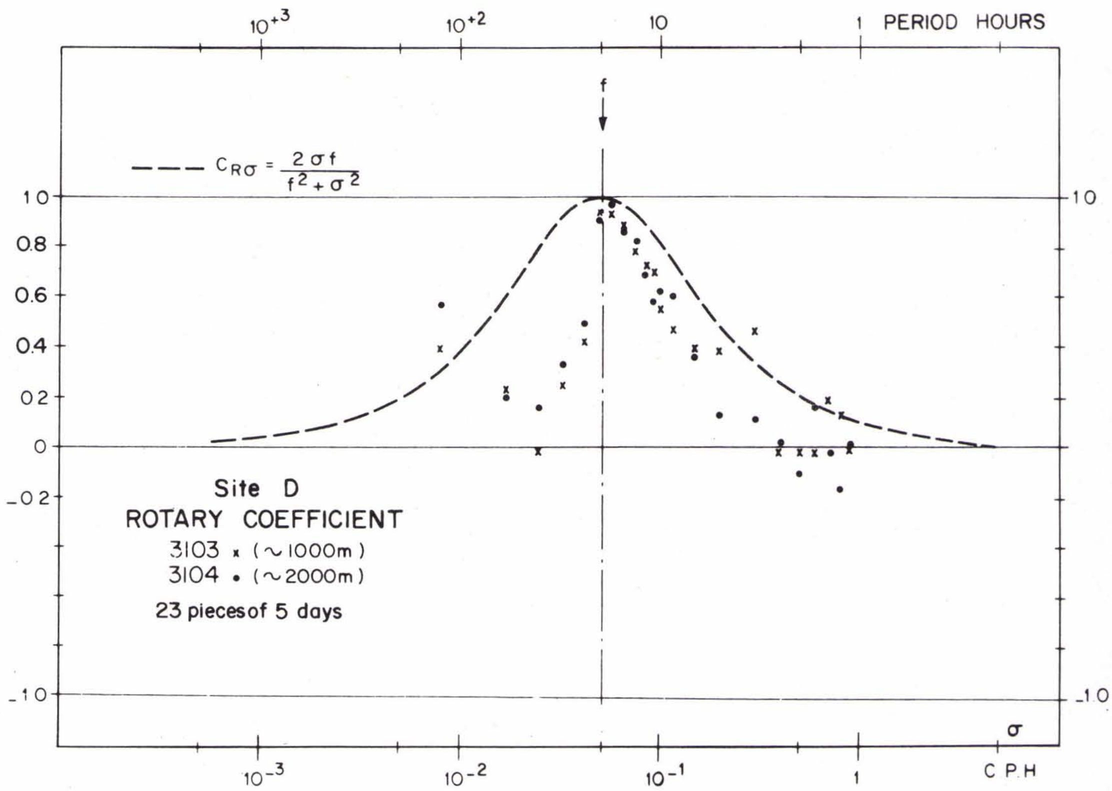
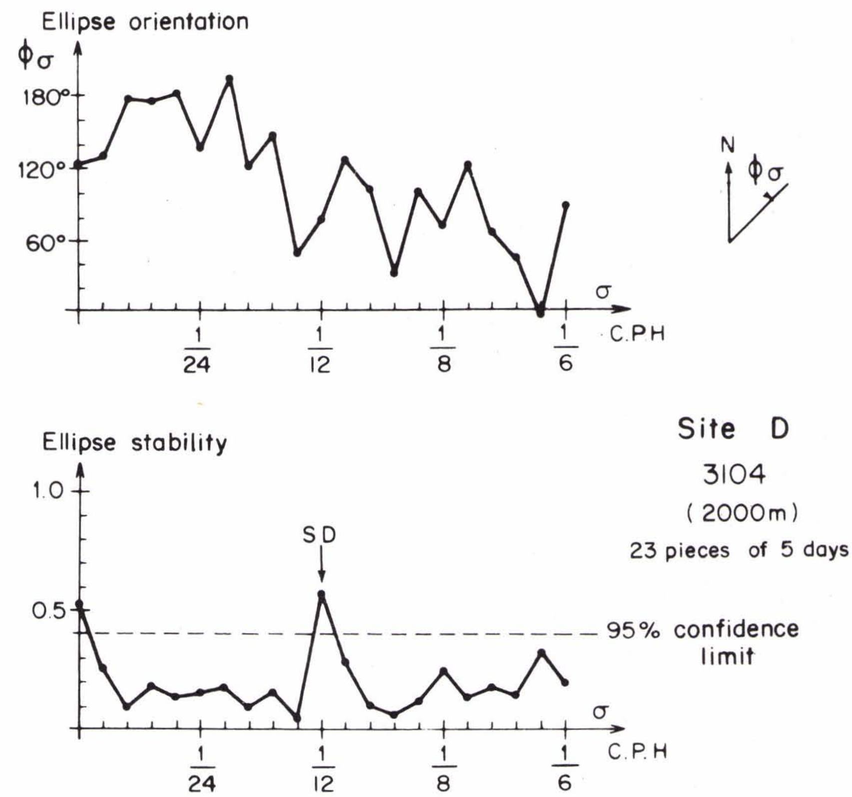

PERIOD - HOURS
10^2
10^1
1
D
f
SD
ENERGY DENSITY (MM/S)^2/C.P.H
10^4
10^3
10^2
10^1
80% confidence band
D.
S.D.
SITE D
3104
(2000m)
23 pieces of 5 days
Total
Clockwise
Anticlockwise
10^{-2}
10^{-1}
σ C.P.H
1

FIG. 5



PERIOD HOURS
f
$C_{R\sigma} = \frac{2\sigma f}{f^2+\sigma^2}$
Site D
ROTARY COEFFICIENT
3103 x (~1000m)
3104 • (~2000m)
23 pieces of 5 days
σ
C.P.H

FIG. 6



Ellipse orientation
ϕ_σ
180°
120°
60°
σ
C.P.H
N
ϕ_σ
Ellipse stability
1.0
0.5
SD
95% confidence limit
Site D
3104
(2000m)
23 pieces of 5 days

FIG. 7

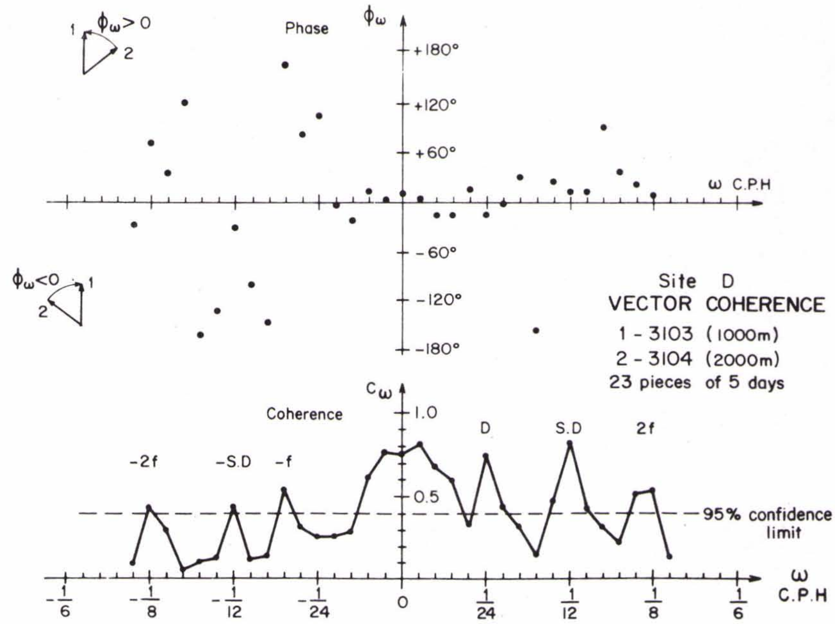


FIG. 8

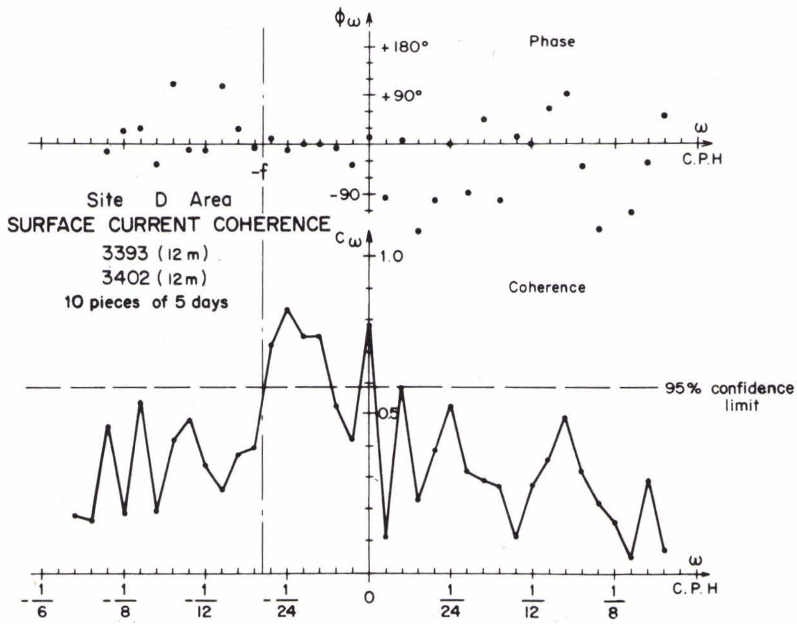


FIG. 9

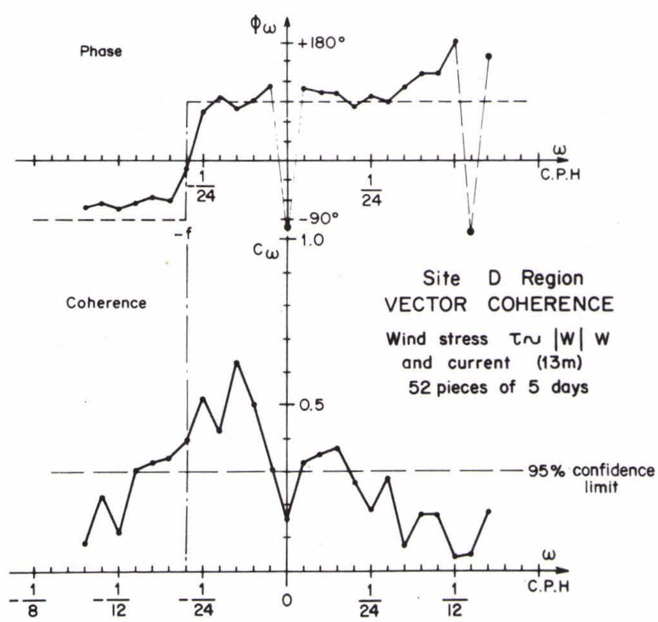


FIG. 10