Geoacoustic Modeling of the Seabed at Higher Frequencies

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Abstract

In propagation at higher frequencies, local fluid motion in "hidden coordinate" systems and local discontinuities due to gas bubbles and bioturbation have a perturbing effect on the wave field. To model these effects, a baseline model that reflects the influence of basic primitive variables such as porosity and mean effective stress is first derived on the basis of the Biot theory and then effects of various discontinuities are introduced as perturbations to the baseline model.

1. Introduction

Since the Biot theory was first applied to the geoacoustic modeling of the seafloor by Stoll and Bryan [1], it has proved to be a powerful tool in studying the response of a wide variety of sediments that make up the ocean bottom. The initial motivation for modeling these particulate materials with a two-component, phenomenological formulation, such as the Biot theory, was to allow more flexibility in incorporating various physical mechanisms that control energy loss, compressibility and shear stiffness in a realistic manner, attributing each to the component (individual grains, fluid or skeletal frame) that is appropriate. The resulting model is much better able to replicate the complex behavior that is observed over a wide frequency range than a straight viscoelastic formulation. However, one pays a price for this flexibility in that the user is required to evaluate over thirteen input parameters, many of which are interrelated in a very specific manner by the mechanics of particulate material.

Much of the early experimental work by geophysicists studying the geoacoustic properties of seafloor sediments was carried out at relatively high frequencies in the range of several kHz to a MHz. On the other hand geotechnical engineers, interested in earthquakes and building vibrations, carried out experimental studies at low frequencies ranging from less than one Hertz to about 100 Hz. If the attenuation measurements made by these two groups are compared and it is assumed that the sediments respond essentially as constant Q materials (i.e., attenuation in dB/m linearly proportional to frequency) there is a major inconsistency particularly when considering the coarse sediments such as sand. It was this inconsistency that motivated the search for a more realistic mechanical model that could account for energy losses of various kinds that would be expected in water-saturated, particulate sediments. In the ensuing application of the Biot theory it was found that for the coarser sediments, motion of the interstitial fluid relative to the skeletal frame produced viscous losses which, when added to frictional losses occurring at the intergranular contacts, resulted in an overall attenuation that matches experimental results at both high and low frequencies.

In finer grained sediments, overall relative motion of the pore fluid in the direction of propagation is much smaller because of low permeability, however viscous dissipation still plays a role in determining overall attenuation because of local fluid motion in a "hidden coordinate system" near the intergranular contacts. Using the principle of viscoelastic correspondence, this kind of viscous loss is incorporated into the model by casting the moduli of the skeletal frame as viscoelastic operators. Biot discusses a number of different kinds of operators in two of his later papers [2,3]. This kind of viscoelastic operator was recently used by Stoll and Bautista to model the soft sediment of Eckernfoerde Bay in the Baltic [4]. Biot also discussed an operator that is appropriate to model the behavior of sediments containing free gas bubbles in [2] and [3]; we will use this kind of operator in a later example.

2. Constructing a Baseline Model

In studying various acoustical phenomena such as scattering, bubble resonance and bottom interaction, it is necessary to supply various geoacoustic parameters that describe sediment compressibility, shearing stiffness and attenuation, usually in the form of complex, frequency dependent moduli. In many cases reference to historical data collections such as those given by Hamilton (e.g., [5]) can provide useful input provided the data base includes entries for a similar sediment type obtained in the appropriate frequency range. However, this is not always possible because of the limited scope of the historical data available and the wide range of frequencies and sediment types that are currently of interest. Furthermore, attempts to extrapolate these data over several decades of frequency or to use a regression curve based on a single independent variable such as grain size have lead to some misleading generalizations . A good example is the persistent and erroneous notion that attenuation can always be assumed to vary as the first power of frequency. Hence a properly formulated baseline model, derived on the basis of the Biot theory and able to predict the correct response over a wide frequency range, becomes an important tool in determining the starting point for studies of the perturbing effects we wish to study.

The first step in establishing a baseline model is to determine complex bulk moduli for the individual grains, K_r, the skeletal frame in a water environment, K_b, and the interstitial fluid, K_f, and the complex shear modulus of the frame, μ, all potentially frequency dependent. These familiar moduli can then be related to the parameters used in the Biot coupled equations of motion using the following equations provided the assumption of homogeneous strain is made (i.e., volumetric strain of pore volume is the same as volumetric strain of frame so that the porosity of the sediment, β, remains unchanged - see [6], pp. 9-10).

$$
\begin{aligned}
\bar{H} &= (K_r - \bar{K}_b)^2 / D + \bar{K}_b + 4\bar{\mu}/3 \\
\bar{C} &= K_r(K_r - \bar{K}_b) / D \\
\bar{M} &= K_r^2 / D
\end{aligned}
\tag{1}
$$

where

$$D = K_r(1 + \beta(K_r/\bar{K}_f - 1) - \bar{K}_b$$

Since each of the moduli in (1) is associated with a specific attribute of the model, it is possible to selectively incorporate realistic mechanisms for energy loss depending on the type of sediment being modeled. For example, in a granular sediment composed of hard, polycrystalline grains of fairly uniform size and shape, the sum of two kinds of energy dissipation result in the overall attenuation that is observed; one is the result of overall relative motion between the fluid and the skeletal frame which is controlled by the permeability of the frame and the other is friction at the intergranular contacts between particles. Since friction is an inherently nonlinear phenomenon, we are forced to use a linear approximation or a truncated constant complex modulus in order to keep the resulting equations of motion linear. Many viscoelastic models that result in an approximately constant value for the quality factor ,Q,have been proposed for this purpose (e.g., see [1] and [3]).

When there is no fluid in this kind of sediment, the attenuation of a propagating plane wave is a linear function of frequency since the energy loss per cycle is constant irrespective of frequency. When fluid is introduced into the voids, additional viscous damping occurs due to relative motion between the fluid and frame and since fluid mobility depends on the permeability, the frequency at which the effects of viscous damping become dominant depends on the intergranular pore size and geometry. In clean sands the effects of viscous damping begin to dominate at frequencies as low as 100 Hz whereas in sediments that contain significant amounts of much finer material, the effects may not be obvious until one reaches the high kHz region. The Biot parameters that control the fluid mobility are the permeability , the viscosity of the pore fluid, a pore-size parameter and a so-called structure factor (also sometimes referred to as an inertial coupling factor).

In addition to the global fluid motion, or "sloshing" as it is sometimes called, there is local fluid motion near the intergranular contacts and between adjacent cells in finer sediments composed of plate-like particles in a "cardhouse structure. This local flow, which occurs in a "hidden coordinate system" as Biot puts it, is generated by relative approach of particles causing squeeze-film motion or by the nonuniform volumetric strain of adjacent cells in the finer sediment. Hence, it is essentially in phase with the volumetric strain of the skeletal frame and its effect can be incorporated into the model by the use of viscoelastic operators which operate on the bulk and shear moduli of the frame. This approach was used by Stoll and Bautista [4] to establish a baseline model for the soft sediments of Eckernfoerde Bay in the Baltic. They used a generalized viscoelastic model based on an adjustable distribution of relaxation times as proposed by Cole and Cole [7]. This formulation is one of many that have been used in the physical sciences to describe rate-dependent processes (e.g., see Gross [8]) and is particularly adaptable to the present problem because of the wide distribution of cell sizes and geometries that exist in real sediment which necessitates a correspondingly wide distribution of relaxation times.

The distribution function used in the Cole-Cole model is of the form

$$F(s)ds = \frac{1}{2\pi} \cdot \frac{\sin \alpha \pi \, ds}{\cosh(1-\alpha)s - \cos\alpha\pi} \tag{2}$$

where s = ln τ/τ_0 and α controls the sharpness of the peak in a bell-shaped symmetrical distribution. (e. g., see Stoll [6], pg. 97). The distribution of relaxation times given by (2) results in a complex modulus with real part N_r and imaginary part N_i given by

$$\begin{aligned} N_r &= N_\infty - \frac{N_\infty [1 + (\omega\tau_0)^{1-\alpha} \sin(\alpha\pi/2)]}{1 + 2(\omega\tau_0)^{1-\alpha}\sin(\alpha\pi/2) + (\omega\tau_0)^{2(1-\alpha)}} \\ N_i &= \frac{N_\infty (\omega\tau_0)^{1-\alpha}\cos(\alpha\pi/2)}{1 + 2(\omega\tau_0)^{1-\alpha}\sin(\alpha\pi/2) + (\omega\tau_0)^{2(1-\alpha)}} \end{aligned} \tag{3}$$

N_∞ is an amplitude factor and τ_0^{-1} is the frequency at which N_i is maximum. The overall complex moduli of the skeletal frame are determined by adding the above modulus to a constant complex modulus that is determined on the basis of the sediment response at low frequency.

In modeling the soft, gassy sediments of Eckernfoerde Bay, parameters input to the Biot model were determined mainly on the basis of field experiments performed during the Coastal Benthic Boundary Layer Special Research Program sponsored by the Naval Research Laboratory. This sediment has an average porosity of 86% and a low permeability of the order of 5 x 10^{-11} cm ^{2} so that global relative fluid motion has a negligible effect on the response at frequencies less than about .5 MHz. Hence simplification to the Gassmann approximation is warranted and only the first of equations (1) is significant. Since the Biot equations reduce to the Gassmann equation when the fluid mobility is very low, no special treatment is necessary in using the full set of Biot equations except to choose the parameters controlling fluid motion (i.e., permeability, pore-size parameter and density coupling factor) to be of the right order of magnitude. The complex shear modulus, μ, was evaluated on the basis of measurements of shear wave velocity and attenuation at low frequency using Love wave measurements [9]. The three parameters for the Cole-Cole modification of the frame bulk modulus were chosen in order to match p-wave attenuation measurements at 58 and 400 kHz by Richardson and Briggs [10]. The attenuation predicted by the model is shown in Fig. 1 along with some historical data for ocean bottom silts and clays from Hamilton [5] and other sources (see [11] for a general review of attenuation measurements in marine sediments). The model predicts an increase of p-wave velocity from 1421 m/s at 1 Hz to 1434 to 1440 m/s at 400 kHz which is in agreement with the range of observed velocities in the Eckernfoerde experiments.

3. Effects of Free Gas as a Perturbation of the Baseline Model

When gas bubbles form in the pore water of a sediment, significant changes in the geoacoustic response occur. Much of the research on this topic through 1980 was summarized in two papers by Anderson and Hampton [12]. From their discussion and several more recent papers [13,14], it is clear that the size and distribution of the gas bubbles can be quite variable, ranging from a uniform distribution of small bubbles in the normal interstices of the sediment to large disruptive bubbles that create their own cavities to distributed regions of free gas that encompass volumes of sediment containing many grains. Hence the problem of modeling the effects of gas in a comprehensive way is quite challenging. In general,

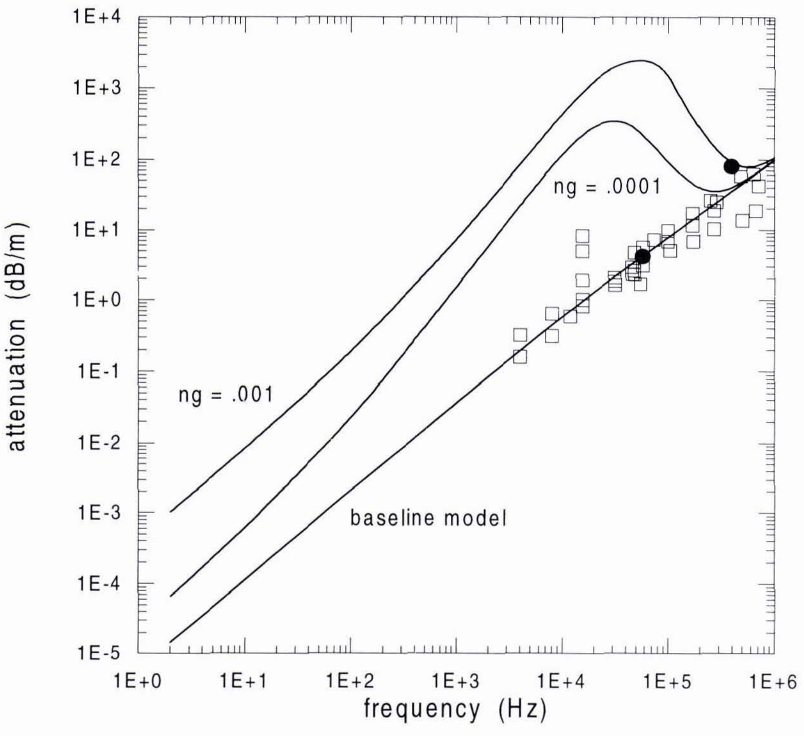

Fig. 1. Attenuation for baseline model and two cases of partial gas saturation. The historical data points are for clays and other fine-grained materials with the two solid symbols corresponding to the soft sediments of Eckernfoerde Bay.

significant changes in compressional wave speed and attenuation are to be expected and a pulsation of the bubbles in the frequency range near their resonance frequency can be expected to produce a marked effect on the geoacoustic response.

In order to incorporate the effects of bubbles into his model, Biot [3] suggested that the fluid compressibility be modified by invoking an operator that produced a phenomenological response similar to a simple harmonic oscillator such as the one shown in Fig. 2. By incorporating mass and damping elements, this model takes into account an inertia effect in the hidden coordinates and viscous dissipation arising from the concentrated radial velocity of the fluid in the vicinity of a bubble. In this model the compressibility of the fluid at frequencies well below resonance is essentially controlled by the elastic elements k_1 and k_2 acting in series so they are related to the quasistatic compressibility of the gassy sediment, $1/K'_w$, by

$$1/k_1 + 1/k_2 = [n_g K_w + (1-n_g) K_g]/K_w K_g = 1/K'_w \tag{4}$$

where n_g is the fraction of sediment pore space occupied by gas and K_w and K_g are the bulk moduli of the water and gas respectively. At frequencies well above resonance the bubbles have insufficient time to compress and the apparent compressibility of the gassy fluid is essentially $1/k_2$ so that $k_2 = K_w$ and $k_1 = K_w K'_w / (K_w - K'_w)$. With these assumptions the model shown in Fig. 2 leads to a complex compressibility of the form

$$1/\bar{K}_f = 1/K_w + 1/[(k_1 - m\omega^2) + i c \omega] \tag{5}$$

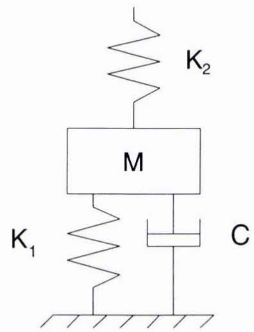

Fig. 2. Schematic of model for compressibility of gassy fluid suggested by Biot.

Since the viscosity, c, and mass, m, are parameters in a phenomenological model, which are not easily quantified in terms of physical variables such as bubble size and resonance frequency, it is useful to recast this equation in the form

$$1 / \bar{K}_f = 1 / K_w + 1 / k_1[1 - (\omega / \omega_n)^2 + i(2D\,\omega/\omega_n)] \tag{6}$$

wherein the damping ratio, D, may be written in terms of the inverse quality factor, 1/Q

$$D = (1/Q)/\sqrt{4 + (1/Q)^2} \tag{7}$$

and the undamped natural frequency, ω_n, related to the resonance frequency using

$$\omega_n = \omega_{resonance}/\sqrt{1 - 2D^2} \tag{8}$$

Hence the complex bulk modulus of the fluid, $\overline{K}_f$, can be introduced into the equations (1) once the resonance frequency and damping for the bubbles in a specific micro-model of the gassy sediment are determined. However, for the reasons mentioned above, this is a formidable task because of the wide variety of different configurations the gas may assume and a number of different approaches to this problem have been attempted . As examples, Anderson and Hampton [12] calculated resonance frequency using a hybrid equation based on historical equations derived for resonance of bubbles in water and in elastic solids whereas Dutta and Ode [15] studied the problem of radially oscillating gas pockets using the Biot theory applied to repeating cells containing a gas pocket in a porous matrix.

As an example of how the above equations modify the predictions of the baseline model, p-wave velocity and attenuation have been calculated for a hypothetical case where bubble resonance occurs at 20 kHz and overall damping of the pulsating bubbles results in a quality factor of approximately 1. While resonance frequencies are found to vary over a wide range depending on bubble size and water depth (i.e., from 2 kHz to over 600 kHz) these values were chosen as typical for soft, fine-grained sediment with bubble radius of the order of 0.5 mm on the basis of calculations given in [12]. Calculations were made for two different gas concentrations, n_g = .001 and .0001 (i.e., voids containing .1% and .01% gas by volume). The effect on attenuation by gas in these concentrations is shown by two curves in Fig. 1 and the

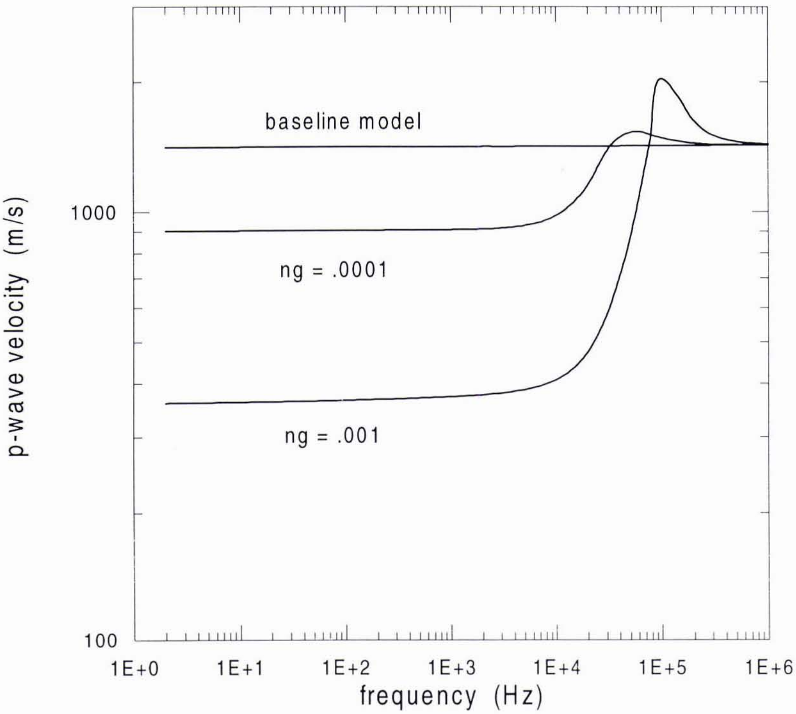

Fig. 3. P-wave velocity for baseline model and for two cases with partial gas saturation. Resonance frequency is 20 kHz and gas fraction of pore space is shown next to curves.

effect on p-wave velocity is depicted in Fig. 3. As can be seen from these figures, even the addition of a small amount of gas results in significant changes in velocity and attenuation. While the cases shown here are hypothetical, they illustrate how the effects of gas can be introduced as a perturbation on a carefully fitted baseline Biot model thus allowing comparative studies of the predictions of various micro-models for evaluation and comparison with experimental results. Moreover the simple model shown in Fig. 2 may be generalized to include the effects of a distribution of bubble sizes by casting the mass, viscous and elastic parameters as distribution functions in a manner similar to that used in Section 2 where a distribution of relaxation times based on the Cole-Cole model was used to modify the complex modulus of the skeletal frame.

4. Use of the Baseline Model in Scattering Studies and Other Cases of Interest

The usefulness of a baseline model in studies of scattering can be seen by considering the parameters used in a recent paper by Lyons et al [13]. The scattering model used in this paper depends on the resonance frequency of the bubbles in the sediment and on the attenuation of waves propagating in the sediment between the pulsating bubbles. These authors considered a scattering model because microscopic and radiographic studies of the pore structure revealed rather large, irregular gas pockets that suggested that scattering would become important at frequencies much lower than in cases where small gas bubbles were contained within the natural interstices of the sediment. Hence the dynamic shearing modulus of the sediment surrounding the bubbles becomes important in determining the resonance frequency, and the propagation characteristics of the sediment (p-wave velocity and attenuation) between larger bubbles, play a key role in constraining the scattering model. Unfortunately the proportion of gas present in bubbles smaller than about 1 mm was unknown, so that it is not clear what would be the most appropriate choice for these propagation characteristics given the important effects of very small amount of gas demonstrated in Section 3 above. In any event, it is felt that use of a baseline model and systematic parametric studies such as those demonstrated in Section 3 would be useful in defining the basic parameters necessary to constrain this kind of study since it forces the user to consider all pertinent input parameters in an integrated way rather than choosing bits and pieces of experimental and historical data in an *ad hoc* manner.

Another perturbation of the basic Biot theory was recently proposed by Leurer [16] who incorporated more damping into the response of montmorillonite-rich fine-grained sediments by modelling the individual composite grains as viscoelastic to account for "squirt flow" from the interlayers of the montmorillonite. In this case the bulk modulus of the grains, K_r becomes complex with a frequency dependence that is determined by the form of viscoelastic operator that is chosen.

5. Summary and Conclusions

Several examples have been described wherein the basic Biot theory can be used as a baseline model in order to study the effects of different kinds of perturbing effects. These include the effects of local fluid motion in "hidden coordinate systems " caused by inter-cell flow or squeeze-film motion, the influence of free gas in the form of bubbles and the effects of anelastic particle response. In each case operator equations of the kind suggested by Biot have been used to modify the moduli of various components in the baseline model which serves as a reference for assessing the relative influence of the perturbing effect. In the case of scattering studies, the baseline model constrains the choice of input parameters which have often been chosen in an *ad hoc* and sometimes unrealistic manner.

Thus the Biot theory allows more realistic modelling of the physical mechanisms that control energy loss, compressibility and shear stiffness, allowing each to be assigned to the most appropriate component of the sediment. Moreover it forces the user to consider the response of the sediment over a wide range of frequencies, taking into account all of the interacting physical mechanisms rather than concentrating on a single mechanism over a narrow bandwidth wherein predominant trends in overall behavior may be overlooked.

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