Seabed Characterization by Inversion of Parametric Sonar Data: Selection of the Cost Function

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Abstract

A class of possible cost functions to be used in model-based estimation of bottom parameters from high-frequency backscatter signals is introduced. The cost is based on the properties of the signal in the wavelet-transformed domain. It is shown, through a simulative study, that the cost can be selectively tuned to be sensitive only to subsets of the parameters to be estimated. This in turn suggests that a "divide-and-conquer" inversion strategy can be applied at least in the case of normal incidence bottom returns from parametric sonar data.

1. Introduction

Model-based inversion of acoustic data has been proved an efficient and valuable approach to the estimation of geophysical properties of the ocean and/or of seafloor sediments when working with deterministic acoustic propagation [1]. This situation usually arises when the acoustic signals propagating in the ocean environment are of low frequency nature (up to 1 KHz). When dealing with higher frequency acoustic, the stochastic nature of the interaction between the propagating signals and the medium inhomogeneities cannot be neglected. This makes the application of model based inversion techniques much more difficult (or even questionable) at the high frequency regime with respect to their low frequency counterparts. This work focuses on the problem of quantitatively estimate the upper seafloor sediment properties (sound speed, density, compressional wave attenuation, seabed rms height, etc.) from bottom backscatter data. The received signals can be modeled as the output of a stochastic process, whose parameters are to be estimated. This makes it meaningless to compare directly the measured data with model-predicted time series. On the other hand, some possible approaches based, for instance, on the computation (from the data) of the backscattering strength vs. grazing angle function may not exploit all the information present in the data.

In this work we discuss some possible choices of cost function, to be minimized in the inversion process, that attempt to exploit as much information as possible from the received data while at the same time avoiding or reducing the effect of the randomness implicit in the physical process. In particular, it is shown how multi-resolution wavelet based data analysis can be used in order to perform the inversion in the wavelet-transformed domain. Moreover, it is possible to tune the cost function in order to make it selectively sensitive only to a subset of the parameter at a time. This in turn suggests that a wavelet-based processing can be well suited for a divide-and-conquer inversion strategy, in which all the parameters of interest are recovered in sequence. Examples are shown from at-sea data collected with the "Topas" parametric sonar system, and with simulated data generated with the "Boris" backscattering stochastic model.

The paper is organized as follows: in the next section some preliminaries on the forward model and on wavelet processing are given. In section 3 it is shown, by means of examples on field data, how wavelet-based processing can effectively be employed to discriminate between different bottom types, at least in a qualitative way. In section 4, a systematic approach to the construction of a class of wavelet-based cost functions is proposed, and it is shown by simulative examples how to select, within the same class, cost functions sensitive to a subset of the sought-for parameters. Finally, some discussion of the current results, open problems, and future work is given.

2. Preliminaries

2.1 Choice of the forward model

A preliminary issue that has to be addressed before even discussing the inverse problem is the selection of the forward model. While the status of ocean acoustic models up to few kHz is relatively mature and well understood [2], the same does not apply to acoustic propagation at high frequency. It is common to have at hand models able to predict some average properties of the measured time signals [3]. Less common is the case in which a prediction of the measured time series is provided. The time series prediction has to be intended, in this context, as a *specific realization* of a stochastic process. It has to be underlined that, even in the case of perfect correspondence between model parameters and physical parameters, model predictions and measured data will in general be different, corresponding to different realizations of the same process. While this situation prevents from a direct time series comparison in any inverse scheme, the availability of a model that does predict data realizations gives, to the designer of a model-based estimation strategy, the greatest flexibility in terms of processing steps. A forward model for bottom backscattering with these capabilities is Boris. A complete description of Boris can be found in [4]-[7]. Boris is able to generate a time series realization given as input the parameters related to: the environment (sound speed in the water column, sound speed, density and attenuation in the sediment, bottom surface roughness as rms elevation, volume inhomogeneity as rms per cent variation of sound speed and density); the geometry (water depth, source-receiver position, grazing angle, tilt); the system (pulse shape transmitted, beam pattern, source level). The output of the model is produced by generating a specific realization of the bottom surface and volume with the specified average properties, and then by computing the specific (deterministic) acoustic return due to the generated surface and volume. The seafloor interface scattering is modeled using the Kirchhoff approximation, and the volume contribution with the small perturbation theory.

2.2 Wavelet processing

Wavelet processing and wavelet transform have received much attention in recent years, essentially due to their ability of producing a time-frequency analysis of a given signal with variable resolution. A general reference text on the subject is [8], while a tutorial paper with specific application of wavelet processing to high frequency backscatter data is [9]. Wavelet processing is based on the choice of a set of basis functions $\phi(t;\tau,\alpha)$ generated in the following form. Let $\phi(t;0,1)=f(t)$; then $\phi(t;\tau,\alpha)=f((t-\tau)/\alpha)$, where τ is a time shift and α is a scale factor. The function $f(t)$ is sometime called the "mother" wavelet, since it generates all the remaining wavelet basis by means of time shifts and scaling operations. It must be remarked that the mother wavelet cannot be any arbitrary function, but has to satisfy several conditions to guarantee orthonormality and completeness of the basis. Note also that if the shift and scale parameters are allowed to be continuous, we talk of wavelet transform; if they are allowed to take a numerable set of values we talk of wavelet series, in analogy with the case of Fourier basis functions. Given an admissible wavelet basis, a generic signal $x(t)$ has the following representation in the τ-α domain:

$$X(\tau,\alpha) = \int_{-\infty}^{+\infty} x(t)\phi(t;\tau,\alpha)dt \tag{1}$$

There are some formal conditions that the basis and the signals have to satisfy to guarantee that the integral in (1) is convergent. In general, these conditions are respected if both the signal and the wavelets have finite energy, as it will be the case in the discussion in the following of the paper. Essentially $X(\tau,\alpha)$ is built as a bidimensional matrix by computing the correlation of the signal $x(t)$ with scaled versions of the mother wavelet $f(t)$. The scale factor α plays a role similar to that of the frequency in the more traditional time-frequency analysis. The similarity becomes a formal equivalence if $f(t)$ is taken as a windowed sine function. In this case, (1) becomes the common expression for the Short Time Fourier Transform. With respect to time-frequency analysis, wavelet processing offer in general more flexibility and variable resolution. However, depending on the specific wavelet basis chosen, it is not always clear what is the physical interpretation of the transform $X(\tau,\alpha)$.

In the following of the paper, we will make use of the wavelet transform with the mother wavelet described by Deaubechies [10], and reported in Figure 1. The reason for this specific choice is mainly because this is the mother wavelet whose shape is closest (at least, among those mother wavelets we are aware of) to that of the Ricker pulse transmitted by the parametric sonar system through which the field data have been obtained. This has some importance in understanding the physical meaning of the specific wavelet transform chosen. In particular, if the mother wavelet is chosen (as in our case) as equal to the transmitted pulse (or close to it as it can be), in the completely deterministic case (no roughness, no inhomogeneities) one would obtain a matrix composed by the deconvolved signal in one row, and zero in the other rows. The effect of the stochastic parameters is to blur the deconvolution results, and the wavelet transform is able to trace the blurring both in the time and scale domain.

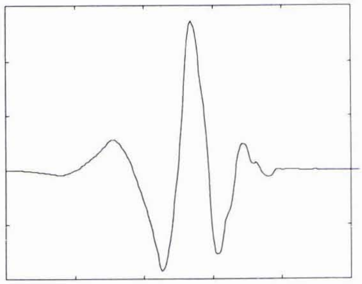

Fig. 1: Deaubechies wavelet used for the transform (1). Time and amplitude axes are not reported, since the function can be arbitrarily scaled and amplified

3. Field Data Analysis with the Wavelet Transform

In this section it is our intention to show how field data taken on different bottom types show systematic and quantifiable differences when processed with the wavelet transform introduced previously. This is a key point in order to attempt a definition of a meaningful cost function for a model-based inversion scheme.

The data to be shown have been collected at the Saclantcen test sites in the gulf of La Spezia, Italy, and have already been partially reported in [7]. The experiment configuration was as follows: the "Topas" 040 parametric sonar from SIMRAD was used in monostatic configuration; the system was transmitting a calibrated Ricker pulse with a center frequency of 8 kHz, and a bandwidth of approximately 8 kHz, steered at normal incidence toward the bottom. Data were acquired at a sampling rate of 100 kHz. The system has been used at three sites, in the following labeled as 'Tellaro', 'Portovenere' and 'Monasteroli', having water depths varying between 12 and 17 meters. At these three sites, the bottom is described as compact sand, silty clay and gravel on sand respectively.

Each signal in the data set has been transformed according to (1), with the following choices: 125 samples (corresponding to 1.25 ms of data) have been selected for each signal; the signals have been aligned, so to avoid any effect due to the different bottom depths; the scale parameter α has been let moving from 1 to 50. The following notational convention is used: $X_{i,j}(\tau,\alpha)$ is the wavelet-transformed signal, with $i=T,P,M$ identifying the location of the original signal, and $j=1,2,...$ identifying the specific signal among those of the location i. The wavelet transform of the different signals are now compared by means of the following difference matrix $\Delta(i,j,k,h)$ defined in the following way:

$$\Delta(i,j,k,h) = \left| \left| X_{i,j}(\tau,\alpha) \right| - \left| X_{k,h}(\tau,\alpha) \right| \right| \qquad (2)$$

Note that Δ is a matrix whose elements are indexed by τ and α; each element in Δ is the modulus of the difference of the moduli of the corresponding elements in the matrices $X_{i,j}$ and $X_{k,h}$. By looking at the difference matrices, it appears systematically that the difference matrix of two signals taken in the same area has elements of about one order of magnitude smaller than the difference matrix of signals taken at different areas. This is shown for instance in fig. 2, where, with the same scale on the z-axis, the difference matrix of two signals from Portovenere and Tellaro is compared with the difference matrix of two signals from Portovenere.

Another example of the same kind is shown in fig. 3. This behaviour of the difference matrix is systematic, at least in the three data sets that we had available. This suggests that the comparison in the wavelet-transformed domain is able to capture the differences between different bottom types. This may be used, for instance, in a qualitative bottom classification scheme, in which the seafloor is characterized as "sand"-"silt"-"clay" types. The purpose of the next section is to explore if more quantitative information can be extracted by comparing the data with model realizations in the wavelet-transformed domain.

4. Cost function selection and parameter sensitivity

In this section we propose a systematic procedure for the definition of a cost function to be used in model-based inversion schemes, relying on the wavelet transform (1). Furthermore, we proceed with a sensitivity analysis of the cost function with respect to the bottom parameters to be estimated. A large use of the Boris model is made in this section, however the procedure in itself is independent of the particular choice of forward model.

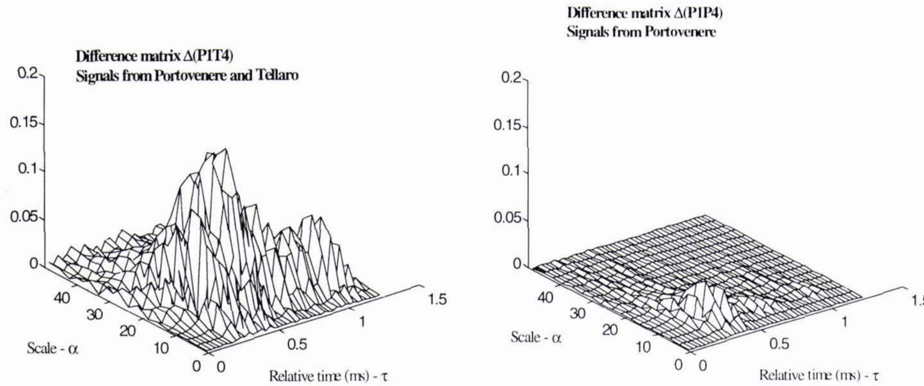

Fig. 2: Difference matrix between signals from Portovenere and Tellaro (left) and difference matrix between two signals from Portovenere (right). Note the difference in magnitude when the signals are coming from different areas.

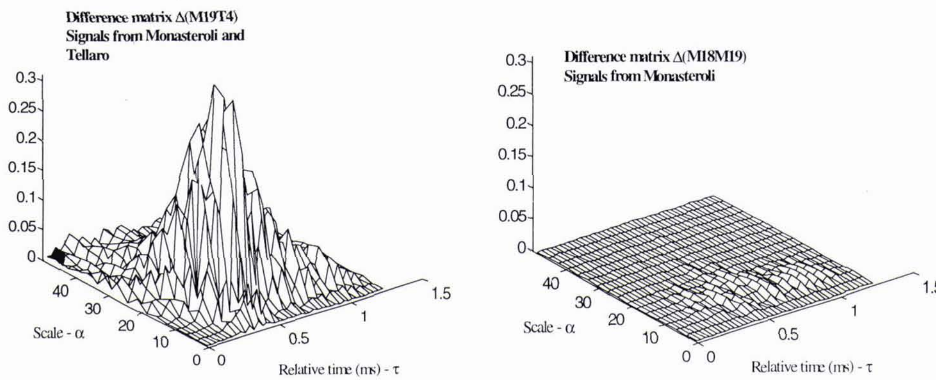

Fig. 3: Difference matrix between signals from Monasteroli and Tellaro (left) and difference matrix between two signals from Monasteroli (right). Note again the difference in magnitude when the signals are coming from different areas.

Let $X_d(\tau,\alpha)$ be the wavelet transform of the data time series $d(t)$. Let $\mathbf{m}$ be the vector of bottom parameters to be estimated. In particular, in the following we will consider $\mathbf{m}=[\sigma, c_p, \nu, \alpha_p]$, where σ is the surface roughness rms in meters, c_p is the sediment sound speed in m/s, ν is the rms sound speed and density volume inhomogeneity, in percent variation w.r. to c_p and ρ, α_p is the compressional wave attenuation in the sediment in db/m. The case of signals at vertical incidence is considered; it is well known that in this case sediment sound speed and sediment density ρ cannot be recovered independently, but only the acoustic impedance $z=\rho c_p$ can be estimated. In the simulation study presented in this section we keep the density linearly related with the sound speed, and we refer to sound speed estimation. However, it has to be clear that what is actually estimated is the acoustic impedance. The assumption of no layering in the sediment is also made. The case of multiple layers is briefly discussed in the next section. The knowledge of the other relevant environmental, geometric and system parameters is assumed.

For a given choice of the bottom parameters $\mathbf{m}$, N realizations of the data $r(t;\mathbf{m},i)$, $i=1,...,N$, are generated through Boris or through any equivalent model. At each realization is associated its wavelet transform matrix $X_{ri}(\tau,\alpha;\mathbf{m})$. The ensemble average of the wavelet transform is then computed:

$$X_r(\tau,\alpha;\mathbf{m}) = \frac{1}{N}\sum_{i=1}^{N} X_{ri}(\tau,\alpha;\mathbf{m}) \qquad (3)$$

At this point we are ready to define the following class of cost functions:

$$J(\mathbf{m}) = \left\| \left(\left| X_d(\tau,\alpha) - X_r(\tau,\alpha;\mathbf{m}) \right| \right) \exp(-\beta / (\left| X_d(\tau,\alpha) \right| + \left| X_r(\tau,\alpha;\mathbf{m}) \right|) \right\| \qquad (4)$$

where the absolute value operations are done element by element, *i.e.*, for each (τ,α) index; $\|\cdot\|$ stands for the usual euclidean matrix norm and the parameter β is a positive real constant selected by the user. The usefulness of the parameter β lies in the fact that, by appropriate tuning, it can be used to increase the sensitivity of the cost function J to some of the parameters in $\mathbf{m}$, and to decrease the sensitivity to some other parameters. The sensitivity of J to a generic element m_i of the vector $\mathbf{m}$ is defined in the usual way as:

$$S(m_i) = \frac{\partial J(\mathbf{m})}{\partial m_i} \qquad (5)$$

Roughly speaking, a "large" value of β gives more weighting to the part of the signal with more energy, which is in general due to the sediment surface return contribution; on the contrary, a "low" value of β tends to preserve the energy content of the original time series (note that $\beta=0$ is a possible choice, which corresponds to no weighting applied to the signal in the wavelet-transformed domain). Note that here "large" and "low" strongly depend on the quantities $|X_d(\tau,\alpha)|$ and $|X_r(\tau,\alpha)|$: in our case, the sum of the two reaches peak values of the order of magnitude of 10^1. This means that, with the choice of β=20 (discussed in the following), the peak values will be weighted by a factor of about 0.1, and the weighting will rapidly decrease moving away from the peak. The idea of the parameter β comes from traditional seismic signal processing, where it is customary to use time-varying adaptive gain on the recorded time series. Here, however, the adaptation gain strategy is performed in the wavelet transformed domain. In particular, we will show, with simulated data generated with Boris, using the calibrated Topas beam pattern, pulse shape and source level, that, by taking β of the order of 10^1, the cost function (4) is such that the sensitivity (5) to the volume parameters ν and α_p is negligible. This, in turn, suggests the following inversion strategy: a cost function (4) is defined with $\beta \geq 10$; the cost is minimized with respect to the parameters σ and c_p. With these estimated values, a new cost function is defined with $\beta=0$, and it is minimized with respect to the remaining parameters ν and α_p.

The (simulated) reference data $d(t)$ have been generated by assuming an environment with the following parameters: σ=0.04 m; c_p=1720 m/s; ν=5%; α_p=0.9 dB/m; ρ=1.9 g/cm^3. The time signals have been generated as in the field data test case, with a sampling of 100 kHz, and only 125 samples have been considered in the wavelet analysis. A value of β=20 has been chosen. The number N of different realizations to be used in computing the cost (4) has been fixed to 10. In fig. 4 we present the cost plotted as a function of σ and ν. This has been obtained by computing J with values of σ ranging from 0.03 to 0.05 m at steps of 0.0015 m, and ν ranging from 0.5% to 11% at steps of 0.75%. The other parameters were kept constant at their reference value.

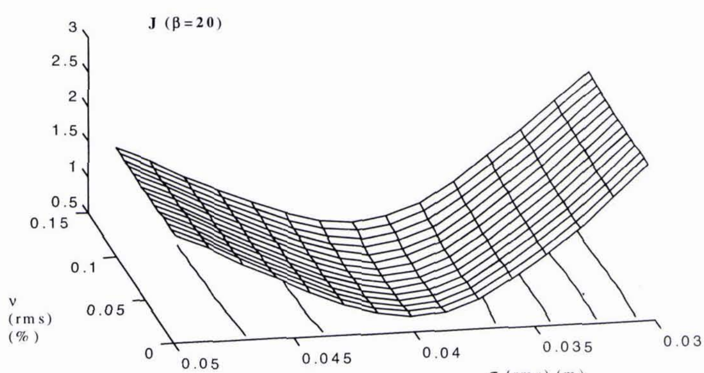

Fig. 4: Low sensitivity of the cost function J with β=20 to the volume inhomogeneity parameter (ν).

It can be seen from the figure that the sensitivity of the cost function to the volume inhomogeneity parameter is negligible (the sensitivity (5) can be evaluated from the figures by looking at the variation of the cost in the direction parallel to that of the axis of the parameter whose sensitivity is investigated; if the contour lines are parallel to some of the axis, the cost is not sensitive to that parameter). Negligible sensitivity is also experienced with the sediment compressional wave attenuation, as it appears in figure 5. Here the cost function has been generated by varying the surface roughness as before, by varying the compressional wave attenuation between 0.3 and 1 dB/m at steps of 0.05 dB/m, and by keeping the remaining parameters fixed.

The cost function is sensitive to both the surface roughness and the sediment sound speed. In fig. 6 the cost as a function of these two parameters is plotted. Note that not only the cost has been generated with different realization of the time series, but also, in this case, the insensitive parameters ν and α_p have been chosen at random for each realization, in order to further verify their passive influence on the cost. In this test the surface roughness has been sampled with the same range and step as before, while the sediment sound speed has been sampled from 1540 m/s to 1960 m/s with a step of 30 m/s. As stated before, the sediment sound speed is *not* independent from the density, since we are treating the normal incidence case. We have fictitiously imposed a dependence of the density from the sound speed of the form $\rho = 1.2*10^{-3}*c_p - 0.1640$. So the sound speed estimation has actually to be intended as an estimate of the acoustic impedance. From fig. 6 it can be seen that there is a unique well defined minimum in the cost function, corresponding to the correct reference data parameters. It is interesting to note that the cost function is very smooth, with a unique minimum.

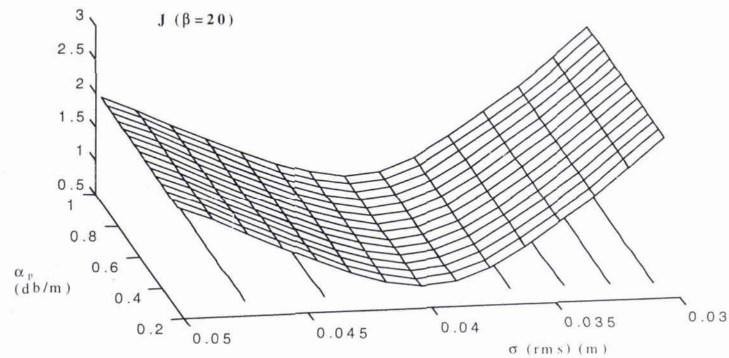

Fig.5: Low sensitivity of the cost function J with β=20 to the sediment compressional wave attenuation (α_p).

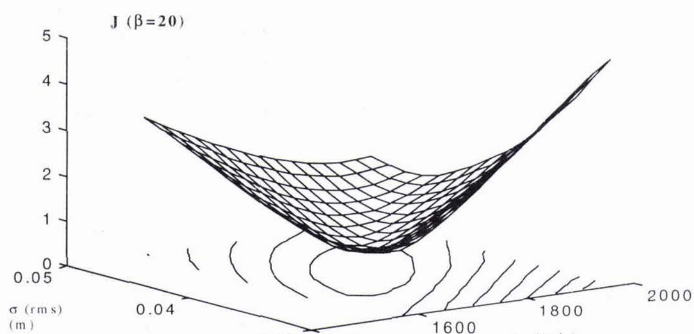

Fig. 6: Cost function J with β=20 w.r. to sediment sound speed and surface roughness

We remark that, although the simulations presented refer to a specific case, they have been run for a number of cases, and these results, at least qualitatively, are systematic. At this point, by selecting a proper value of the parameter β, it is assumed that a correct estimate of the surface roughness has been obtained. Having fixed the surface roughness, a new cost function belonging to the class (4) is defined, this time by choosing β=0. In fig. 7 the cost function is shown with respect to sediment sound speed and volume inhomogeneity. Also in this case, there is a unique minimum at the expected position, although the cost function is less smooth.

In this case, in which the volume gives significant contributions to the cost function, the compressional wave attenuation is also expected to play a significant role. However, in the example reported in fig. 7, the bidimensional cost has been obtained by fixing the attenuation to a constant value of 0.5 dB/m, different from that of the reference data. It can be seen that this has no influence on the correct determination of the other two parameters. This is confirmed if we plot the cost as a function of volume inhomogeneity and compressional wave attenuation, with surface roughness and sediment sound speed fixed to their correct (reference) values. As it can be seen in fig 8, the cost function has low sensitivity to the attenuation parameter; however, it has to be remarked that the sensitivity to the compressional wave attenuation is not entirely negligible, as it was in the previous cases discussed. In any case, it is not sufficient to correctly identify the attenuation parameter. Note that the volume inhomogeneity parameter is correctly recovered.

It is not clear to us at the moment why the cost function is not sensitive to the sound speed attenuation; it may be due that, by taking only 1.25 ms of data, the travel time within the bottom is not sufficient to consistently show the effect of the attenuation; or, it may be that the wavelet-based cost function itself masks the attenuation.

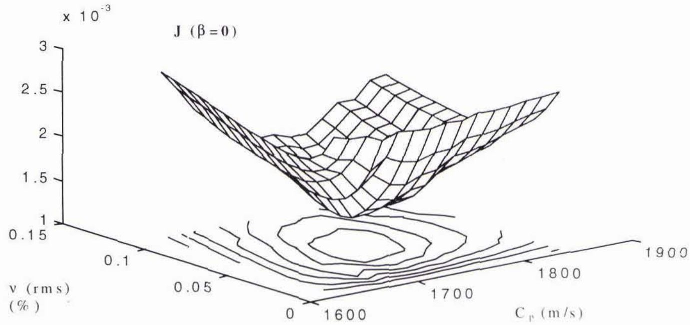

Fig.7: Cost function J with β=0 w.r. to sediment sound speed and volume inhomogeneity

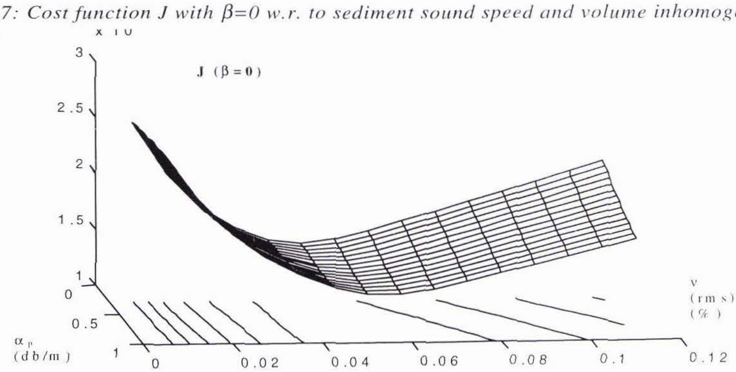

Fig 8: Low, although not entirely negligible, sensitivity of the cost function J with β=0 to the sediment compressional wave attenuation α_p

5. Discussion and Conclusions

Wavelet processing of normal incidence backscatter signals from parametric sonar seems to have the potential for a proper classification and model based inversion of both deterministic and stochastic sea bottom parameters. Many questions, however, remain open. One is the physical explanation of why should wavelet basis show this property. We are currently exploring other approaches to time-frequency and time-scale analysis (in particular, with the Wigner-Ville distribution) in an attempt to clarify this aspect.

An implementation of a "divide and conquer" inversion strategy, relying on the class of cost functions defined in (4), is currently under development. It is expected, however, that even better results, in terms of selectively reducing the sensitivity to some of the parameters, may be obtained by using backscatter signals at different frequencies. In particular, using a parametric sonar, the primary frequency may be used for the surface roughness and acoustic impedance determination, without recurring to the tuning of the parameter β. Note that, in the case of the Topas system, the primary frequency is at about 40 kHz, while the difference frequency signals taken as reference in this work have center frequency at 8 kHz.

The case of multiple layers can be treated with the same approach underlined here, by inverting the parameters of one layer at a time (a technique known as "layer stripping" in the seismic community). It has to be said, however, that all the inversion methods of the divide-and-conquer type, including layer stripping, may suffer of severe inaccuracies from early errors: any inaccurate result is in fact carried over in the subsequent steps of the inversion procedure. This implies that an accurate analysis of the robustness of the proposed scheme will also be necessary in the near future.

Acknowledgements

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