DETERMINATION OF THE INTENSITY OF SOUND AT ARBITRARY POINTS
IN THE SOUND-FIELD OF A SOURCE IN A HORIZONTALLY LAYERED MEDIUM

by

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The following method is based on finding all the rays through a point P and to add the corresponding intensities.

Consider a layered medium with a linear depth-velocity profile. The ray-path and the corresponding intensity losses are calculated according to a number of well-known formulae (Fig. 1).

To determine the intensity at P, we add an extra layer through P to the velocity/depth profile. When two rays, leaving the source with starting-angles close to each other, intersect the level through P on both sides of P, there will be generally at least 1 ray with a starting angle between the two mentioned ones, which reach the level at P.

An iterative process will give us the value of $\theta_0$.

Of course we need a set of good starting values for this iterative process. Therefore, we define the characteristic velocities of the depth-velocity profile. These are the greatest velocities of each layer, provided that this value is greater than the velocity at the source-depth, $C_B$, and greater than all values occurring between the source and the layer considered. The corresponding characteristic values of $K(\theta_0) = \frac{C_B}{\cos \theta_0}$ gives the start angles of the rays, which will turn back just at the limit of a layer.

The horizontal distance between the source and a point on the ray, S, is a function of $K(\theta_0) : S = S(K(\theta_0))$. We consider one specified
depth level. S consists of a number of pieces of the type $\Delta S_1$, $\Delta S_2$, $\Delta S_3$ (see Fig. 1). We can write in general

$$
S = \sum_{i=1}^{A_1} (\Delta S_1)_i + \sum_{i=1}^{A_2} (\Delta S_2)_i + \sum_{i=1}^{A_3} (\Delta S_3)_i
$$

Consider the derivatives of $S$ [Fig. 2]. We see that $\frac{dS}{dK(\theta_0)}$ can be written as the sum of two monotonic functions, one increasing and the other decreasing. From this we can conclude that in the interval between two characteristic values of $K(\theta)$, $\frac{dS}{dK}$ has two zero points: $K(\theta*)$.

Now we add all the values $K(\theta*)$ for which $\frac{dS}{dK} = 0$ to the array of characteristic values $K(\theta)$. This means that in the interval between two successive values of array $K$, the function of the horizontal distance $S$ at a certain level is monotonic. When two rays with successive values $K[i]$ and $K[i+1]$ intersect the considered level on both sides of $P$, there will be exactly 1 ray with a starting angle $\theta$ between $K[i]$ and $K[i+1]$, intersecting the level in $P$.

In our computer model the ray path will be symmetric, fixed by three values $a$, $b$, $c$ [see Fig. 3].

As you can see, we make a difference between the direct and indirect rays. The $n^{th}$ intersection with the level through $P$ of the ray with value $K[i]$ lies at

$$
\text{DISTANCE} (K[i],n) = a + \left[ \frac{n}{2} \right] * b + \left[ \frac{n-1}{2} \right] * c
$$

for the indirect rays:

$$
\text{INDIST} (K[i],n) = d + \left[ \frac{n}{2} \right] * b + \left[ \frac{n-1}{2} \right] * c
$$

$K[i]$: the array of characteristic starting values.

For every value of $n$ we decide if array $K[i]$ must be completed with values for which $\frac{dS}{dK} = 0$. 

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Define: \( \text{MAX}(n) = \max_i \text{DISTANCE}(K(i), n) \) \( \text{MIN}(n) = \min_i \text{DISTANCE}(K(i), n) \)

When for the horizontal distance between the source and the point \( P \), range \( P \), the following relation holds:

\[ \text{MIN}(n) \leq \text{range } P \leq \text{MAX}(n), \]

then there will be rays intersecting the level considered for the \( K \)th time in \( P \).

When we repeat this process for \( n=1, \ldots, N \) both for direct as well as indirect rays, then we will find \( M \) rays going through \( P \). For each of these rays we determine the intensity and finally we find for the total transmission loss at \( P \)

\[ N_{spr} = 10 \log_{10} \left[ \sum_{i=1}^{N} \left( \frac{I}{I_P} \right) \right] \]

Of course there are some restrictions in the present computer model. Except for the restriction of a linear depth-velocity profile the most important assumption is that the sea surface is a flat plate in order to obtain a symmetric ray path. However, there is the possibility of giving an attenuation factor for each surface reflection. For the bottom similar assumptions are made.

When we introduce a waving surface, the simplicity of the computation disappears since the function \( S \) becomes much more complicated.

**DISCUSSION**

The author said that no comparisons had yet been made with measurements.
Transmission loss

\[ R = R(e_0, k) : \text{horizontal distance between transmitter and receiver} \]

\[ \frac{1}{R} = \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} \]

Transmission loss in dB

\[ H = 10 \log \left( -\frac{k}{\cos \theta} \right) \text{ dB} \]

\[ k = \frac{c}{\cos \theta} \quad \text{constant for 1 ray} \]

\[ g = \text{velocity gradient} \]

\[ k = k_0 \]

\[ S_l = S_0 \left[ \sin \theta - \sin \theta_0 \right] \]

\[ = \frac{1}{2} \left[ \sqrt{k^2 + c_n^2} - \sqrt{k^2 + c_m^2} \right] \quad \text{ray down} \]

\[ S_h = S_0 \left[ \sin \theta - \sin \theta_0 \right] \quad \text{ray up} \]

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\[ S = \sum_{i=1}^{n} (S_{l1})_i + \sum_{i=1}^{n} (S_{l2})_i + \sum_{i=1}^{n} (S_{h1})_i \]

\[ dS_i = \frac{1}{2} \left[ \sqrt{k^2 + c_m^2} - \sqrt{k^2 + c_n^2} \right] \]

\[ d(S_i) \]

furthermore:

- SECOND derivative always POSITIVE
- THIRD derivative always NEGATIVE

\[ d(S_i) \]

is a monotonically increasing function

\[ d(S_i) \]

is a function with two zero-points

Horizontal distance \( S \) as a function of \( k(\theta) \) on a certain depth-level

FIG. 1

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FIG. 2

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Direct Ray

--- Indirect Ray

FIG. 3

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