Report no. changed (Mar 2006): SR-132-UU

SACLANTCEN REPORT
serial no.: SR-132

SACLANT ASW RESEARCH CENTRE REPORT

Numerical modeling results for mode propagation in a wedge

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November 1987

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Abstract: A PE (parabolic equation) code is used to study propagation in a shallow water wedge with a penetrable bottom. Particular attention is given to 'wedge modes' that have wave fronts that are arcs of circles centered on the wedge apex. For downslope propagation and for upslope propagation before cutoff, the wedge modes propagate independently without coupling. For upslope propagation through cutoff, there is a small amount of coupling to the next lower mode and a trace of coupling to the next higher mode. The wedge modes can be considered the natural modes of the wedge.

Keywords: mode coupling ○ mode cutoff ○ normal modes ○ numerical modeling ○ parabolic equation ○ range-dependent propagation ○ sloping bottoms ○ wedge modes

Contents

This report is reprinted from the *Journal of the Acoustical Society of America*, **82**, 1987: 211–216.

Numerical modeling results for mode propagation in a wedge

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(Received 1 December 1986; accepted for publication 27 March 1987)

A PE (parabolic equation) code is used to study propagation in a shallow water wedge with a penetrable bottom. Particular attention is given to "wedge modes" that have wave fronts that are arcs of circles centered on the wedge apex. For downslope propagation and for upslope propagation before cutoff, the wedge modes propagate independently without coupling. For upslope propagation through cutoff, there is a small amount of coupling to the next lower mode and a trace of coupling to the next higher mode. The wedge modes can be considered the natural modes of the wedge.

PACS numbers: 43.30.Bp

INTRODUCTION

Recent theoretical[1] and experimental work[2,3] on sound propagation in a shallow water wedge with a penetrable bottom suggests that there exists a set of natural modes that propagate in the wedge with negligible coupling. In the present work, we use a PE (parabolic equation) code to study "wedge modes" and to examine their properties for both downslope and upslope propagation. Numerical modeling has the great advantage that it is easy to vary the parameters and, in particular, to model steep slopes.

In the past few years there has been much interest in underwater sound propagation over a sloping bottom. The PE calculations of Jensen and Kuperman[4] for upslope propagation showed that, as a mode reaches its cutoff depth, its energy is transferred to the bottom as a well-defined beam with negligible transfer of energy to other modes. This absence of mode coupling when it might be expected to be largest was interesting and led to the search for a mode description that would be valid as a mode passed through its cutoff depth. Arnold and Felsen[1] have formulated a solution in terms of "intrinsic modes" that is valid through mode cutoff. However, more important, the formulation shows that the intrinsic modes propagate independently, without coupling to other modes. The intrinsic modes are assumed to be the natural modes of the wedge, but as yet there are no numerical results which show their detailed properties.

Buckingham[5,6] showed that an exact modal solution can be formulated for an isovelocity wedge with perfectly reflecting pressure-release boundaries. The natural modes for this idealized model are sine functions in a cylindrical coordinate system centered on the wedge apex. Buckingham[6] suggested that the natural modes of a real wedge would be similar and gave an approximate solution for a wedge with a penetrable bottom in terms of an "effective wedge" containing a displaced pressure-release bottom.

In recent articles,[2,3] experiments to examine the propagation of normal modes over a sloping bottom were reported. These experiments were for downslope propagation and showed that it is possible to generate uncoupled modes in a wedge. These "wedge modes" have wave fronts that are curved into arcs of circles centered on the wedge apex, and to generate a wedge mode experimentally it was necessary to curve the line source array. The experimental results were in general agreement with Buckingham's[5,6] solutions.

We shall here pursue a simple heuristic approach to constructing a natural mode of a wedge with a penetrable bottom. We assume that the pressure amplitude distribution perpendicular to the free surface is identical to that of a standard vertical mode. Next, we take account of the fact that a cylindrical coordinate system centered on the wedge apex is the natural coordinate system for the wedge. The vertical mode is mapped onto cylindrical coordinates by associating with the vertical mode a curved wave front centered on the wedge apex. We shall demonstrate that the wedge mode so constructed does indeed propagate through the wedge with negligible coupling and, hence, closely represents a natural mode of the wedge.

The PE model is first used to examine the effect of wave front curvature on the downslope propagation of modes in a wedge and we confirm the results of Tindle *et al.*[2,3] The model is then used to look at upslope propagation.

The upslope case is difficult to study experimentally for two reasons. First, it is dominated by a transfer of energy into the bottom as modes reach cutoff. Therefore, for a full investigation, it is necessary to make measurements in the bottom. Second, to generate a wedge mode in deep water, it is necessary to have a line array with a large number of source elements. Our results will show that the wedge modes propagate upslope without coupling, except for a very small transfer of energy to adjacent modes as they reach cutoff. Almost all the energy in a given mode enters the bottom as a beam as originally described.[4]

I. THE PARABOLIC EQUATION MODEL

The particular PE code[7] used in the present study solves the Thomson–Chapman wide angle parabolic equation.[8] Since it is extremely difficult to assess the overall numerical accuracy of a numerical solution to a complex propagation

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problem, we have used a parabolic equation that is particularly well suited for wide angle propagation problems. Moreover, to improve the accuracy of the PE solution for the strongly range-dependent situations considered, we have implemented the procedure for updating the PE reference wavenumber with range, as suggested by Pierce.[9] This procedure minimizes accumulated phase errors due to the choice of reference wavenumber. The final check, however, on the accuracy of the PE solution was a comparison with results generated by an entirely different numerical code based on coupled (conventional) normal modes.[10,11] There was very close agreement between the two solutions and we are confident that the PE results presented here are accurate solutions of the problems studied.

In the results to be presented in this article, the environmental model has cylindrical symmetry about a vertical axis through the source position and consists of a "wedge" of water lying over a fluid bottom. The bottom forms a conical hill for downslope propagation and a conical bowl for upslope propagation. The water has density ρ_1 and sound speed c_1 and the bottom has density ρ_2, sound speed c_2, and attenuation coefficient α. The parameter values are $c_1 = 1500$ m s^{-1}, $c_2 = 1800$ m s^{-1}, $\rho_1 = 1000$ kg m^{-3}, $\rho_2 = 2000$ kg m^{-3}, $\alpha = 1$ dB/λ.

In all cases shown, the bottom slopes at 10°. Both smaller and greater slopes have been examined and they give

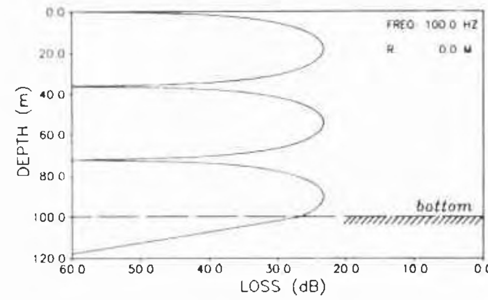

FIG. 2. Pressure amplitude versus depth for the source fields of Fig. 1.

rise to the same effects. Except for Fig. 4, the source frequency is 100 Hz throughout.

II. DOWNSLOPE PROPAGATION

A. Wedge modes and vertical modes

In the work of Ref. 2, a line source was used to generate individual modes in a wedge. It was found experimentally that if the line source was vertical, the mode generated was not uncoupled and the sound field showed structure due to mode interference. To generate an uncoupled mode, it was necessary to align the source array to follow an arc of a circle centered on the wedge apex. The corresponding displace-

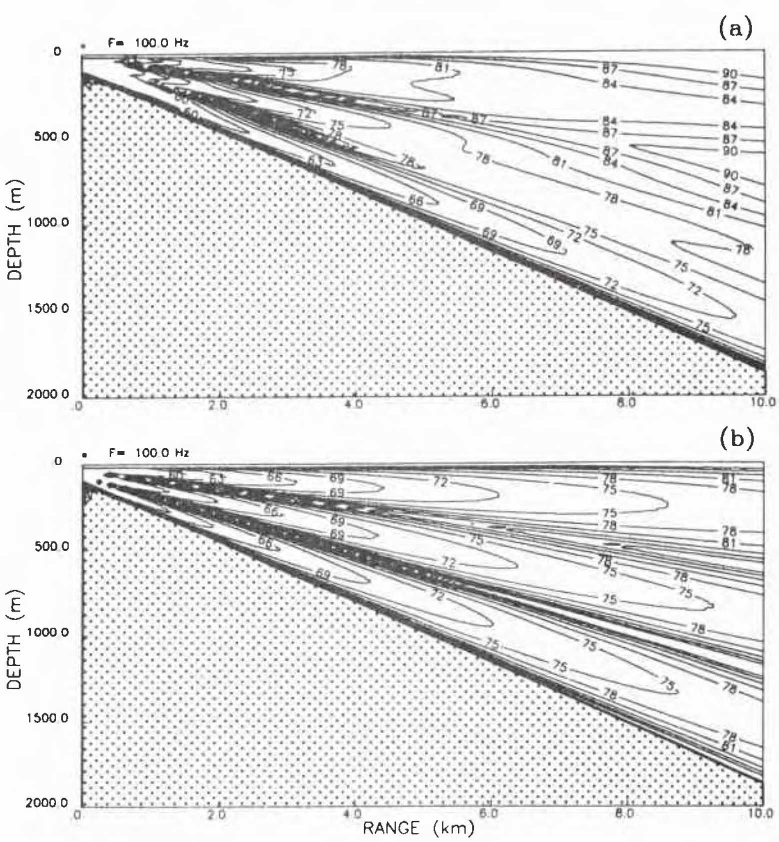

FIG. 1. Contours of dB loss for downslope propagation in a wedge. The source fields have the depth dependence of mode 3 and stimulate (a) a vertical mode, and (b) a wedge mode. The model parameters are given in the text.

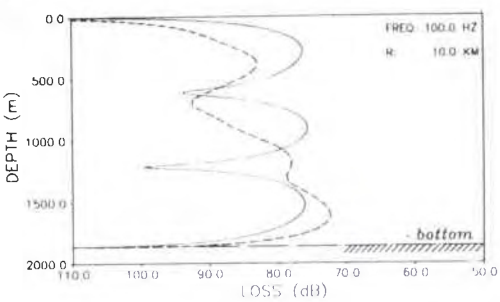

FIG. 3. Loss versus depth at a range of 10 km in Fig. 1(a) (--- vertical mode source) and 1(b) (—— wedge mode source).

ments of the source elements away from vertical were small but of crucial importance.

The effect of line source alignment is readily examined using the PE model and the results are shown in Fig. 1. The model has a harmonic line source at a frequency of 100 Hz and the water depth at the source is 100 m. The contoured field solutions show propagation loss for a 10° downslope situation using two different source distributions, each with the depth dependence of mode 3.

In Fig. 1(a), the source distribution is a conventional vertical mode, and, in Fig. 1(b), the source distribution is a wedge mode. It is clear that the two acoustic fields look quite different in the two cases, even though the two source distributions have identical pressure amplitude dependence as a function of depth. The difference in the two fields is caused entirely by the change in the wave front of the mode from being vertical in Fig. 1(a) to being circular, centered on the wedge apex, in Fig. 1(b). While the vertical mode is observed to couple strongly during downslope propagation, the wedge mode solution [Fig. 1(b)] is seen to propagate in an uncoupled manner preserving the vertical pressure distribution typical of mode 3.

Figure 2 shows the initial pressure amplitude distribution over depth for both the vertical mode and wedge mode source fields of Fig. 1. For the conventional vertical mode, the source field has vertical wave fronts, i.e., the acoustic pressure has the same phase at all depths (apart from the usual phase reversal between adjacent standing-wave lobes). The pressure level is shown as loss in dB relative to the pressure 1 m from a point source.

For wedge modes, the wave fronts are arcs of circles centered on the wedge apex. In Ref. 2, the line source was curved by arranging the source elements along an arc of a circle. Since the PE code is set up to start with a given vertical acoustic pressure distribution specified by the user, it is easier to simulate curvature by phasing the line source as a function of depth. The phase at each depth was, therefore, adjusted to simulate wave fronts centered on the wedge apex.

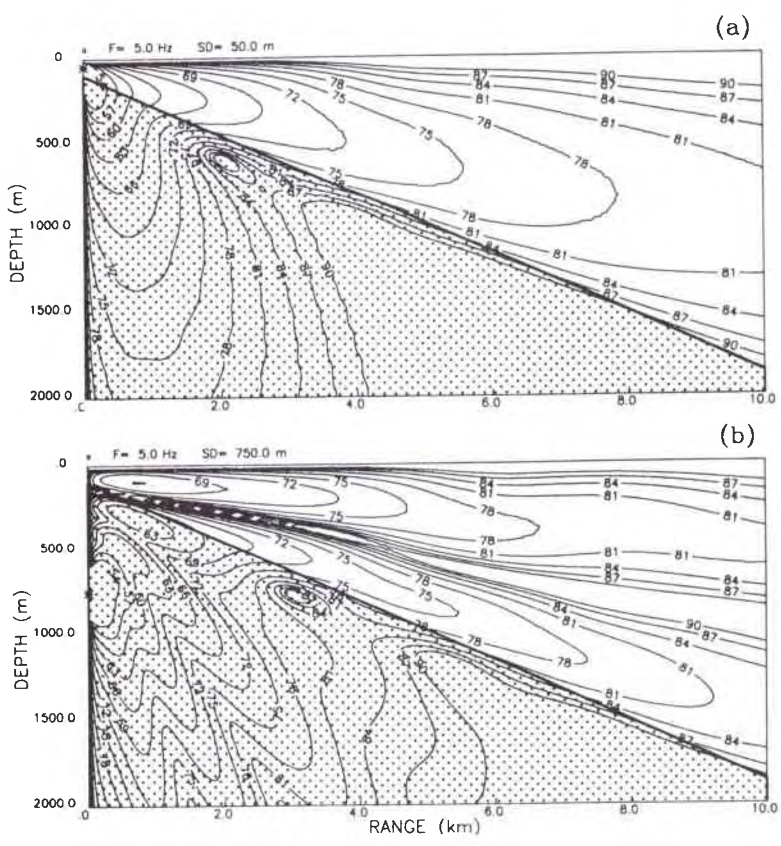

FIG. 4. Contoured loss versus depth and range at a frequency of 5 Hz. There are no trapped modes in the water depth of 100 m at the source. The point source is at a depth of (a) 50 m, and (b) 750 m, giving rise to capture of mode 1 and mode 2, respectively.

The phase adjustment $\Delta\phi$ is found from the geometry of the wedge and is given by

$$\Delta\phi = \pm (\omega/c_1)(\{z^2 + H_0^2/[4\tan^2(\theta)]\}^{1/2} - H_0/[2\sin(\theta)]), \quad (1)$$

where ω is the angular frequency of the line source, H_0 is the water depth at the source position, z is the depth of the source element, and θ is half the slope angle. The $\pm$ sign is chosen according to whether the propagation is upslope or downslope. For the vertical line source of Fig. 1(b), the phase adjustment which simulates the curved wave front mode varies smoothly from $+54°$ at the surface to $-156°$ at the bottom.

The essential difference between the vertical mode field and the wedge mode field of Fig. 1 is illustrated in Fig. 3. The figure shows the vertical pressure distribution at a range of 10 km in Fig. 1. The dashed line is for the vertical mode and corresponds to Fig. 1(a), and the solid line is for the wedge mode and corresponds to Fig. 1(b). The wedge mode result shows a very clean uncoupled mode 3 with equal intensity on all three lobes. In contrast, when the vertical mode propagates 10 km down the 10° slope, the pressure distribution over depth has very little resemblance to a mode 3 pattern.

The results in Figs. 1–3 show clearly that allowance for wave front curvature has enabled us to generate a natural mode of the wedge in complete agreement with the experimental results of Ref. 2.

B. Mode capture

Mode capture can occur in downslope propagation and is the appearance of a trapped mode down range when the water depth at the source is not sufficient to support that mode. The effect was pointed out in theoretical calculations[12] and verified experimentally in a tank.[3]

Mode capture is readily demonstrated using the PE model and the results are illustrated in Fig. 4, which shows fields obtained for a single point source. It should be noted that no phasing of the source is possible here because we are dealing with a point source rather than the vertical line source in Fig. 1. The model parameters are the same as before except that the source frequency has been reduced to 5 Hz. At this frequency, there are no trapped modes in the 100-m water depth at the source position. The point source conditions are simulated using the standard Gaussian PE source which has a -3-dB half-width of 35°. For a source depth of 50 m [Fig. 4(a)], we see that energy becomes trapped in the duct beyond a range of 1 km, propagating almost entirely in mode 1.

In Fig. 4(b), the source has been placed in the bottom at a depth of 750 m. This unusual source position is chosen because, if the propagation were upslope, it would be in the middle of the beam formed by the transfer of the energy of mode 2 into the bottom as it passes its cutoff depth. Since acoustic transmission is reciprocal, this source position might be expected to preferentially excite mode 2. This is

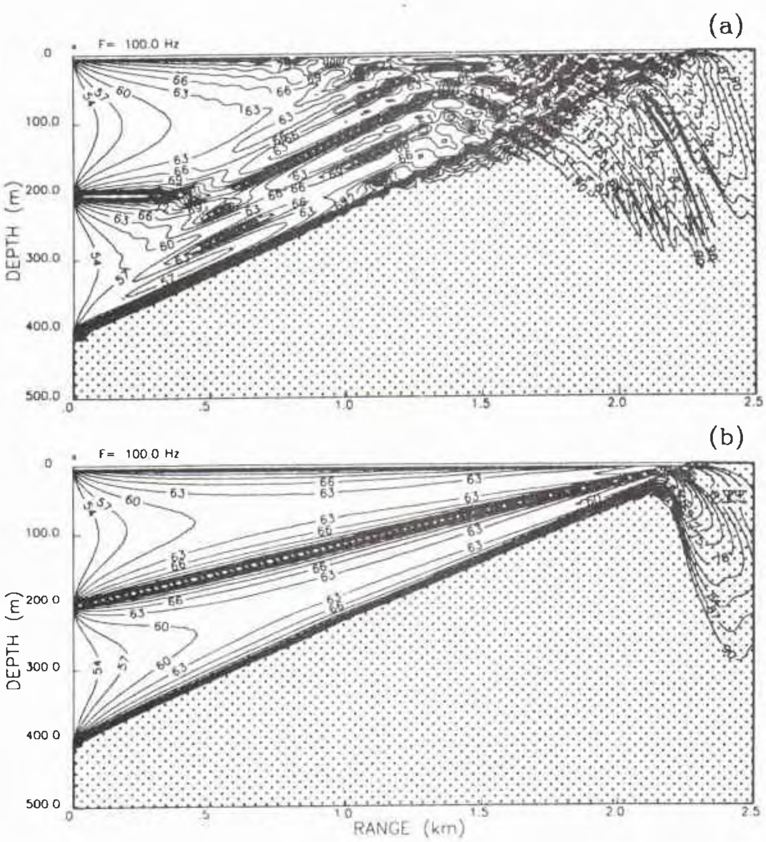

FIG. 5. Contours of dB loss for upslope propagation in a wedge. The model parameters are given in the text. The source fields have the depth dependence of mode 2 and simulate (a) a vertical mode, and (b) a wedge mode.

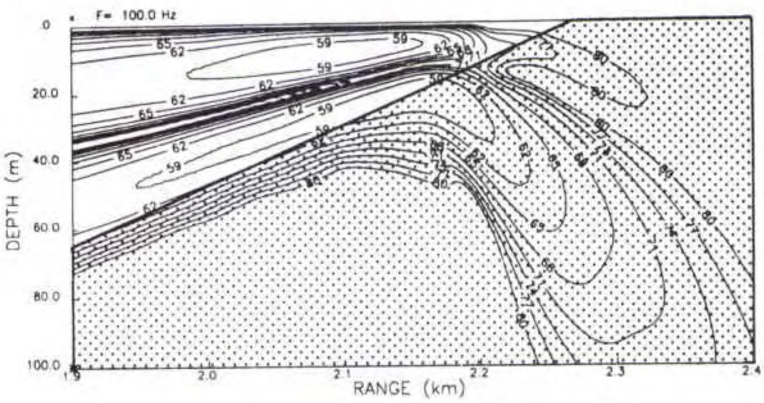

FIG. 6. Expanded section of Fig. 5(b). The cutoff range for mode 2 is 2.15 km.

indeed the case, as is shown in Fig. 4(b). Beyond a range of about 2 km, the field has a deep null at about midwater depth with smooth maxima above and below. The field is thus predominantly mode 2. However, the fact that the two maxima are not equal in magnitude and some of the contours in the water oscillate shows that there is slight interference from other modes.

The results in Fig. 4 confirm that mode capture must be allowed for in any downslope calculation of mode propagation. However, for a point source in the water column, only one mode can in practice be captured from the continuous spectrum.

III. UPSLOPE PROPAGATION

Experimental investigation of the upslope propagation of wedge modes is difficult and no results have been reported. The PE model, however, is readily applied to the upslope case so the investigation can be continued through numerical calculations.

The contour plots of Fig. 5 show a comparison of the upslope propagation of a vertical mode and a wedge mode. In both cases, the pressure amplitude field at the source has the depth dependence of mode 2 in 400 m of water. The only difference between the two source distributions is that Fig. 5(a) simulates a conventional vertical mode whereas Fig. 5(b) simulates a wedge mode with a curved wave front. The wave front is again an arc of a circle centered on the wedge apex exactly as described in connection with Fig. 1 for downslope propagation.

The complicated field of Fig. 5(a) shows that the simple field structure at the source is rapidly lost and it is clear that many modes are interfering. As the modes pass through cutoff, energy is transferred to the bottom. There is interference structure present in the bottom, also, and this further indicates that there are multiple modes present.

In complete contrast, the acoustic field in Fig. 5(b) is very simple and shows that while the mode is trapped, the wedge mode behaves as a natural mode of the wedge with no interference from other modes. As the mode passes through cutoff and enters the bottom as a beam, there is virtually no indication in Fig. 5(b) of the presence of mode coupling.

The details of the cutoff process can be examined in Fig. 6, which shows the same field as Fig. 5(b) on an enlarged scale around the mode 2 cutoff range of 2.15 km. The major part of the energy of the wedge mode 2 leaks into the bottom after the cutoff range is passed. A very small fraction of the

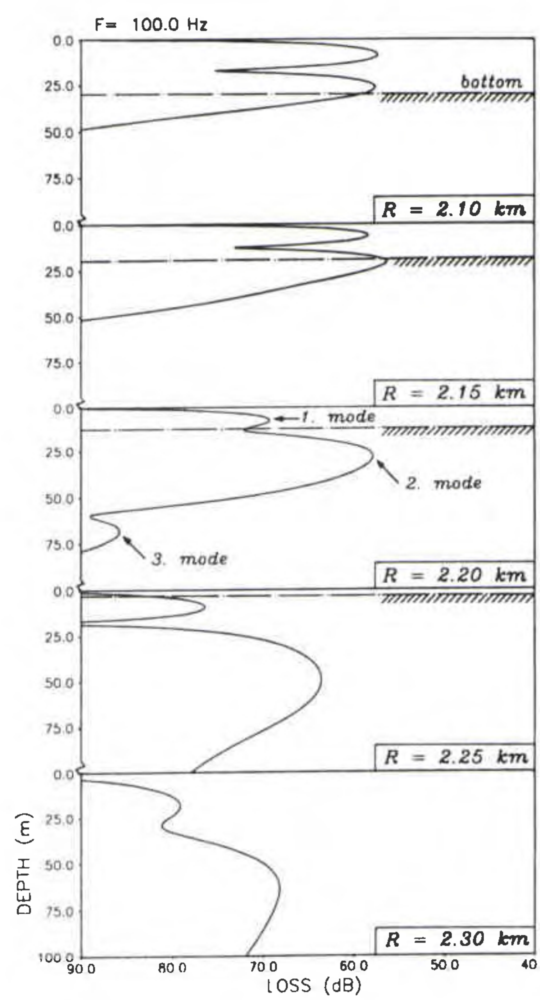

FIG. 7. Loss versus depth at the ranges indicated for the field of Fig. 6.

energy couples into mode 1 and leaks into the bottom closer to the wedge apex after the mode 1 cutoff range of 2.23 km is passed. The secondary cutoff beam has an approximately 18-dB lower level than the primary beam, corresponding to 98.5% of the energy being in the primary beam and only 1.5% being in the secondary beam. Hence, for practical purposes, it appears that coupling of energy near cutoff is negligible for wedge mode propagation.

To verify that the occurrence of a small but finite coupling of energy near cutoff is not an artifact associated with the PE solution, a coupled mode result[10,11] was generated for the same propagation situation shown in Fig. 6. The initial wedge mode starting field was decomposed into a set of vertical modes which in turn were propagated upslope through the wedge. The result of the coupled mode calculation with backscattering included was in such close agreement with the PE result of Fig. 6 that we feel confident that the finite coupling of energy into mode 1 is a real effect.

The coupling of energy between modes at cutoff is also illustrated in Fig. 7. The graphs show the acoustic pressure distribution over depth at selected ranges around the cutoff of wedge mode 2. At 2.10-km range, the depth dependence shows a pure mode 2. At 2.15 km, the mode is just at cutoff and the lower peak has just passed into the bottom. At a range of 2.20 km, the beam associated with mode 2 is propagating well into the bottom, while a little energy remains in the water column propagating as a well-trapped mode 1. We also notice the peak at 70-m depth. This is the depth at which a mode 3 beam would appear if wedge mode 3 were used as the source field. We assume, therefore, that the peak at 70-m depth indicates that energy from mode 2 has also coupled into mode 3. The amount of coupling is, however, extremely small, and the peak level is 28 dB below the level of the main mode 2 peak. At a range of 2.25 km, in Fig. 7, mode 1 has just passed cutoff, and, moving on to the final range of 2.30 km, we see two peaks in the bottom which correspond to the mode 1 and 2 cutoff beams.

IV. DISCUSSION AND CONCLUSIONS

We have demonstrated that a wedge mode can be constructed through a simple geometrical transformation of a standard vertical mode, i.e., by associating with a vertical mode amplitude distribution over depth a circular wave front centered on the wedge apex. Here, PE simulations for both downslope and upslope propagation in an isovelocity wedge confirm that wedge modes propagate with negligible mutual coupling and, hence, constitute a close approximation to the natural modes of the wedge.

The modeling results presented here are in complete agreement with earlier experimental results by Tindle *et al.*[2] for downslope propagation.

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