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VARIABLE SEA LEVEL AND STRAIT FLOWS  
IN THE MEDITERRANEAN:  
A THEORETICAL STUDY OF  
THE RESPONSE TO METEOROLOGICAL FORCING

by

Christopher J.R. GARRET

15 MAY 1983

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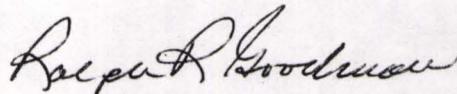
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RALPH R. GOODMAN  
Director





Mediterranean  
Sea level  
Strait of Gibraltar  
Strait of Sicily  
Atmospheric pressure  
Méditerranée  
Niveau moyen  
Détroit de Gibraltar  
Détroit de Sicile  
Pression atmosphérique

# Variable sea level and strait flows in the Mediterranean: a theoretical study of the response to meteorological forcing

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## ABSTRACT

Changes in atmospheric pressure over the Mediterranean are thought to account for a significant part of low frequency sea level changes and associated flow through the Strait of Gibraltar. However, Crepon's (1965) examination of sea level data suggested that both sea level in the Western Mediterranean and inflow through the Strait were in phase with minus the atmospheric pressure, in apparent contradiction to mass conservation. This apparent paradox can be resolved if allowance is made for flow through the Strait of Sicily in response to different sea levels in the Eastern and Western Mediterranean basins. A two-basin, two-strait model is developed, using a simple approximate formula for strait flow that is likely to be adequate for the 10 day periodicity that is emphasized. The phase difference between flow through the Strait of Gibraltar and sea level in the Western basin becomes fairly small if allowance is made for the eastward propagation of atmospheric pressure systems, but a significantly non-inverse-barometer response of sea level in the Eastern Mediterranean is predicted for periods of several days. The effect on Mediterranean sea level of low frequency wind-induced set-up on the continental shelf outside the Strait of Gibraltar is estimated to be less than the effect of atmospheric pressure. The great need is for a statistical study of data on sea level in the Eastern Mediterranean, and of flow through the two straits, in relationship to the large-scale low frequency weather patterns.

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## RÉSUMÉ

Variations du niveau de la mer et des courants  
dans les détroits en Méditerranée :  
une étude théorique sur la réponse de l'océan aux forces atmosphériques

On estime généralement que les changements de pression atmosphérique sur la Méditerranée jouent un rôle important sur le niveau moyen et sur le flux associé à travers le détroit de Gibraltar. Cependant les travaux de Crepon (1965) suggèrent que le niveau moyen du bassin occidental et le flux à travers le détroit sont en phase avec l'opposé de la pression atmosphérique, ceci en apparence contradiction avec l'équation de continuité. Ce paradoxe peut être résolu si l'on admet un flux à travers le détroit de Sicile provoqué par une différence de niveau entre le bassin occidental et oriental. Un modèle comportant deux bassins et deux détroits est proposé, utilisant une formulation simplifiée pour le flux dans les détroits qui semble en accord avec les phénomènes périodiques d'une dizaine de jours, qui sont particulièrement retenus. Le déphasage entre l'écoulement à travers le détroit de Gibraltar et le niveau moyen du bassin occidental devient assez petit si l'on admet que les perturbations atmosphériques se déplacent vers l'est. Cependant les résultats du modèle font apparaître dans le bassin oriental une variation du niveau moyen non statique pour des périodes de quelques jours. L'influence du vent sur les variations à basses fréquences du niveau moyen sur le plateau continental en dehors du détroit de Gibraltar est estimée inférieure à l'effet statique de la pression atmosphérique. Au vu de l'étude il apparaît qu'il serait nécessaire d'effectuer un travail statistique sur les données concernant le niveau moyen du bassin oriental, les flux dans les détroits, et les données météorologiques.

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## INTRODUCTION

The exchange of water, and water properties such as salt and heat, through the Strait of Gibraltar is one of the classical problems of physical oceanography (see Defant, 1961 and Deacon, 1971). An excess of evaporation over precipitation for the Mediterranean Sea requires that there be a net inflow through the Strait of Gibraltar to achieve a mass balance. A salt balance, and observed salinities of Atlantic and Mediterranean waters, require that this net flux arise from the difference between a surface inflow and a bottom outflow that are each about twenty times larger than the net flux. A recent discussion (Bethoux, 1980) of budgets for the Mediterranean arrives at an inflow of 1.68 Sv and an outflow of 1.60 Sv.

Confirmation of these estimates by direct measurement has always been difficult due to the strength of the currents and their considerable spatial and temporal variability. Lacombe (1961) and Lacombe *et al.* (1964) in fact estimated the mean fluxes in and out to be about 75% of the figures given above; neither set of figures can yet be regarded as definitive.

Direct current measurements (or casual observation by mariners) have also demonstrated the importance of tidal currents, mostly semi-diurnal, and of lower frequency fluctuations. While the measurements of Lacombe (1961) and Lacombe *et al.* (1964) showed that the mean flow, tidal currents and sub-tidal flows all have considerable spatial variability, currents of the order of 0.5 to 1 m. sec.<sup>-1</sup> are representative of all three types of flow. Lacombe (1961) suggested that the local wind in the Strait could be responsible for a current of up to 0.25 m. sec.<sup>-1</sup> in the top 20 m or so of the water column, and Lacombe and Richez (1961), Boyce (1975) and others have also drawn attention to the large internal waves of tidal and higher frequencies. The sub-tidal fluctuations in the flow through the Strait are perhaps the least predictable features of the current regime, and it is with them that the present paper is concerned. On the seasonal time scale fluctuations may be associated with the seasonal cycle of evaporation minus precipitation for the Mediterranean, or with seasonal changes in water stratification. However, fluctuations with periods of days to months are most likely to be associated with meteorological forcing, as the Atlantic and Mediterranean respond to large scale patterns of atmospheric pressure and wind.

For many straits [e. g. the Straits of Dover, see Bowden (1956) and Alcock and Cartwright (1977), and the Strait of Belle Isle, see Garrett and Toulany (1982)] the flow arises from a difference in sea level, between opposite ends of the strait, attributable to wind-induced set-up. Indeed Garrett and Toulany (1982) used multiple regression analysis to show that wind driven sea level changes both on the Labrador shelf and in the Gulf of St. Lawrence contribute to flow through the Strait of Belle Isle, but that the direct effect of atmospheric pressure is negligible. They attributed this to the presence of a much larger opening, Cabot Strait, connecting the Atlantic and the Gulf of St. Lawrence. This allows a rapid equilibration, between opposite ends of the Strait of Belle Isle, of sea level changes caused by atmospheric pressure.

The Mediterranean Sea, however, has but a single connection, the Strait of Gibraltar, to the Atlantic Ocean, so that one would expect significant flows through the Strait to occur in response to changes in the average atmospheric pressure over the Mediterranean. This has long been recognised; Lacombe (1961) argued that an "inverted barometer" response of the Mediterranean would require fluctuations in the flow through the Strait of Gibraltar that are of the same order of magnitude as those observed in direct current measurements, or in the surface flow as estimated from sea level differences across the strait assuming a geostrophic balance.

The problem was investigated in more detail by Lacombe *et al.* (1964) and Crepon (1965). The latter examined data, for several separate periods of one month each, on daily mean atmospheric pressure and daily mean sea level for a number of ports in the Western Mediterranean. He found the fluctuations of sea level and minus the atmospheric pressure to be correlated, with a ratio consistent with an inverted barometer response. He also found the sea level difference between the south and north sides of the Strait of Gibraltar, representing surface inflow, to be correlated with minus the average atmospheric pressure over the Western Mediterranean.

It should be mentioned that the records examined by Crepon (1965) tend to be dominated by fluctuations with a period of about 10 days, but his conclusions cannot be regarded as particularly definite even at that period due to the lack of a quantitative statistical analysis. However, if we accept his broad conclusions,

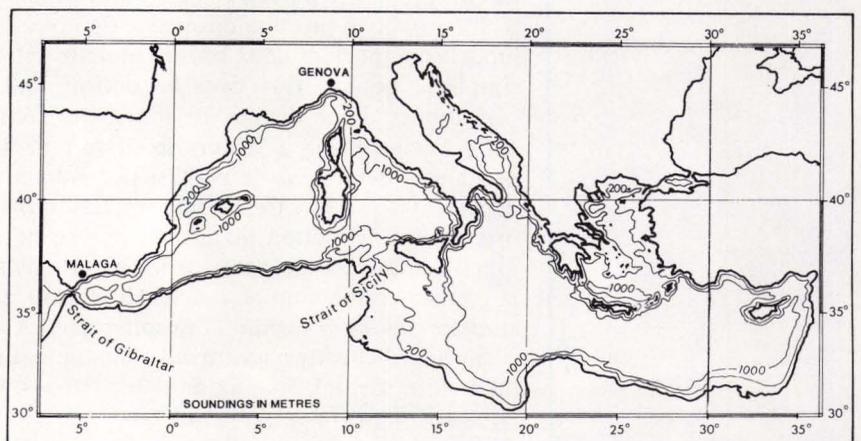


Figure 1  
The Mediterranean Sea, showing the 200 and 1000 m isobaths.

which are the same as those reached by Lacombe *et al.* (1964), they imply an inflow through the Strait of Gibraltar that is proportional to sea level in the Western Mediterranean. As recognized by Lacombe *et al.* (1964) and Crepon (1965), this appears to be contrary to simple requirements of mass conservation, which would require the rate of change of sea level in the Mediterranean, rather than the sea level itself, to be correlated with the inflow.

If one accepts Crepon's (1965) results, the only reasonable conclusion is that significant fluctuations in flow occur through the Strait of Sicily (see Fig. 1), and that sea level in the Eastern Mediterranean is significantly different (particularly for periods of the order of 10 days) from that in the Western Mediterranean.

The Strait of Sicily is certainly a significant constriction. Although it appears much wider than the Strait of Gibraltar, much of it has a depth of less than 200 m. The sill depth is 430 m (Frassetto, 1964) compared with 320 m at the Strait of Gibraltar, but this is in a very narrow and tortuous channel. It seems that the Strait of Sicily may be at least as constrictive as the Strait of Gibraltar. This is demonstrated by the difference in water properties of the two basins (see Béthoux, 1980, for a recent discussion of budgets and exchanges due to the mean flow through the Strait of Sicily). In the present study of the response of the Mediterranean to low frequency meteorological forcing it is treated as made up of two basins separated from the Atlantic by one strait and from each other by another. This does not allow adequately for the separate response of areas such as the Adriatic or Aegean Seas, but is probably a good first approximation.

The main purpose of this paper is to predict, *via* a simple model, the response of sea level in the two Mediterranean basins and flow through the Straits of Sicily and Gibraltar, to low frequency changes in atmospheric pressure. An important ingredient of the model is a simple formula for the flow through a strait in terms of the difference in sea level of the two basins it connects. This formula is introduced in the next section, which summarizes the simple model, for strait flow, of Garrett and Toulany (1982). In the third section the formula is used to study the response of one basin connected to an exterior ocean via a single strait, and in the fourth section a two-basin model, representing the Mediterranean, is discussed. The fifth section allows for the difference in mean atmospheric pressure between the two basins. In the sixth section the theoretical predictions are related to observations, and in the seventh section the role of wind forcing, particularly by longshore wind on the continental shelf of northwest Africa outside the Strait of Gibraltar, is discussed. The last section reviews the type of data, and data analysis, that will be required for validation and improvement of the theory, and for its use in any predictive sense for strait flows.

#### A SIMPLE MODEL OF FLOW THROUGH A STRAIT

Consider a shallow strait connecting two larger, deeper, ocean basins in which the elevations are

$\text{Re } \zeta_0 \exp(-i \omega t)$  and  $\text{Re } \zeta_1 \exp(-i \omega t)$ . These elevations will, of course, be functions of position, but presumably with much smaller horizontal gradients than occur in the vicinity of the strait; we regard  $\zeta_0, \zeta_1$  as the values taken near the ends of the strait, but away from its perturbing influence. Let  $\zeta_{0r}, \zeta_{0l}, \zeta_{1r}, \zeta_{1l}$  [all times  $\exp(-i \omega t)$ ] be the elevations at each end of the strait, on the right and left sides facing into the strait, as sketched in Figure 2.

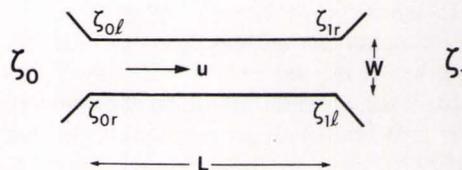


Figure 2

Schematic of a strait of length  $L$  and width  $W$  connecting two water bodies with levels  $\text{Re } \zeta_0 \exp(-i \omega t)$  and  $\text{Re } \zeta_1 \exp(-i \omega t)$ .  $\text{Re } u \exp(-i \omega t)$  is the average surface current.

In the absence of the earth's rotation one might assume that  $\zeta_{0l} = \zeta_{0r} = \zeta_0$ ,  $\zeta_{1l} = \zeta_{1r} = \zeta_1$  and that the slope along the strait is balanced by the acceleration of the water in the strait and by friction. In the presence of rotation a geostrophic balance across the strait requires a cross-strait difference of elevation. However, it seems reasonable to suppose that we still have  $\zeta_{0r} = \zeta_0$  and  $\zeta_{1r} = \zeta_1$  on the grounds of Kelvin wave propagation from each basin [though, as stressed by Garrett and Toulany (1982) who presented the same model, this assumption needs confirmation through the use of an appropriate Green's function in matching the flow through the strait to the exterior ocean basins].

Let  $\text{Re } u \exp(-i \omega t)$  be the average (across the width of the strait) surface current through the strait. Cross-strait geostrophy then requires:

$$\zeta_{0r} - \zeta_{0l} = \zeta_{1l} - \zeta_{1r} = f W u / g, \quad (2.1)$$

where  $W$  is the width of the strait. The equation of motion along the strait (ignoring the advective term for the moment) is:

$$\partial u / \partial t + g \partial \zeta / \partial x + F = 0, \quad (2.2)$$

where  $F$  represents friction, which we linearise to  $\gamma u$ . Simple algebra then leads to:

$$u = g L^{-1} (\zeta_0 - \zeta_1) [-i \omega + \gamma + f W / L]^{-1}, \quad (2.3)$$

where  $L$  is the length of the strait. When  $f = 0$  the flow is clearly limited by acceleration and friction; when  $f W / L \gg \omega, \gamma$  the flow is limited by geostrophy, in the sense that the sea level difference across the strait cannot be greater than the sea level difference between the two ocean basins.

The volume flux  $Q$  corresponding to (2.3) is:

$$\begin{aligned} Q &= W H u \\ &= g H (\zeta_0 - \zeta_1) [(-i \omega + \gamma)(L/W) + f]^{-1}, \end{aligned} \quad (2.4)$$

if the strait has a constant depth  $H$ , and ignoring variations of  $u$  with depth. For a real strait, with varying depth and width and a rather ill-defined length, one could try to determine empirical values of  $H$ ,  $\gamma$  and  $L/W$  for use in (2.4) by running a numerical model for the real topography.

We may estimate the order of magnitude of the terms in (2.4) for the Straits of Gibraltar and Sicily. We take  $f = 8.6 \times 10^{-5} \text{ sec.}^{-1}$  and for a 10 day period  $\text{sec.}^{-1}$ . We represent the friction coefficient  $\gamma$  by  $C_D u_0/H$ , where  $C_D \cong 2.5 \times 10^{-3}$  is a bottom drag coefficient,  $u_0$  an average speed, say  $0.5 \text{ m. sec.}^{-1}$ , and the depth  $H$  is taken as 250 m. This gives  $\gamma \cong 5 \times 10^{-6} \text{ sec.}^{-1}$ . It is difficult to guess an appropriate value for  $L/W$ , but 3 seems reasonable. In this case, at a period of 10 days,  $\omega(L/W)$  is only about  $(1/4)f$  and can reasonably be neglected as a first approximation. We note that if bottom friction is important it has the same effect in (2.4) as reducing  $H$ ; unlike the  $\omega$  term it does not affect the phase of the flux with respect to  $\zeta_0 - \zeta_1$ .

An advective term  $uu_x$  has been omitted from (2.2). Its magnitude compared with  $\partial u/\partial t$  can be estimated as  $O(u\omega^{-1}L^{-1})$  or, for our parameter values,  $O(1)$ .

However, it can be argued that  $\int uu_x dx$  from one ocean basin to the other is very small, so that it does not enter into an equation, such as (2.4), relating the flux to  $\zeta_0 - \zeta_1$ ; the advective term merely leads to a lowering of sea level in the strait and a recovery at the other end.

As a first approximation to (2.4) for the low frequency motion of interest it will generally be assumed in this paper that we may approximate (2.4) by:

$$Q = g H f^{-1} (\zeta_0 - \zeta_1). \quad (2.5)$$

## A SINGLE BASIN

Consider the response of a basin of area  $A_1$ , separated by a strait of effective depth  $H_1$  from an ocean which has oscillations of level, near the strait, given by  $\text{Re } \zeta_0 \exp(-i\omega t)$ . This may be due to the inverted barometer response of the ocean or to some other cause (Fig. 3). It is assumed that long waves travel quickly enough inside the basin to make  $\zeta_1$  spatially uniform on the time scales being considered (this is valid for the Mediterranean Sea, for which the length scale/long wave speed  $\cong 2 \times 10^6 \text{ m}/150 \text{ m. sec.}^{-1} \cong 1.3 \times 10^4 \text{ sec.} \cong 4 \text{ hours}$ ).

Volume conservation then requires:

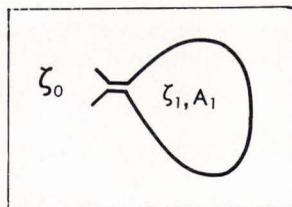


Figure 3  
Schematic of a basin of area  $A_1$  and sea level  $\text{Re } \zeta_1 \exp(-i\omega t)$  connected by a strait to an ocean with sea level  $\text{Re } \zeta_0 \exp(-i\omega t)$ .

$$A_1 \partial/\partial t [\zeta_1 \exp(-i\omega t)] = g H_1 f^{-1} (\zeta_0 - \zeta_1) \exp(-i\omega t), \quad (3.1)$$

so that:

$$\zeta_1 = \zeta_0 \left( 1 - \frac{i\omega f A_1}{g H_1} \right)^{-1}. \quad (3.2)$$

We note that the key parameter here is  $\varepsilon_1 = \omega f A_1 / g H_1$ . If we take  $H_1 = 250 \text{ m}$  and  $\omega = 2\pi/10 \text{ days}$ , we get  $\varepsilon_1 = 0.64$  for  $A_1 = 2.5 \times 10^{12} \text{ m}^2$ , the area of the whole Mediterranean (apart from the Black Sea), but  $\varepsilon_1 = 0.22$  if  $A_1 = 8.4 \times 10^{11} \text{ m}^2$  corresponding to the western basin only.

This solution shows that  $\zeta_1$  lags  $\zeta_0$  and is less than it. The flux  $Q$  leads  $\zeta_1$  by  $90^\circ$ , but leads  $\zeta_0$  by a smaller amount. If we retain  $-i\omega$  in (2.4), but still ignore  $\gamma$  or include it with  $f$ , we have:

$$\zeta_1 = \zeta_0 \left( 1 - \frac{\omega^2 A_1 L}{g H_1 W} - \frac{i\omega f A_1}{g H_1} \right)^{-1}. \quad (3.3)$$

This may permit a Helmholtz-like resonance at a frequency:

$$\omega_R = \left( \frac{g H_1 W}{A_1 L} \right)^{1/2}. \quad (3.4)$$

With  $L/W = 3$  and  $H_1 = 250 \text{ m}$  the corresponding period would be 4 days for the whole Mediterranean, or 2.3 days for the Western basin alone.

A single basin is clearly an inadequate model for the Mediterranean Sea, but before moving on to a two-basin model we consider the relationship between the forcing function  $\zeta_0$  and atmospheric pressure in the situation where the latter is not spatially uniform.

## Spatially varying atmospheric pressure

If the atmospheric  $p_a$  varies over the basin, we assume that fast-propagating gravity waves remove any spatial variation in the sub-surface pressure  $p_a + \rho g \zeta_1$ . Hence, if an overbar denotes a spatial average over the basin, we have

$$p_a(\mathbf{x}, t) + \rho g \zeta_1(\mathbf{x}, t) = \bar{p}_a(t) + \rho g \bar{\zeta}_1(t). \quad (3.5)$$

Indeed, if  $\mathbf{x}_s$  denotes the position of the strait, we have:

$$\zeta_1(\mathbf{x}_s, t) = \bar{\zeta}_1(t) + (\rho g)^{-1} [\bar{p}_a(t) - p_a(\mathbf{x}_s, t)]. \quad (3.6)$$

Ignoring gradients of atmospheric pressure along the strait we write the sea level just outside the strait as the sum of an inverted barometer response,  $-(\rho g)^{-1} p_a(\mathbf{x}_s, t)$ , and some extra term  $\zeta'_0(t)$ . Then the difference of sea level which controls the flow through the strait is:

$$\begin{aligned} \zeta_0(\mathbf{x}_s, t) - \zeta_1(\mathbf{x}_s, t) \\ = \zeta'_0(t) - (\rho g)^{-1} \bar{p}_a(t) - \bar{\zeta}_1(t). \end{aligned} \quad (3.6)$$

The mass conservation equation (3.1) now applies with  $\partial \bar{\zeta}_1 / \partial t$  instead of  $\partial \zeta_1 / \partial t$ , and so (3.2) becomes:

$$\bar{\zeta}_1 = [\zeta'_0 - (\rho g)^{-1} \bar{p}_a] \left( 1 - \frac{i \omega f A_1}{g H_1} \right)^{-1}. \quad (3.7)$$

Local changes in sea level may then be obtained from (3.5).

This result makes it clear that the forcing function for the basin is minus the average atmospheric pressure over the basin, together with any departure from an inverted barometer response in the ocean just outside the strait. One could also, of course, add to  $\zeta'_0$  an extra term  $-\zeta'_1$  due to forcing by other mechanisms inside the strait.

The single basin model serves to illustrate the fundamental physics, but is clearly inadequate as a model for the Mediterranean unless the Strait of Sicily is either very much more, or very much less, constrictive than the Strait of Gibraltar, in which cases the above theory would apply with  $A_1$  equal to the area of the western basin or of the whole Mediterranean respectively. Neither of these extremes seems likely as the Strait of Sicily seems comparable to the Strait of Gibraltar, at least in terms of the factor  $g H f^{-1}$  in (2.5) relating the flux through a strait to the sea level difference between the basins it connects. We thus move on to a more elaborate model with two basins.

## TWO BASINS

We now consider the response of two basins, of areas  $A_i$ , as sketched in Figure 4. The elevation in the ocean just outside the first strait is  $\zeta_0$  [or rather  $\text{Re } \zeta_0 \exp(-i \omega t)$ ] and the two basins have average elevations  $\zeta_i$  in response to atmospheric pressure which has basin averages  $\bar{p}_i$ . As in the preceding section, we assume that gravity waves remove any spatial variation in sub-surface pressure within each basin, so that:

$$\zeta_i = \bar{\zeta}_i + \bar{p}_i - p_i \quad (4.1)$$

[dropping the factor  $(\rho g)^{-1}$ , i. e. taking the atmospheric pressure to be measured in equivalent units of sea level]. Using (2.5) and the analysis of the preceding section we see that the flow from the ocean into the first basin, through the strait of depth  $H_1$ , is given by:

$$Q_1 = g H_1 f^{-1} (\zeta'_0 - \bar{p}_1 - \bar{\zeta}_1), \quad (4.2)$$

where  $\zeta'_0$  is the oceanic sea level change in addition to an inverted barometer response. The flow through the second strait, of depth  $H_2$ , from the first basin to the second, is given by:

$$Q_2 = g H_2 f^{-1} (\bar{p}_1 - \bar{p}_2 + \bar{\zeta}_1 - \bar{\zeta}_2). \quad (4.3)$$

Volume conservation for the two basins now requires:

$$A_1 \partial \bar{\zeta}_1 / \partial t = Q_1 - Q_2, \quad A_2 \partial \bar{\zeta}_2 / \partial t = Q_2. \quad (4.4)$$

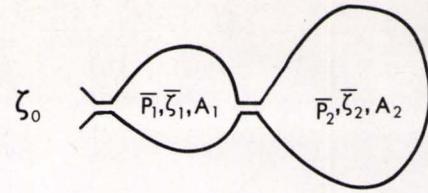


Figure 4

Schematic of two basins and basins in series. The overbar refers to a spatial average over each basin of the atmospheric pressure ( $p$ ) and sea level ( $\zeta$ ). All variables fluctuate with frequency  $\omega$ .

Hence, after some algebra:

$$\bar{\zeta}_1 = [(1 - i \varepsilon_2) (\zeta'_0 - \bar{p}_1) - i \varepsilon_1 R (\bar{p}_2 - \bar{p}_1)] \times [1 - \varepsilon_1 \varepsilon_2 - i \varepsilon_1 (1 + R) - i \varepsilon_2]^{-1} \quad (4.5)$$

$$\bar{\zeta}_2 = [(\zeta'_0 - \bar{p}_1) - (1 - i \varepsilon_1) (\bar{p}_2 - \bar{p}_1)] \times [1 - \varepsilon_1 \varepsilon_2 - i \varepsilon_1 (1 + R) - i \varepsilon_2]^{-1} \quad (4.6)$$

where:

$$\varepsilon_i = \omega f A_i (g H_i)^{-1}, \quad R = A_2 / A_1. \quad (4.7)$$

The spatially averaged sea levels of the two basins thus demonstrate a response to two types of forcing. If  $\bar{p}_2 = \bar{p}_1$ , the response is proportional to  $\zeta'_0 - \bar{p}_1$ , showing that the basins respond in the same way to minus the average atmospheric pressure as to a non-inverted barometer response of the exterior ocean. The terms proportional to  $\bar{p}_2 - \bar{p}_1$  show that the difference in average atmospheric pressure over the two basins is important.

As for the single basin discussed in the preceding section, we see the critical importance of the parameters  $\varepsilon_i$ . For the Mediterranean we have  $\varepsilon_1 = 0.22$  at a period of 10 days if  $H_1 = 250$  m. Also,  $A_2 / A_1 = 2.0$ , and  $\varepsilon_2 = 0.44$  if we take  $H_2 = H_1 = 250$  m. With these values, and taking  $\bar{p}_2 = \bar{p}_1$ , we have:

$$\bar{\zeta}_1 = (0.77, 27^\circ) (\zeta'_0 - \bar{p}_1), \quad (4.8)$$

$$\bar{\zeta}_2 = (0.70, 51^\circ) (\zeta'_0 - \bar{p}_1) \quad (4.9)$$

and the flow through the first strait is proportional to:

$$\zeta'_0 - \bar{p}_1 - \bar{\zeta}_1 = (0.47, -48^\circ) (\zeta'_0 - \bar{p}_1), \quad (4.10)$$

where the angle is the lag of the response behind the forcing  $\zeta'_0 - \bar{p}_1$ .

These values clearly do not account for Crepon's (1965) results; quite apart from anything else the flow through the first strait leads  $\bar{\zeta}_1$  by  $75^\circ$ , almost as much as the  $90^\circ$  for a single basin model. Of course the values we have chosen for  $\varepsilon_1, \varepsilon_2$  are suspect. Figure 5 shows the amplitude and phase of the coefficients of  $\zeta'_0 - \bar{p}_1$  for  $\bar{\zeta}_1, \bar{\zeta}_2$  and  $1 - \bar{\zeta}_1$ . As both  $\varepsilon_1, \varepsilon_2$  are linearly

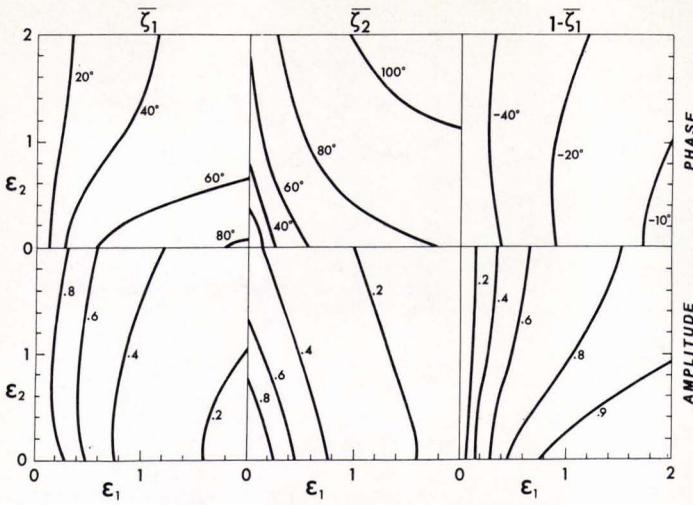


Figure 5

Contour plots against  $\epsilon_1$  and  $\epsilon_2$  of the amplitude and phase lag of the average sea level in each basin in response to unit forcing by external sea level or spatially uniform atmospheric pressure (an amplitude of 1 and phase of zero corresponds to sea level changes equal to those in the exterior ocean, or to an inverted barometer response to atmospheric pressure). Also shown is  $1-\zeta_1$  which gives a measure of the flow through the first strait (Gibraltar).

proportional to frequency a cut through these diagrams in a given direction can be regarded as a prediction of

the response to  $\zeta'_0 - \bar{p}_1$  as a function of frequency, although as  $\epsilon_1, \epsilon_2$  (and so  $\omega$ ) increase the neglect of  $\omega(L/W)$  compared with  $f$  in (2.4) becomes more and more inappropriate. It is simple to allow for  $\omega(L/W)$  in calculating the response, but in view of the uncertainty in appropriate values for  $L/W$  for the two straits it seems more profitable to proceed to a consideration of the importance of  $\bar{p}_2 - \bar{p}_1$ , still neglecting  $\omega(L/W)$  compared with  $f$  so that the theory is valid only for low frequencies.

## RESPONSE TO TRAVELLING PRESSURE SYSTEMS

The results presented in the preceding section may be used to estimate the response of the basins and straits to forcing by  $\zeta'_0$ , which might be a wind-induced set-up on the continental shelf outside the Strait of Gibraltar. This, and a discussion of its importance compared with direct forcing by atmospheric pressure, will be pursued in the seventh section. In this section we take  $\zeta'_0 = 0$  and propose a simple relationship between  $\bar{p}_2$  and  $\bar{p}_1$  which permits an evaluation of  $\bar{\zeta}_1, \bar{\zeta}_2$  in terms of a single input parameter  $\bar{p}_1$ .

In general, weather systems in the earth's atmosphere travel eastward. We thus represent the atmospheric pressure by  $p_a = \exp(ikx)$ , and take the phase speed  $c = \omega/k$  to be constant. If we now represent the two Mediterranean basins by channels of length  $L_1$  and  $L_2$  meeting at the Strait of Sicily, taken as  $x=0$ , we have

$$\bar{p}_1/\bar{p}_a = \delta_1^{-1} \sin \delta_1 \exp(-i\delta_1),$$

$$\bar{p}_2/\bar{p}_a = \delta_2^{-1} \sin \delta_2 \exp(i\delta_2), \quad \delta_i = \frac{1}{2} k L_i \quad (5.1)$$

and:

$$(\bar{p}_2 - \bar{p}_1)(-\bar{p}_1)^{-1} = 1 - (\delta_1 \sin \delta_2)$$

$$(\delta_2 \sin \delta_1)^{-1} \exp[i(\delta_1 + \delta_2)]. \quad (5.2)$$

If we take  $L_1 = 1000$  km,  $L_2 = 2000$  km, and  $c = 15$  m. sec.<sup>-1</sup> [Papa (1978) cites typical propagation speeds of 4 to 9 m. sec.<sup>-1</sup> in summertime, but 11 to 18 m. sec.<sup>-1</sup> in winter, which covers most of Crépon's (1965) data], we have  $\delta_1 = 0.24$ ,  $\delta_2 = 0.48$  at a period of 10 days.

For these values  $\sin \delta_i/\delta_i$  is very close to 1, so that the basin average pressure, at this frequency, does not differ much from the local pressure. In general, assuming  $\delta_2 = 2\delta_1$ , we have:

$$(\bar{p}_2 - \bar{p}_1)(-\bar{p}_1)^{-1} = 1 - \cos \delta_1 \exp(3i\delta_1), \quad (5.3)$$

with  $\delta_1$  proportional to frequency. For our choice of parameters  $\delta_1/\epsilon_1 \cong 1$ , but we will examine other ratios to cover other phase speeds or other values of  $H_1$  in  $\epsilon_1 = \omega f A_1 (g H_1)^{-1}$ .

Figure 6 shows the amplitude and phase of  $\bar{\zeta}_1, \bar{\zeta}_2$  and  $1-\bar{\zeta}_1$  as functions of  $\epsilon_1$  for a range of ratios  $\epsilon_2/\epsilon_1$  and  $\epsilon_1$ . The amplitude and phase of  $\bar{\zeta}_1$  and  $1-\bar{\zeta}_1$  are given relative to  $-\bar{p}_1$ , i. e.  $\bar{\zeta}_1 = (1.0, 0^\circ)$  would correspond to an inverted barometer response in the first basin. The phase of  $\bar{\zeta}_2$  is taken relative to that of  $-\bar{p}_2$  (which lags  $-\bar{p}_1$  by  $3\delta_1$ ). The flow through the Strait of Gibraltar (relative to  $-\bar{p}_1$ ) is given by  $1-\bar{\zeta}_1$ ; this is the predicted sea level difference (south shore minus north shore) across the Strait. The flow through the Strait of Sicily into the eastern basin of the Mediterranean leads  $\bar{\zeta}_2$  by  $90^\circ$ , and the sea level difference across the Strait should be  $\epsilon_2 |\bar{\zeta}_2|$  in magnitude.

The results shown in Figure 6 are rather insensitive to the ratio  $\epsilon_2/\epsilon_1$ , but do depend significantly on the ratio of  $\delta_1/\epsilon_1$ . We see that allowing for the phase difference between the average atmospheric pressure over the two basins reduces the phase lag of  $\bar{\zeta}_1$  and also brings the phase of  $1-\bar{\zeta}_1$  closer to zero for the relevant values of  $\epsilon_1$ . In particular, if we take  $\epsilon_2 = 2\epsilon_1$  and  $\delta_1 = \epsilon_1$ , then at  $\epsilon_1 = 0.22$  (the estimate corresponding to a period of 10 days) we have:

$$\bar{\zeta}_1 = (0.64, 14^\circ), \quad 1-\bar{\zeta}_1 = (0.41, -23^\circ). \quad (5.4)$$

Hence both the sea level in the western basin and the flow through the Strait of Gibraltar are much more in phase with  $-\bar{p}_1$  than the solution in (4.8,10) for  $\delta_1 = 0$ , and hence more in accord with Crépon's (1965) result.

However, the amplitudes of both  $\bar{\zeta}_1$  and  $1-\bar{\zeta}_1$  are less than the value of order unity claimed by Crépon (1965).

Figure 6  
Amplitude and phase of the sea levels in the two basins, and of the measure  $1 - \bar{\zeta}_1$  of flow through the first strait, as functions of  $\varepsilon_1$  for  $\delta_1/\varepsilon_1 = 0, 1/2, 1, 2$  and for (a)  $\varepsilon_2 = \varepsilon_1$ , (b)  $\varepsilon_2 = 2\varepsilon_1$ , (c)  $\varepsilon_2 = 3\varepsilon_1$ . The phases of  $\bar{\zeta}_1$  and  $1 - \bar{\zeta}_1$  are relative to  $p_1$ ; the phase of  $\bar{\zeta}_2$  is relative to  $p_2$ .

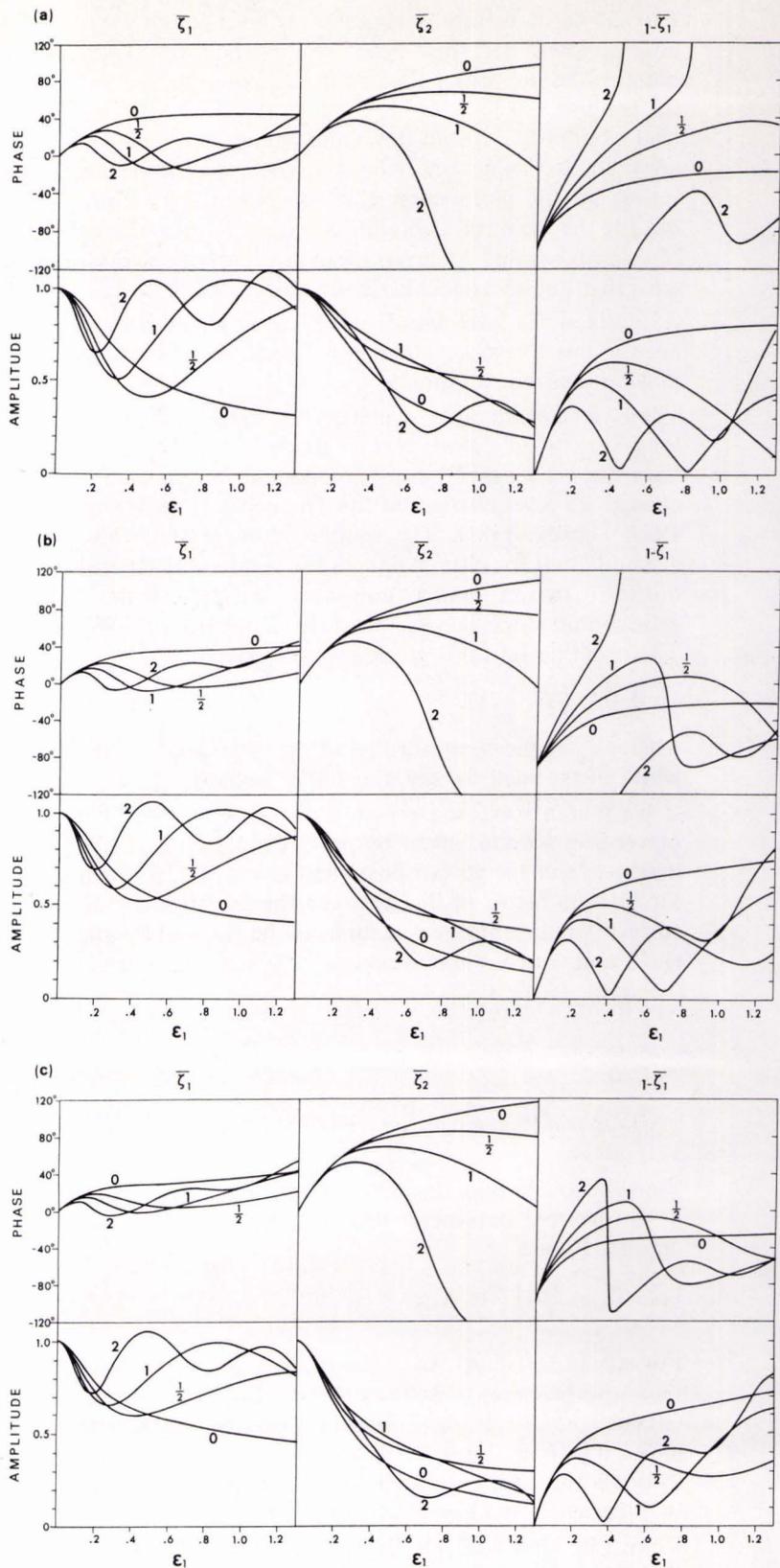
At these same parameter values  $\bar{\zeta}_2 = (0.78, 48^\circ)$ , so that the sea level response in the Eastern basin is not much less than one in magnitude, but lags minus the average pressure over the Eastern basin by  $48^\circ$ . Figure 6 suggests that the magnitude of the response should decrease quite rapidly with frequency, and that the phase continues to increase unless  $\delta_1$  is larger than estimated.

## COMPARISON WITH DATA

As discussed in the preceding section, the results of a simple model suggest that Crépon's (1965) apparent paradox can be accounted for (in phase if not magnitude of the responses) by allowing for the presence of the Strait of Sicily and the likelihood of a different average atmospheric pressure over the Eastern basin. However, Crépon's (1965) study was based solely on visual examination of sea level and atmospheric pressure data. What is required is a statistical study of the relationships, as functions of frequency, of atmospheric pressure, sea level and strait flows.

Garcia *et al.* (1980) and Lafuente and Admetlla (1982) have performed a cross-spectral analysis between sea level and atmospheric pressure at Malaga (Fig. 1) as well as other locations in the Strait of Gibraltar and outside the Mediterranean. There is a hint of a small ( $\cong 15^\circ$ ) lag of sea level behind minus the atmospheric pressure at low frequency, and the gain of sea level on pressure appears to be less than one. Both of these results are in accord with the predictions of this paper, but can certainly not be regarded as verification of the theory, given the uncertainty in the gain and phase calculated from the data, and the lack of any allowance for wind-induced changes in sea level. Other studies of sea level in the Western basin have been reported by Stocchino and Scotto (1970) and Papa (1978) for Genoa (Fig. 1), but without any cross-spectral analysis. Palumbo and Mazzarella (1982) have studied the response of sea level at Naples to atmospheric pressure and wind, finding the frequency-dependent regression coefficient of sea level on pressure to have a phase of  $180^\circ$  and a magnitude of about 0.8 at a period of ten days, increasing, surprisingly, to about 1.4 at much longer periods.

A key test of the theory could clearly come from an observational study of the relationship of sea level to atmospheric pressure in the Eastern basin of the Mediterranean, where a response significantly different from that of an inverted barometer is predicted. Unfortunately, no study for the frequencies of interest seems to have been carried out.



## WIND-DRIVEN CHANGES

There are certainly wind-driven changes in sea level in shallow areas of the Mediterranean, such as the northern Adriatic Sea (Robinson *et al.*, 1972) and the Egyptian Mediterranean coast (El Din, Moursy, 1977), particularly on time scales of several hours. However, the large depth of most of the Mediterranean suggests

that the basin-average changes in sea level due to wind, and the associated strait flows, will not be large. There remains the possibility that significant wind-driven set-up just inside or just outside either of the two straits will lead to significant strait flows and changes in mean sea level. In particular, one wonders whether wind-driven set-up on the continental shelf of North Africa just outside the Strait of Gibraltar could produce a forcing of strait flows and Mediterranean sea level comparable with that due to atmospheric pressure.

Analysis of sea level data from this area would help to answer this question, but it also seems worthwhile to make a theoretical estimate.

There is considerable evidence to suggest that the longshore wind component is much more important than the cross-shelf wind component in producing coastal sea level changes at low frequency (e. g. Wang, 1979; Hickey, 1981). The simplest model, which also accounts well for data, balances the wind stress against bottom friction on a longshore current, with a geostrophic slope across the shelf (Sandstrom, 1980). The resulting set-up is given approximately by:

$$\zeta \cong 0.03 (f W_s / g) V_s, \quad (7.1)$$

where  $V_s$  is the longshore wind at 10 m and  $W_s$  the width of the shelf to, say, the 200 m isobath.

If we take a travelling pressure disturbance, as in the preceding section, given by  $p_a = a \exp i(kx - \omega t)$ , the magnitude of the geostrophic wind is  $k (f \rho_a)^{-1} a$ . With a reduction factor of 0.7 between the geostrophic and surface wind, which we assume to be parallel to the shelf, and with  $k = \omega/c$ , we have:

$$V_s = 0.7 \omega a (c f \rho_a)^{-1}, \quad (7.2)$$

so that:

$$\zeta \cong 0.02 \omega a W_s (c g \rho_a)^{-1}. \quad (7.3)$$

We wish to compare this with the magnitude  $a (g \rho_w)^{-1}$  of an inverted barometer response, with  $\rho_w$  the water density. Hence:

$$\zeta_{\text{wind}} / \zeta_{\text{pressure}} \cong 17 (\omega W_s / c). \quad (7.4)$$

For  $\omega \cong 2\pi / (10 \text{ days})$ ,  $W_s \cong 40 \text{ km}$  and  $c \cong 15 \text{ m. sec.}^{-1}$ , this ratio becomes 0.3. Thus the wind-induced, set-up outside the Strait of Gibraltar appears to be less important than atmospheric pressure in inducing flows through the Strait, though not negligible. At higher frequencies it could be more important.

It must be remarked, however, that the associated flow through the Strait of Gibraltar would in fact just be a deflection through the Strait of a wind-driven current on the Atlantic continental shelf south of the Strait. We have implicitly assumed this to extend to a depth of

200 m, but it might be rather less. In any event the net transport through the Strait might be associated with a smaller effective depth  $H_1$  than for the pressure-driven flows.

The phase of this contribution deserves discussion. The set-up on the shelf arises from a northward wind, which lags local atmospheric pressure by  $90^\circ$ . The lag of the set-up behind the wind and the lead of Gibraltar pressure over Western basin pressure are both small and approximately cancel, so that the net effect is that the forcing term  $\zeta'_0$  in (4.2) leads the forcing term  $p_1$  by about  $90^\circ$ . Hence the effect of  $\zeta'_0$  is to reduce the phase lag of the basin response behind local atmospheric pressure, but to increase the phase lead of flow through the Strait of Gibraltar.

## DISCUSSION

We have seen how a simple two-basin, two-strait model of the Mediterranean shows a response to travelling atmospheric pressure systems that goes part-way towards explaining some initially paradoxical observations by Crépon (1965). There are, of course, major uncertainties in the parameterisation of low-frequency flows through the straits of Gibraltar and Sicily. These could be removed, to some extent, by the development of barotropic models for these straits, with real topography and careful attention to boundary conditions.

The most pressing need, though, is for further statistical studies of sea level response to atmospheric pressure, particularly in the Eastern basin of the Mediterranean, where a non-hydrostatic response is predicted. The use of data from tide gauges close to deep water would minimize the effect of a local wind, but in any case a multiple regression (in frequency space) of sea level on geostrophic winds as well as atmospheric pressure should be performed (see, e. g., Garrett, Toulany, 1982). Similar analysis of strait flows should also be carried out, if data are available.

If low frequency sea level changes and strait flows are found to be correlated with large scale meteorological forcing, though with regression coefficients being functions of frequency, a statistical prediction scheme for the strait flows would become possible, through the use of an empirically-adjusted model of the sort developed in this paper.

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