Source and Receiver Responses in Boreholes

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Abstract

This report considers the acoustic interaction between the acoustic field in a borehole and the elastic field in the surrounding formation. The problem with acoustic-seismic radiation of energy from a source in a fluid filled borehole with a drillstring surrounded by an elastic formation of infinite extent is first considered. The exact solution is established and simplified expressions are derived for the low frequency case. The source produces a compressional field and a shear wave field. The compressional field has a maximum value at the direction perpendicular to the axis of the borehole, the shear field will in most cases has a maximum approximately at an angle of 45°. *However, in special cases the radiation out into the formation may be completely dominated by shear radiation in a very narrow angle. Secondly, the response of a sensor in the borehole due to an incoming plane wave is considered and the complete solution valid for all frequencies is determined. Numerical examples shows that the receiving response can be very complicated and composed of several modes and each of them may have strong resonances.*

1. Introduction

In many instances borehole measurements are used to determine in situ values for the geo-acoustic properties of marine sediments, either cross-hole measurements or in-hole measurements. Correct interpretation of such results requires an understanding of how acoustic energy in a fluid filled borehole is coupled to the field in the formation in reception and transmission, both in time and frequency domain. This is particularly important with so high frequencies that wavelengths approximately equal the diameter of the hole.

This paper presents theory and examples of transmitting and receiving responses in boreholes. First, we study the field set up in the formation due to a cylindrical source in the borehole. Secondly, we consider the response of a sensor in the borehole due to a plane wave arriving at an arbitrary angle in the formation.

This study is based on previous works reported in the literature. The axisymmetric radiation from a cylindrical source in a homogenous elastic formation has been treated by Heelan [1], Abo-Zena [2], Lee and Balch [3] and Winbow [4]. Lee [5] extended to non axisymmetric excitation. The receiver problem has been thoroughly investigated by Schoenberg [6] and his formulations were later used by Peng et al.[7] to study borehole effects on downhole seismic measurements as function of frequency, angle of incidence and polarization. Borehole coupling and resonances were discussed by Lowell and Hornby [8] in the axisymmetric case with a sensor on the axis of the borehole. We extend the theory to include a hard drillstring in the borehole and nonaxisymmetric sensor locations and we focus the attention to so high frequencies that borehole resonances may play important roles.

2. Mathematical modeling

Consider a cylindrical borehole with a coaxial drillstring with radius respectively a and b, Figure 1. The surrounding formation is assumed to be elastic and homogenous with P-wave and S-wave velocities v_p and v_s and density ρ. Between the drillstring and the formation there is a fluid with sound velocity v_f and density ρ_f. On the drillstring there is mounted a cylindrical source with same radius as the drillstring and with length L. For simplicity we shall in the following assume that the length L is short compared with the wavelength so that transmitter directivity can be neglected.

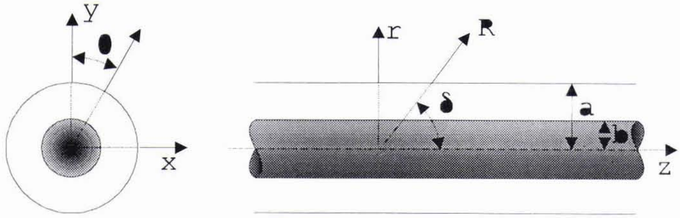

Figure1. Model.

2.1 Transmitter

The radiated waves in the formation are described by a scalar potential for the compressional wave and a vector potential for the shear wave. In this axisymmetric problem the vector potential has only one component and therefore the fields can be described by two scalar functions ϕ and ψ. In cylindrical coordinates the solution to the wave equations for the radiated waves in the formation are

$$\begin{aligned}\phi &= AH_0^{(2)}(\gamma_p r)\exp(i\omega t - ikz)\\ \psi &= BH_1^{(2)}(\gamma_s r)\exp(i\omega t - ikz)\end{aligned} \tag{1}$$

Without the presence of a source the field in the fluid is

$$\phi = \left[A_0 J_0(\gamma_f r) + B_0 N_0(\gamma_f r)\right]\exp(i\omega t - ikz) \tag{2}$$

$H_0^{(2)}(\cdot)$ and $H_1^{(2)}(\cdot)$ are Hankel functions of the second kind, order zero and one, representing outgoing radiated P- and S-waves. $J_o(\cdot)$ and $N_o(\cdot)$ are the Bessel and Neumann functions. The axial wavenumber is k and the radial wavenumbers are γ_p and γ_s, for P and S-waves in the formation, the radial wavenumber in the fluid annulus is γ_f. The time dependency $\exp(i\omega t)$ will hereafter be assumed and omitted.

$$\begin{aligned}\gamma_p^2 &= (\omega/v_p)^2 - k^2\\ \gamma_s^2 &= (\omega/v_s)^2 - k^2\\ \gamma_f^2 &= (\omega/v_f)^2 - k^2\end{aligned} \tag{3}$$

The source function has the form

$$\phi_0 = Q\frac{1}{L}H_0^{(2)}(\gamma_f r) \tag{4}$$

Q is a measure of the source strength and related to the volume displacement of the source V (m^3) by

$$Q = -\frac{V}{2\pi\gamma_f b}\frac{1}{H_1^{(2)}(\gamma_f b)} \tag{5}$$

In the equations above A, B, A_0 and B_0 are unknown coefficients and functions of wavenumber and frequency to be determined by the boundary conditions. These require continuity of normal stress and displacement on the surface of the drillstring and at the borehole wall. We shall assume that the drillstring is infinitely hard so that the displacement at the surface of the drillstring is zero. Having found the coefficients the wave fields are determined by integration over axial wave numbers k.

The pressure in the borehole fluid is

$$p_0(\omega,r,z) = -\frac{\omega^2\rho_f}{2\pi}\int_{-\infty}^{\infty}\left[A_0(\omega,k)J_0(\gamma_f r) + B_0(\omega,k)N_0(\gamma_f r)\right]\exp(-ikz)dk \tag{6}$$

The radial and tangential displacements in the formation are

$$u_r(\omega,r,z)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[-\gamma_p A(\omega,k)H_1^{(2)}(\gamma_p r)+ikB(\omega,k)H_1^{(2)}(\gamma_s r)\right]\exp(-ikz)dk \tag{7}$$

$$u_z(\omega,r,z)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[-ikAH_0^{(2)}(\gamma_p r)+\gamma_s BH_0^{(2)}(\gamma_s r)\right]\exp(-ikz)dk \tag{8}$$

The expressions above are valid for all frequencies and used for numerical calculation. For low frequencies one can develop approximate results that are more suited to give insight in the physical processes. The low frequency results are valid as long as the first resonance frequency of the fundamental mode is higher and outside the band of excitation of the transmitting element. This will be discussed more later. To arrive at the low frequency approximation it is first assumed that only the farfield solution is of interest and therefore the Hankel functions in (6), (7) and (8) are replaced with their asymptotic values for large arguments [9]. Furthermore, the integrals are approximated with their stationary phase value [10]. New components of the displacements are introduced by

$$\begin{aligned} u_R &= u_r \sin\delta + u_z \cos\delta \\ u_\delta &= u_r \cos\delta - u_z \sin\delta \end{aligned} \tag{9}$$

and the Hankel functions are replaced with their low frequency approximations. The end result of this is that the transmitted field in the formation is decomposed into two parts. A compressional field with a particle displacement in the direction of propagation of

$$u_R=\frac{V}{4\pi R}\frac{i\omega}{v_p}\frac{\rho_0}{\rho_1}\left(\frac{a^2}{a^2-b^2}\right)\frac{v_T^2}{v_s^2}\frac{1-2\dfrac{v_s^2}{v_p^2}\cos^2\delta}{1-\dfrac{v_T^2}{v_p^2}\cos^2\delta}\exp(-i\frac{\omega}{v_p}R) \tag{10}$$

and a shear wave with particle displacement normal to the direction of propagation

$$u_\delta=\frac{V}{4\pi R}\frac{i\omega}{v_s}\frac{\rho_0}{\rho_1}\left(\frac{a^2}{a^2-b^2}\right)\frac{v_T^2}{v_s^2}\frac{2\sin\delta\cos\delta}{1-\dfrac{v_T^2}{v_s^2}\cos^2\delta}\exp(-i\frac{\omega}{v_s}R) \tag{11}$$

R is the distance from the center of the source to the field point (Figure 1).

The pressure in the borehole is by the same approximation

$$p_0(\omega,z)=\frac{i\omega V}{2\pi(a^2-b^2)}\rho_0 v_T \exp(-i\frac{\omega}{v_T}z) \tag{12}$$

This is the so called tube wave propagating in the cylindrical annulus between the drillstring and the formation. The low frequency approximation to tube wave velocity, which also appears in (10) and (11), is

$$v_T=\frac{v_0}{\left(1+\dfrac{a^2}{a^2-b^2}\dfrac{\rho_0}{\rho}\dfrac{v_0^2}{v_s^2}\right)^{1/2}} \tag{13}$$

Normally the tube wave velocity is lower than the shear and compressional wave velocities in the formation and the tube wave sets up an evanescent field in the formation which is not included in (10) and (11).

2.2 Receiver

We will determine the response of a sensor at the surface of the drillstring due to an incoming plane P-wave or SV-wave (vertically polarized shear wave). The calculation of the receiver response is almost the same for the two cases, we outline the derivation for an incoming plane P-wave. The transmitting problem we studied previously is an axisymmetric problem but the receiving problem is not, because of the incoming plane wave and the assumption that the receiving sensor may be in an non-axial position and located on the surface of the drillstring. We consider an incoming plane P-wave with velocity amplitude U, frequency ω, incident angle δ, and azimuth angle θ (Figure 1). The plane incoming wave is expressed as a sum of cylindrical waves [9]

$$\phi_p = -i\frac{v_p U}{\omega}\exp\left(i\gamma_p r\cos\theta\right)\exp(i\omega t - ikz) = -i\frac{v_p U}{\omega}\left[J_0\left(\gamma_p r\right) + 2\sum_{m=1}^{\infty} i^m J_m\left(\gamma_p r\right)\cos m\theta\right]\exp(-ikz) \tag{14}$$

The incoming wave sets up a field in the borehole

$$\phi_f = \left[A_0 J_0(\gamma_f r) + a_0 N_0(\gamma_f r) + 2\sum_{m=1}^{\infty} i^m \left[A_m J_m(\gamma_f r)\cos m\theta + a_m N_m(\gamma_f r)\cos m\theta\right]\right]\exp(-ikz) \tag{15}$$

and two scattered waves in the formation, one SV-wave expressed as

$$\varsigma = \left[B_0 H_0^{(2)}\left(\gamma_S r\right) + 2\sum_{m=1}^{\infty} i^m B_m H_m^{(2)}\left(\gamma_S r\right)\cos m\theta\right]\exp(-ikz) \tag{16}$$

and a scattered P-wave

$$\phi = \left[D_0 H_0^{(2)}(\gamma_p r) + 2\sum_{m=1}^{\infty} i^m D_m H_m^{(2)}(\gamma_p r)\cos m\theta\right]\exp(-ikz) \tag{17}$$

In the expressions above ϕ, ϕ_p and ξ are scalar displacement potentials. There are now four unknown coefficients, A_m, a_m, B_m, and D_m that are determined by the same boundary conditions as before and, in addition, the tangential stress is zero at the borehole wall.

The pressure in the borehole is $p_0 = \rho_f \omega^2 \phi_f$ and examples of numerical solutions for the pressure in the borehole will be presented later. In the same way as for the transmitter problem one can derive simple expressions for the receiver response in the low frequency case. Each mode must be treated separately, for the fundamental mode m=0 the low frequency approximation of the pressure on a sensor at $r=b$ for an incoming P-wave in the formation with a displacement amplitude U_p is

$$p_0 = i\omega U_p(\rho_0 v_p)\frac{a^2}{a^2-b^2}\frac{v_T^2}{v_s^2}\frac{1-2\frac{v_s^2}{v_p^2}\cos^2\delta}{1-\frac{v_T^2}{v_p^2}\cos^2\delta} \tag{18}$$

With an incoming plane SV- wave with displacement amplitude U_s the pressure will be

$$p_0 = -i\omega U_s(\rho_0 v_s)\frac{a^2}{a^2-b^2}\frac{v_T^2}{v_s^2}\frac{2\sin\delta\cos\delta}{1-\frac{v_T^2}{v_s^2}\cos^2\delta} \tag{19}$$

The solutions (18) and (19) are the reciprocal solutions to the transmitting responses of (10) and (11).

3. Numerical results

In this section we will presents numerical results of transmitting and receiving responses in boreholes. The parameters of the examples are given in Table 1.

Parameter	Formation 1	Formation 2
P-wave velocity	3500 m/s	5000 m/s
S-wave velocity	2000 m/s	1000 m/s
Density of formation	2500 kg/m^3	2500 kg/m^3
Fluid velocity	1500 m/s	1500 m/s
Fluid density	1300 kg/m^3	1000 kg/m^3
Borehole radius	0.11 m	0.115 m
Drillstring radius	0.08 m	0.05 m

Table 1. Parameters for the numerical examples

3.1 Transmitting response

Figure 2 shows the transmitted field in a Formation 1 of Table 1 which may be typical for a rather hard and fast formation. Figure 2a shows the directivity patterns for the compressional and the shear components of the particle displacements. The compressional displacement has a maximum perpendicular to the borehole axis, the shear component has a maximum at approximately 45° in this case. The directivity patterns are normalized so that the maximum compressional displacement is set equal to unity, the figure therefore shows that maximum amplitude of the shear displacement may be larger than the maximum amplitude of the compressional displacement. Figure 2b shows an illustration of the evolving radiated field in time and space. The scales are in meters and the field is shown out to a distance of 12 meters from the source. The transmitted signal is a short Ricker pulse and the figure shows a snapshot of the field at a time $T = 3\,ms$ after the transmission. The two components traveling with the compressional and shear speeds respectively are clearly seen.

The direction for maximum shear displacement is given by the value of tube wave velocity of (13). When this velocity is much higher than the shear velocity of the surrounding formation the direction of maximum shear displacement will be close to 45°, as can be seen from (11). In most situations this will be the case and Figure 2 is representative for the normal case with a tube wave velocity of $v_T = 1078\ \mathrm{m/s}$, much smaller than the shear wave velocity of $v_s = 2000\ \mathrm{m/s}$. However, there are cases where (13) gives a value for the tube wave velocity that exceeds the shear wave velocity in the formation. Physically this means that there no longer exist a guided tube wave in the borehole since the tube wave is radiating out in the formation mainly in the direction given by

$$\delta_{max} = \arccos\left(\frac{v_s}{v_T}\right) \tag{20}$$

An example of this is shown in Figure 3 for the parameters of Formation 2 in Table 1. The radiation is totally dominated by shear radiation at a very narrow angle given by (18), in this case $v_T = 1033\ \mathrm{m/s}$, $v_s = 1000\ \mathrm{m/s}$ and $\delta_{max} = 14.5°$. On the scale used in Figure 3b the compressional field would not have been visible.

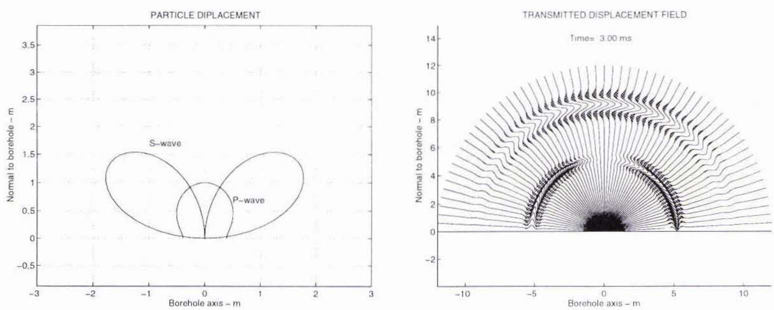

Figure 2 Transmitting response in Formation 1

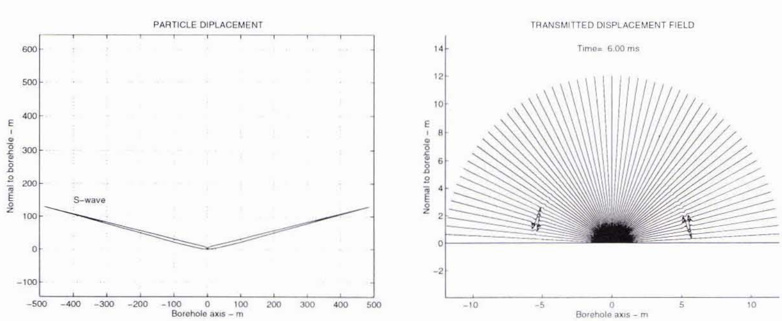

Figure 3 Transmitting responses in Formation 2

3.2 Receiving response

A computer code has been developed that can compute the full waveform field in the borehole by (15), both in time and frequency domain for any type of incoming plane wave. Figure 4 shows an example of the output from this program. The incoming P-wave has the form of a Ricker wavelet and the incidence angle is $\delta = \pi / 2$. The time signal of the incoming wave is shown in the upper right panel and its frequency spectrum is shown in lower right panel. The time response of a sensor located on the drillstring at $r = b$ and $\theta = 0$ is shown in the upper left panel and the frequency response for the borehole signal is shown in the lower left panel. The received signal is quite complicated and spread out in time, the frequency spectra indicates a number of sharp resonances.

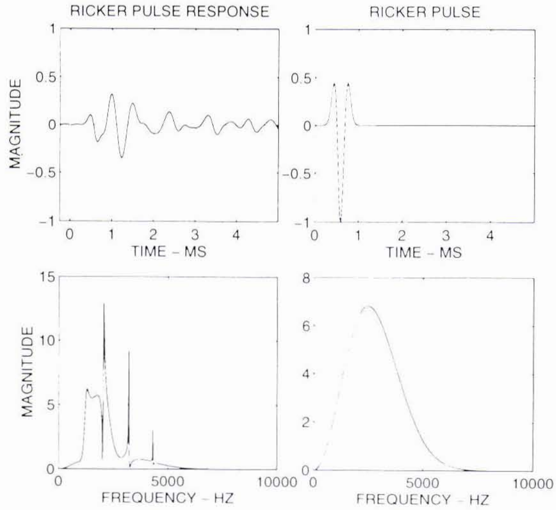

Figure 4. The response of a sensor on the drillstring caused by an incoming plane P-wave arriving from $\theta = 0$ and $\delta = \pi / 2$. Summation of modes $m = \{0, \cdots 5\}$

Equation 15 shows that the response of a sensor located at an angular position θ in respect to the direction of the incoming wave is given by a sum of mode functions $e_m(t)$ weighted by the cosine of the angular separation between the direction of the incoming wave and sensor position. Therefore the received time signal of a sensor can be expressed as

$$s(t) = \sum_{m=0}^{M-1} e_m(t) \cos(m\theta) \tag{21}$$

In principle the number of modes is infinite, in practice only a few modes are significant. The number of significant modes depends on the parameters of the formation, the size of the borehole, the drill string and the frequency spectrum. For realistic parameters, e.g. as given in Table 1, the number of significant modes may be quite small. This is illustrated in Figure 5, showing the time and frequency functions for the first 6 modes, $m = \{0, \cdots 5\}$ for the same example as in Figure 4. The sum of the traces of Figure 5a yields the received signal shown in the upper left panel of Figure 4. In this case only the first 3-4 modes are significant.

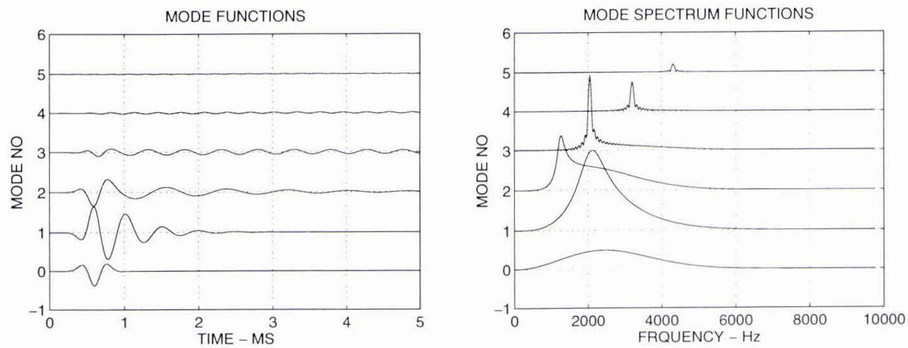

Fig. 5. Mode functions for modes $m = \{0, \cdots 5\}$ Parameter set no.2, Table 1
a) Time functions, b) Frequency spectra functions.

The received signal may have a number of resonances caused by waveguide modes in the fluid annulus between the drillstring and the formation. Borehole resonances were discussed by Lowell and Hornby [8], but only resonances in the received signal of a sensor at the center of a borehole without drillstring. In that case only the fundamental mode m=0 contributes. For a sensor positioned non-symmetrically in the borehole there will be a number of different vibrational modes, each with potentially many resonance frequencies. From Figure 5b it can be observed that the modes m= 5, 4 and 3 have sharp resonances at the approximate frequencies of 4.2 kHz, 3.2 kHz and 2.1 kHz respectively and mode m=2 has a weaker resonance at approximately 1.25 kHz. The resonances are caused by poles in the transfer function and are given by the zeroes of the determinant of the set of linear equations that determines the unknown coefficients. The expression for the determinant is quite complicated and requires numerical analysis. However, an understanding is obtained by considering the simpler case, where both the drillstring and the formation are infinitely hard. With this boundary condition, the radial displacements equal zero at radius $r = a$ and $r = b$. The characteristic equation for borehole modes can easily be obtained from (2) and attains the form

$$\begin{aligned} &\left[mJ_m(\gamma_f a) - \gamma_f aJ_{m+1}(\gamma_f a)\right]\left[mN_m(\gamma_f b) - \gamma_f bN_{m+1}(\gamma_f b)\right] \\ &-\left[mN_m(\gamma_f a) - \gamma_f aN_{m+1}(\gamma_f a)\right]\left[mJ_m(\gamma_f b) - \gamma_f bJ_{m+1}(\gamma_f b)\right] = 0 \end{aligned} \tag{22}$$

Consider first the fundamental mode m=0. The first zero of (22) occurs when $\gamma_f a = 0$ and gives the mode normally referred to as the Stoneley mode or the tube wave mode. The second zero for m=0 is for $\gamma_f a = 11.6$ for the values of a, b and v_f of Formation 1 in Table 1. For normal incidence, $\delta = \pi / 2$, this mode gives a resonance frequency where the thickness of the annulus *(a-b)* is approximately equal to half the wave length in the fluid. This is normally a quite high frequency, in the current case the resonance frequency is 25.2 kHz, which is outside of the frequency range covered by the incoming wave. Therefore the approximate low frequency result derived for the transmission response is valid up to quite high frequencies. The fact that the mode m=0 is only slightly dispersive means that its waveform is nearly an exact replica of the incoming wave. The resonances of the higher modes may occur at rather low frequencies which is important for the shape of the receiving responses. For mode m=1 (22) predicts a resonance at 2.7 kHz which is difficult to recognize in Figure 5. However, in the low frequency approximation the time function of this mode is the negative time derivative of the time signal of mode m=0 (Schoenberg, [6]). The resonance frequencies are also dependent on the angle δ and the dependency is easily determined from (3). When $f_{res}(\delta = \pi / 2)$ is any resonance frequency for normal incident wave, this resonance will for any other incident angle move to the frequency

$$f_{res}(\delta) = \frac{f_{res}(\delta = \frac{\pi}{2})}{\sqrt{1 - \left(\frac{v_f}{v_p}\cos(\delta)\right)^2}} \tag{23}$$

It follows from (21) that the total response will be quite different depending on the angular position of the sensors with respect to the incoming wave. This fact is illustrated by the simulation in Figure 6 which shows the responses of 8 sensors located at equal spacing of 45° on the surface of the drillstring, the incoming plane P-wave arrives at angle of $\theta_{in} = -20^\circ$ with respect to the position of sensor no. 0.

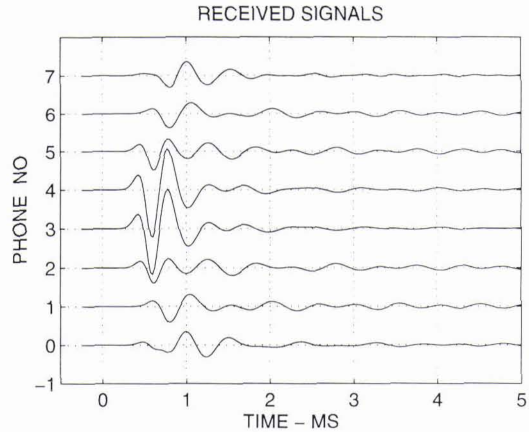

Fig. 6 Simulated received time signals on 8 equally spaced sensors. Plane wave arriving with $\theta_{in} = -20^0$ and $\delta = 90^0$

4. Conclusions

The radiation and receiving responses for acoustic sources and receivers in fluid-filled boreholes have been studied theoretically and numerically.

Approximate expressions for the radiation from a cylindrical source have been derived. These are valid for frequencies lower than the first resonance frequency of the fundamental mode, which cover most frequencies of interest.

The receiver response of a non axially position sensor on the drillstring due to an incoming plane wave has been established and solved numerically. The results show that the high frequency response is very complicated and composed of many vibrational modes and that the higher modes may have strong resonances at relative low frequencies.

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