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An autonomous undersea surveillance vehicle: Concept and operational feasibility

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An autonomous undersea surveillance vehicle: Concept and operational feasibility

R.H. Riffenburgh

Executive Summary: The goal of this report is to present the concept of an autonomous undersea surveillance vehicle (AUSV) and analyze its operational use to learn if it could effectively assist the NATO navies' ASW mission. The sensors posed are active sonar and magnetic anomaly detection (MAD).

This report gives a functional description of the vehicle, a specification of control logic, and a concept of operation. The operational analysis shows that the AUSV described would be an effective ASW asset, especially in shallow water area search, a mission for which current assets are inadequate. For small areas, detection probabilities were much higher than those by current assets. MAD resulted in higher detection probabilities than active sonar and shows considerable promise.

The AUSV effectiveness was examined for sensors of current active sonar and MAD capabilities and a more optimistic MAD capability as shown likely by contemporary development in magnetics. Detection probabilities using both sensors are computed for area search and barrier search. Areas are examined from 100 to 1000 n.mi^2 and barriers from 2 to 100 n.mi in length. The effect of submarine evasiveness is calculated. Other uses for an AUSV-type platform are listed. Appendices include a dynamic enhancement table (relative speed of two randomly moving vehicles) and a documentation of some arguments on the numerous small *vs* few large combatant controversy.

It is recommended that the AUSV be further examined from the physical, engineering, and economic viewpoints and that other operational missions for AUVs be analyzed.

Report no. changed (Mar 2006): SM-215-UU

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An autonomous undersea surveillance vehicle: Concept and operational feasibility

R.H. Riffenburgh

Abstract: The concept of autonomous undersea vehicles (AUVs) is becoming feasible through the increasing practicality of artificial intelligence, decreasing size and cost of computers, and improved technical capabilities. This paper proposes a self-controlled undersea unmanned vehicle for anti-submarine surveillance. Given is a functional description of the vehicle, a specification of control logic, and a concept of operation. The effectiveness of this autonomous undersea surveillance vehicle (AUSV) is analysed, yielding further specification as related to detection probabilities. The AUSV is shown to be practical from an operational research viewpoint. It is suggested that the AUSV be examined further from the physical, engineering, and economic viewpoints. Appendices include a dynamic enhancement table (relative speed of two randomly moving vehicles) and a documentation of some arguments on the numerous small *vs* few large combatant controversy.

Keywords: active sonar ○ area search ○ AUSV ○ AUV ○ artificial intelligence ○ barrier search ○ control logic ○ dynamic enhancement ○ high-frequency sonar ○ MAD ○ ocean surveillance ○ operational effectiveness ○ probability of detection ○ RPV ○ short-range sonar ○ sonar ○ submarine evasiveness ○ surveillance ○ UMV

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Preface

During 1987, there emerged considerable interest in the capabilities of remote undersea vehicles from two NATO organizations: the SACLANT Undersea Research Centre and the Defence Research Group's Panels 7 (on operational research) and 1 (on long-range scientific development). Remotely operated vehicles (RPVs) appeared to be well advanced in development and deployment by the nations, e.g. in guided torpedoes and remotely controlled minehunters. On the other hand, autonomous undersea vehicles (AUVs) showed evidence of neglect.

In June 1987, Panel 7 asked the author of this report to look into the status of AUVs, including the operational feasibility of AUVs as maritime platforms to carry out certain NATO missions. Three informal papers for Panels 1 and 7 were prepared: a sampling of reports on UMVs to illustrate the status, a documentation of the numerous small *vs* few large combatant arguments, and a concept-and-feasibility study on a possible AUSV. The last paper composes the present report with the second paper as an appendix.

Separately, SACLANTCEN has initiated a pilot study of a joint engineering and operational nature to examine the physical character of possible AUVs and identify the missions for which they show evidence of being feasible. This study began in June 1988 carrying on from the earlier draft of the present AUSV report.

Contents

1. Introduction

This paper deals with an autonomous underwater vehicle (AUV). Its purpose is to describe a conceptualized specific AUV dedicated to surveillance, i.e. an autonomous undersea surveillance vehicle (AUSV), and to examine this AUSV's operational effectiveness.

AUVs are hardly new. Homing torpedos and submarine-launched mobile mines are certainly AUVs.* However, until recently, more sophisticated AUVs have been limited by technology and have remained only concepts. Recently technological progress has been made in a number of areas which might lift the advanced AUV from concept to reality (Corell, 1987). These areas include improvement in power (energy storage and transmission), control systems, navigation (including reduced cost and size of inertial navigation), sensors, the artificial intelligence triumvirate of evaluation/decision/execution, and in some situations communication. Perhaps it is an idea whose time has come.

The proposed AUSV exhibits a seemingly unique 'win-win' capability in that it has a very high probability of mission accomplishment whatever action the opponent may take. A submarine ignoring an AUSV will be detected with a very high probability and the AUSV's mission is accomplished. To avoid the AUSV, a submarine must stay out of the area of its targets and the AUSV's mission is accomplished. If a submarine attacks the AUSV, the best of options for us, the AUSV acts as a flaming datum, signalling the detection, location, and classification of the submarine; even should the submarine survive, it costs the opponent an expensive weapon and reduces its stock of threatening weapons. Submarines carry a very limited stock of weapons. Not only are these weapons costly, but replenishment may cost the opponent weeks out of action and risks of loss to transit to base for rearming and return. A more complete discussion of these issues, in fact a more general examination of the arguments associated with numerous small *vs* few large combatants, appears as Appendix A.

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* Some might claim a modern mine to be an AUV, but I take 'vehicle' to imply mobility or transportation.

2. Functional description of AUSV

Some AUSVs have been proposed in the past, but they were of rather complicated natures. It has been my observation that the realization of new concepts meets so many problems, usually underpredicted or unanticipated, that the majority fail. The best chance of realizing a new concept is to keep it as simple as possible.

Bergersen (1974) conceived of a medium-sized (12 m) platform, but it was designed for transocean convoy escorting operations, at convoy speeds naturally, a bigger task than here suggested. Tudor-Craig conceived of a (10 m) AUSV, but it was an armed long-range, very high-speed, surveillance-plus-attack platform, considerably more involved than what is envisioned here.

The AUSV here proposed would patrol a limited area at slow to moderate speeds, as high speed would require too much power and would give away its nature and position. It could be put in the water and retrieved by a 1–2 ton crane on a small ship, possibly as small as a PCer, but more practically a medium-sized frigate to provide helicopter prosecution and attack and for handling in heavy seas. Methods of handling, storage space on the mother platform, and the sea state in which it could be handled should be addressed in a later report in conjunction with total cost estimation. Included should be considerations of whether the mother could be a ship of opportunity or must be (temporarily) dedicated. The AUSV would carry enough power, probably electric, for, perhaps, a 72 h patrol. Thus, it would be a small platform, about the size of a heavy-weight torpedo (4 m or so). Also, it could be launched from torpedo tubes.

Absent would be pressurizing, human facilities, self-defence-oriented sensors, offensive and defensive weapons, and complicated communication devices.

Present would be power, a sensor system, control devices, a navigation system, a computer facility for calculation/decision logic, and releasable buoys to signal surface or air supporting units.

Two types of sensor are postulated: a magnetic anomaly detector (MAD) and a short-range active sonar.

MAD In deep water, an AUSV patrol at half the greatest usual operating depth of the submarine would maximize the volume of water examined for magnetic anomalies by an omnidirectional MAD. The only false targets to be expected would be surface vessels, and these could be eliminated as targets through range gating with a triaxial magnetometer. In shallow water, the AUSV would patrol at half the water depth to maximize volume of water examined. False targets would result from earth magnetism and bottom clutter on the seafloor. The MAD could be approximately range gated, but would miss possible targets resting on the bottom.

Sonar The AUSV would emit occasional (i.e. not continuous) high-frequency signals, chosen to occur at random. In deep water, the AUSV would patrol deep, below the operating depth of potential targets, looking upward with its active sonar. The short-range, high frequency (perhaps 50 kHz) sonar would beamform for better localization with each beam range-gated to stop short of the surface (perhaps 25 m). Suitable baffling at these higher frequencies could provide a high back-to-front ratio, allowing an upward look without interference from downward looking beams. The high frequency, the upward look, and the subsurface range gating would almost eliminate a false-contact problem. In shallow water, the AUSV would patrol close to the bottom.

3. Concept of AUSV operations

Let us define an AUSV patrol region as a volume of ocean in the shape of a box. The AUSV is programmed to move in a random fashion within the box. The sonar AUSV is further programmed to emit acoustic signals, or 'pings', with a random interval between them. The probability distributions from which the random control inputs are chosen are derived mathematically to maximize the success probability of the AUSV's mission.

The AUSV is launched probably by a surface vessel, but possibly by aircraft or submarine. A monitor platform remains within contact and response distance; the monitor could be an escort with helicopter for response, an MPA, or even an airship.

The AUSV operates randomly within its box until its patrol is finished. (The user specifies the nature of the patrol path, but it must contain enough randomness to be unpredictable by a target that is aware of its existence.) The MAD AUSV would be covert. A potential target could receive the sonar AUSV's emission, but would not know if it had been 'seen'. Upon making a contact, the AUSV signals its monitor. Present thinking is that the AUSV would eject a buoy which sends a radio signal upon reaching the surface; other possibilities exist. The monitor then prosecutes the target. An alternative might be for the AUSV to approach the target and clamp onto its hull, remora style. While a more difficult feat, the rewards would be much greater: elimination of false targets, verified detection, and precise and continual tracking.

When the patrol is concluded, the AUSV could surface and be retrieved or not. This decision depends on cost and asset availability: if AUSVs are plentiful and low cost or if there is not surface support available, it could be abandoned. An alternative would be for the AUSV to home in on its mother when its mission is complete and 'come home' to be retrieved.

The ability to signal when destroyed is a requirement. It is not included in the control logic of the next section because I have assumed a very simple automatic mechanism. The AUSV tows a buoy with the connexion reaffirmed by the control. If the control ceases, the connexion ceases to be held; the buoy surfaces and signals uniquely. Other mechanisms may easily be postulated.

The control logic can be extended to include a number of functions likely to be useful. When a contact is received by a sonar AUSV, it may be programmed to devolve to continuous active sonar to confirm, classify, and locate before signalling the monitor. However, the target would then know it had been seen and could take evasive action or begin countermeasures.

Another option would be to use passive sonar continuously on either the MAD AUSV or active sonar AUSV; while passive sonar would have very low probability of detection in the future time of this concept, any enhancement of detection probability would be welcome. In the active sonar AUSV, the addition of passive sonar would cost only signal processing capability and a little power.

A third option would be the ability of the AUSV itself to attack the target. This would require a warhead of considerable size, increased decision-making capability, much lower probabilities of error, and a homing device. A warhead implies increased size and power, implying a bigger motor and more fuel/energy, which in turn implies increased size and power – a converging series of increases resulting in a much larger and more complex craft. Torpedo-homing devices exist, but other options are also available and might well be examined for feasibility. For example, such a homing device could be developed from Cyr's (1987) Terrain and Obstacle Avoidance Sonar (TOAS) reprogrammed to seek certain obstacles and avoid others. After classifying and locating the target, the AUSV would either release its (homing) weapon or home itself upon the target. In the latter case, the AUSV might well increase speed and move on the target from astern. A small charge, or even the impact of the AUSV itself, could damage the screws sufficiently for, at minimum, mission abort.

4. Control logic

Figures 1 and 2 depict the gross, i.e. high level, control logic of the simplest MAD and sonar AUSVs, respectively. This control logic is not configured to include certain sophisticative capabilities discussed in Sect. 3:

- To further investigate a perceived contact.
- To use passive or non-acoustic information.
- To attack a target.
- To signal when destroyed.

The figures depicts flow charts of logic that describe the initiation and conduct of a patrol in a random pattern within a prescribed area and, for the sonar AUSV, independently provide randomly timed emissions. The AUSV signals when it perceives a target or when its patrol is finished.

The random numbers are drawn from probability distributions input by the operator. For example, when a constant depth is desired, the operator puts in a constant as distribution f_3 and this constant is drawn for each new setting. For another example, the patrol path within the patrol area A may be specified to some extent, but must contain enough randomness to be unpredictable. It has been shown repeatedly (e.g. USNA, 1977) that a patterned or exhaustive search produces better detection probabilities than a random search, so long as the target moves randomly, i.e. is not aware of the search pattern. A compromise between pattern and unpredictability is suggested here: The search legs would be usually long, either, say, $\sqrt{A}$ or drawn at random from a half-normal with mode $\sqrt{A}$, and the angle of turn to start a new leg would be drawn randomly from a uniform distribution of angles (excepting 0).

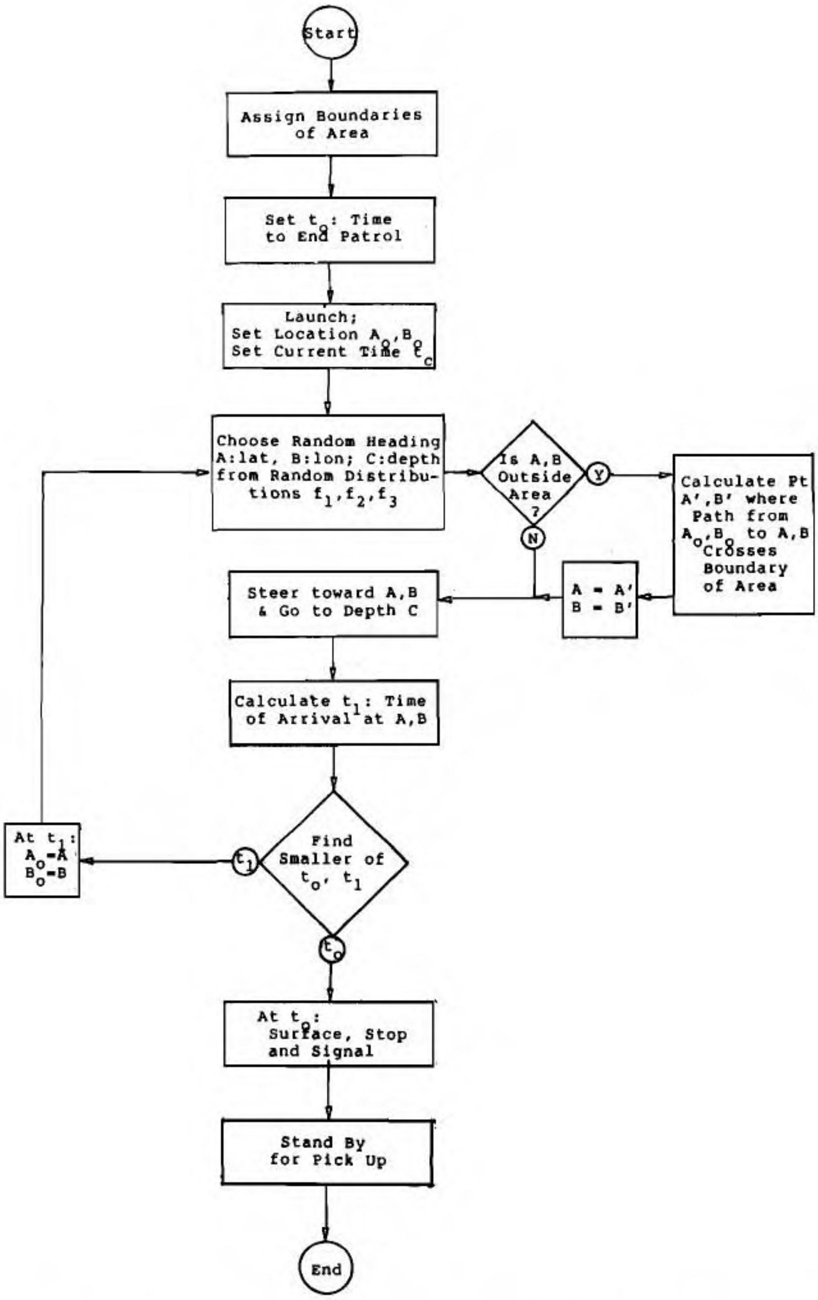

Fig. 1. Gross control logic of simplest MAD AUSV. Whenever a contact is made, a signal buoy is released, regardless of logic below.

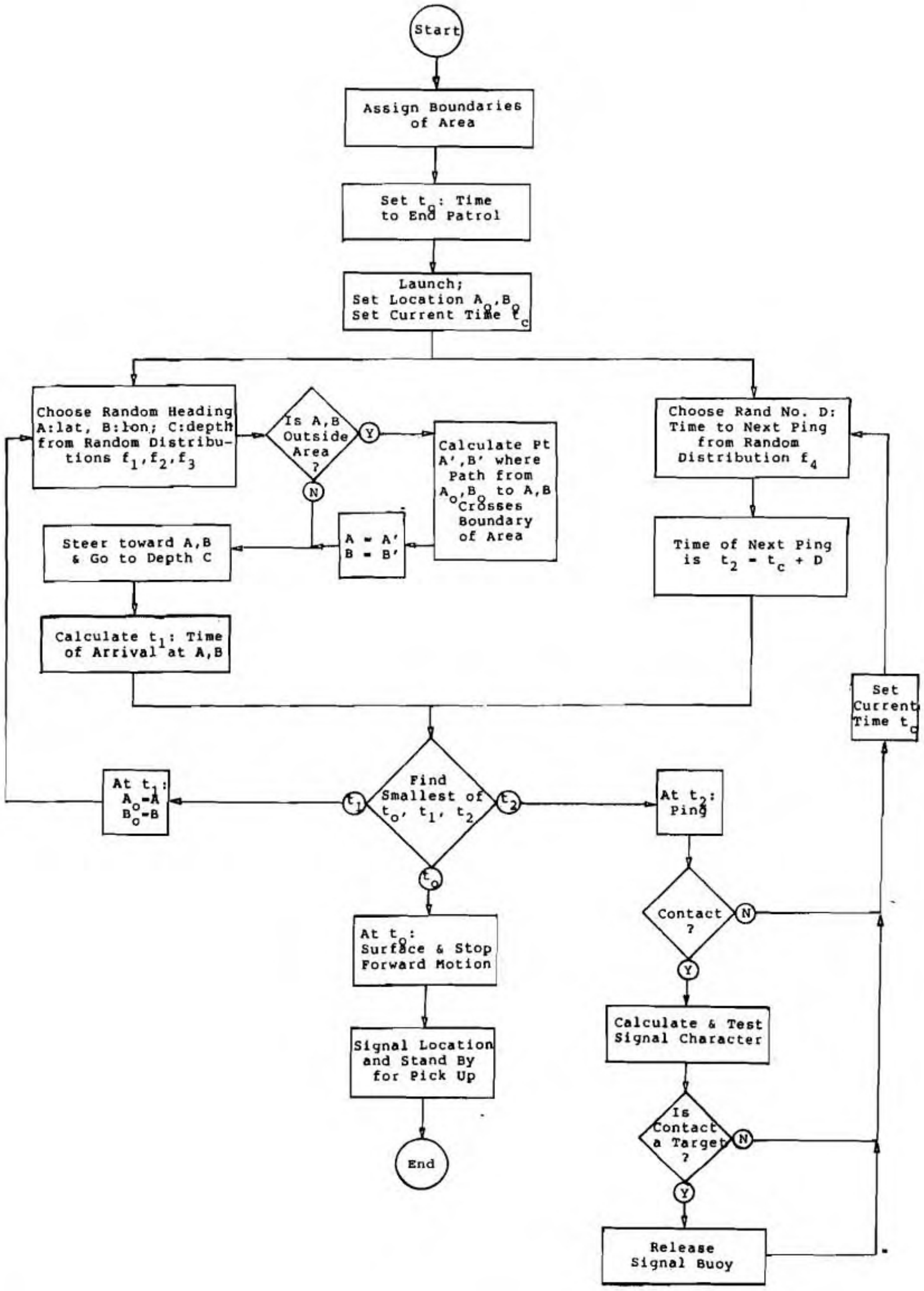

Fig. 2. **Gross control logic of simplest sonar AUSV.**

5. Operational effectiveness definitions

Equations expressing detection probabilities in naval search operations have become almost folkloric. The classical work is Koopman's 1946 book. The 1980 revision is also referenced. Other classical works are found in the naval operational research books of the US Naval Academy (1977), in Zehna (1971), and in Washburn (1981).

Table 1 contains a collection of definitions which may be helpful in following the development of and using the models in this paper.

From the plan (or overhead) view, the target and the AUSV are taken as positioned and moving random uniform in a plane figure of area A or length L. At time of contact, the sensor lies in the center of a circle whose radius is detection range R, such that the probability of detection of a submarine is 1 if it lies within this circle and 0 otherwise. Distance is given in nautical miles (n.mi), speed in knots (kn), and time in hours (h).

- Areas to be searched could be a (partially) confined water area like the Golfo di Napoli (with approaches, about 200 n.mi^2), the Folla–Frohavet Gap (about 500 n.mi^2), or the SWAP (about 1000 n.mi^2), or alternatively the area could be a small section of open ocean. In the open ocean, the 'mother' ship could lie at the intersection of four adjacent rectangular areas. AUSV area search could even fit into a resurrected concept of a defended convoy lane.

  For patrolling submarines in areas, 48 h is assumed as the time S remains in the area (of size A), as this is a common time given ASW forces to 'sanitize' an area before operations in that area commence.

- Barriers are generally laid approximately perpendicular to the expected path of a transitter, as an orthogonal barrier maximizes the barrier length for a given number of assets. Barriers to be searched could lie across a natural choke point, such as Gibraltar (8 n.mi at narrowest), the Skagerrak (about 50 n.mi), or the Sicily Strait (about 75 n.mi), or alternatively they could lie across a man-made choke point like a safe passage between mine fields.

Table 1
Definitions

S	a submarine (the target)
A	area (plan view) of antisubmarine search (n.mi^2)
t	time S spends in A (h)
v	speed of S (kn)
u	speed of platform carrying sensor (kn)
P_d	probability of one or more detections while S is in A, possible evasions by the target not considered
E	event that intended track of S comes within range of a given sensor (falls in detection circle) ('E' for encounter)
F	event that S fails to avoid detection circle
G	event that sensor emits a pulse
R	detection range of sensor used in models (n.mi)
R_s	effective range of sensor assumed
D_T	largest usual operating depth of potential targets
D_A	patrol depth of AUSV
c	expected time between active sonar pulses (h)
$I(k)$	complete elliptic integral of the second kind depending on $k = 2\sqrt{(uv)/(u+v)}y$
t'	time available for search lost due to target's evading
$P_{d\|\text{poss}(e)}$	probability of detection considering target will evade *if* warned, but *may not* be warned
$P_{d\|e}$	probability of detection for case in which target *is* warned and *does* evade
$P_{d\|\text{not}(e)}$	probability of detection for case in which target is *not* warned (and therefore does not evade)

6. Effectiveness in area search using MAD

In recent developmental work on MAD, France (Devaud, 1987) and the United States (Texas Instruments, 1987) have quoted significantly improved MAD performance. Their results show MAD ranges exceeding those of high-frequency active sonar. These ranges provide a rationale for including a MAD AUSV in this paper. However, because these results seem somewhat optimistic technically, the model used in this paper will be carried out as well for more pessimistic ranges.

Consider an AUSV in fixed position for a moment. A MAD search by the AUSV may be thought of as the random track of a submarine patrolling at speed v kn through the search area with the radius of MAD detection radiating perpendicularly out from the track so that a detection sleeve, centered on the track, is formed. The probability of detection is based on the volume of the sleeve divided by the volume of the patrol region. If A_d denotes the (detection) area of a cross section of the sleeve, the detection probability, as in Koopman (1946), p. 28, is

$$P_\mathrm{d} = 1 - \exp(-A_\mathrm{d} vt/AD_\mathrm{T}). \tag{1}$$

When the AUSV is moving (at speed u kn) as well, the search speed is dynamically enhanced. The development of this enhancement appears in a number of places, an early one of which is Koopman (1946), p. 7. This enhancement leads to replacing v in Eq. (1) by

$$w = 2(v+u)\,I(k)/\pi. \tag{2}$$

To save the reader from the inconvenience of calculating w using an elliptic integral table, values of w as depending on u and v appear in Appendix B. The more general probability of detection becomes

$$P_\mathrm{d} = 1 - \exp(-A_\mathrm{d} wt/AD_\mathrm{T}). \tag{3}$$

To derive the value of A_d, consider Fig. 3, a diagram of the cross section of the detection sleeve. A_d is obtained by subtracting from πR^2 the areas of the segments of the detection circle lying above the sea surface and below the maximum usual operating depth of S. The area of a segment is the area of the sector containing it less the area of the remaining triangle. Then, taking angular measure in degrees,

$$A_\mathrm{d} = \pi R^2 - \pi R^2(\phi_1+\phi_2)/180 + D_\mathrm{A} R\sin\phi_1 + (D_\mathrm{T} - D_\mathrm{A})R\sin\phi_2. \tag{4}$$

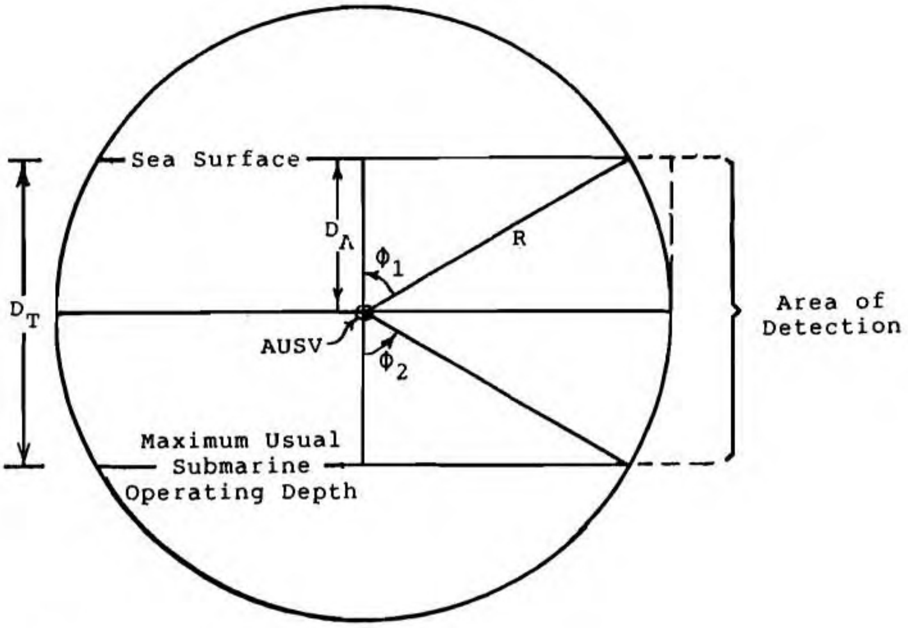

Fig. 3. Front view of cross section of MAD search sleeve.

Since A_d is maximized by taking $D_A = D_T/2$, this depth for the AUSV will be usual and will be assumed in this paper. Then, taking $\phi_1 = \phi_2 = \phi$,

$$A_d = \pi R^2(1 - \phi/90) + 2D_A R \sin\phi. \tag{5}$$

Note that this area is not far from the area of the rectangle $2RD_T$. It might be easier to calculate A_d by finding the proportion error, say α, from using this rectangle and multiplying the rectangular area by this error.

$$\alpha = A_d/2RD_T = \frac{\pi R}{4D_A}(1 - \phi/90) + \tfrac{1}{2}\sin\phi. \tag{6}$$

Then P_d becomes

$$P_d = 1 - \exp(-2\alpha Rwt/A). \tag{7}$$

It may also be of interest that the rate of surface area coverage (from the plan view) is given by $2\alpha Rw$.

7. Effectiveness in area search using active sonar

In the case of a high-frequency active-sonar AUSV, the relationship is not so simple. It is possible that S will be within range of the sensor during a time when it does not emit a signal. Furthermore, the AUSV is not covert as with MAD; it is possible that S hears the signal before coming within detection range and therefore evades. It is even possible that S recognizes the AUSV for what it is and moves at greater speed to a (relatively) distant location, thus violating the randomness assumptions and invalidating the mathematics. Detection occurs if S is encountered, if S does not evade given it has been encountered, and if the sensor emits a signal given S has been encountered and has not evaded. As probabilities, the probability of detection on a single encounter is given by

$$P(E)\,P(F|E)\,P(G|F,E). \tag{8}$$

In a recent manuscript, Goodman showed some advantages of a distributed active sensor field with a very low pulse repetition rate. The detection success of Goodman's model of sensors with very low detection probability per sensor was surprising. It suggested to me that a low pulse repetition rate might be beneficial in the case of an AUSV.

While an exact formulation of sonar detection probability would have to use volumes based on a detection cone extending from the AUSV upward to the depth above which a submarine would be too vulnerable to surface detection to operate, the qualities of high frequency, short ranges, and upward looking permit approximation to the cone by a cylinder with vertical axis, as seen in side view in Fig. 4. This cylinder has depth D_{T} defined as the largest usual operating depth of potential targets and a radius R minimizing the errors of this assumption, i.e. intersecting the detection cone at $D_{\mathrm{T}}/2$. Such a cylinder will then permit the detection probability to be calculated based on detection circles superposed on the surface plane. From Fig. 4, it can be seen that

$$R = (1 - D_{\mathrm{T}}/2D_{\mathrm{A}})\left(R_s^2 - D_{\mathrm{A}}^2\right)^{1/2}. \tag{9}$$

For the sonar AUSV, an emission is taken as an acoustic pulse. The pulse repetition rate (PRR) is determined by the time between pulses c denoted here as duty cycle. This duty cycle may be taken as constant in some cases, but unless specified otherwise is taken as the expectation of a random uniform distribution of pulses.

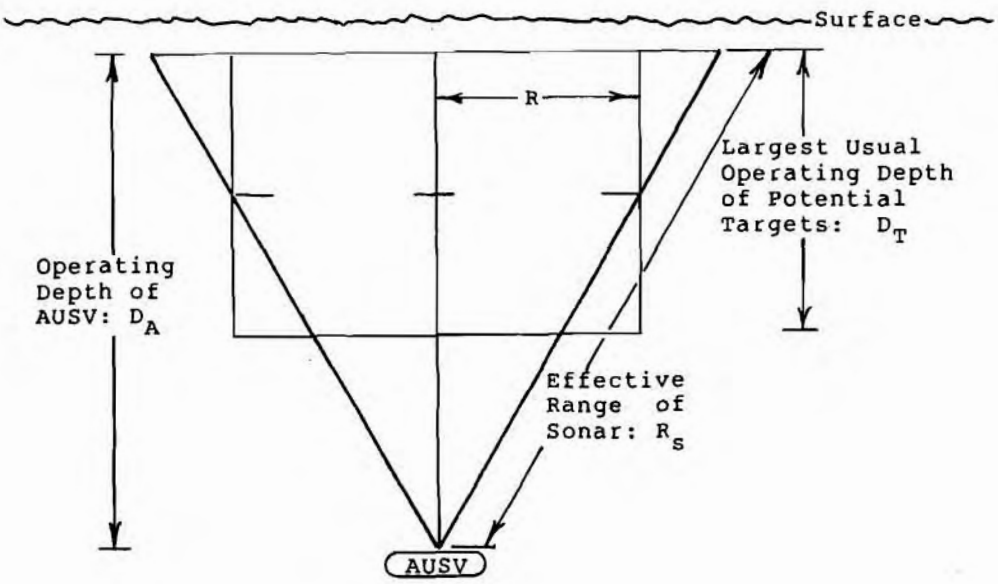

Fig. 4. Side view of AUSV sonar coverage.

7.1. $P(E)$

The probability that S's intended track comes within range of emission of a given sensor, in fixed position for the moment, i.e the probability that the (randomly positioned) sensor lies within detection range of S's path, is given by the area 'swept' by S divided by the total area, less overlaps, or approximately

$$P(E) = 2Rvt/A. \tag{10}$$

This relationship may be visualized as in Fig. 5.

Again, letting both units move, we have w as in Eq. (2) and the more general $P(E)$ becomes approximately

$$P(E) = 2Rwt/A = 4R(v+u)\,tI(k)/\pi A. \tag{11}$$

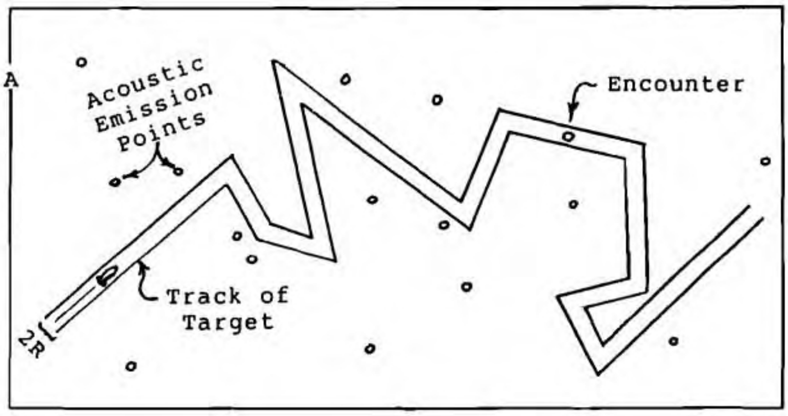

Fig. 5. **Representation of target's random path through and random encounters with a set of randomly positioned fixed sensors.**

7.2. $P(F|E)$

If R is the range at which sound travels to the target, reflects, and returns to the source in strength sufficient to be perceived, then the range at which a target can perceive the source is greater than R. How much greater this range is will depend on type of receiver, target strength, and physics of the local water. For the purpose of this feasibility investigation, let us make the usual assumption: S can perceive the source at distance $2R$.

Under the assumption that S will sense the signal if within range and will evade if it does sense the signal, it will evade if the acoustic emission occurs while it is within the shaded area of Fig. 6. The probability that S fails to evade, given encounter, is 1 − *this evasion probability*, or 1 − *(area shaded)/(area swept during duty cycle).* The shaded area is *(area of outer circle of radius 2R − area of inner circle of radius R − area of two segments cut off by the path)/2*. In symbols, the shaded area is

$$[\pi(2R)^2 - \pi R^2 - 2(4\pi R^2/3 - \sqrt{3}\,R^2)]/2 = (\pi/6 + \sqrt{3}\,)R^2. \qquad (12)$$

We divide Eq. (12) by the area swept during the duty cycle, $2R$ wide by wc long, and subtract from 1:

$$P(F|E) = 1 - [R(\pi/6 + \sqrt{3}\,)]/2wc. \qquad (13)$$

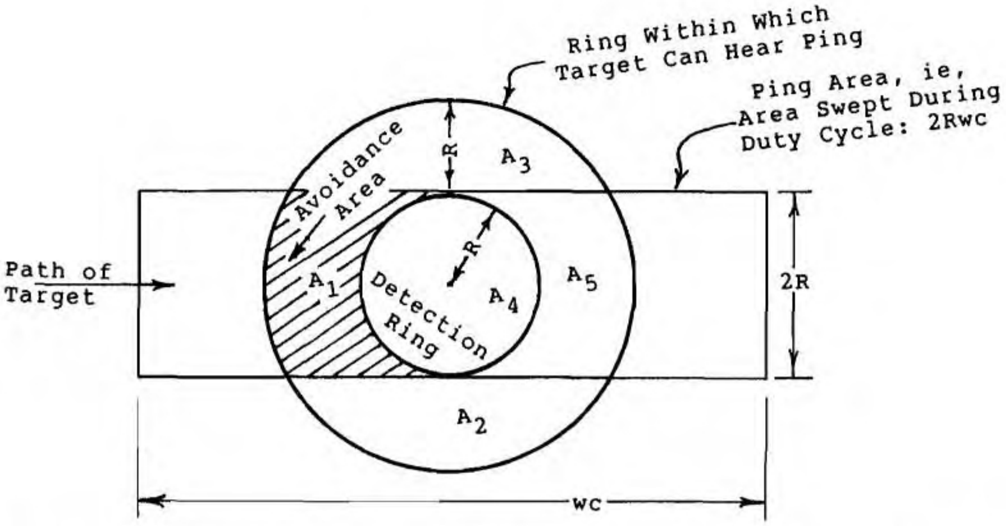

Fig. 6. **Representation of target's possible warning and avoidance of AUSV relative to detection area.**

7.3. EFFECT OF SUBMARINE EVASIVENESS

The cost to P_{d} of evasion by the target can be calculated, but only given an assumed evasion policy. Given an evasion policy that can be translated into the probability of detection given a warning is definitely received, i.e. $P_{\mathrm{d|e}}$, the adjusted probability of detection where a (single) evasion might or might not occur becomes

$$P_{\mathrm{d|poss(e)}} = [1 - P(F|E)]\, P_{\mathrm{d|e}} + P(F|E)\, P_{\mathrm{d|not(e)}}. \tag{14}$$

In Sect. 10, an evasion policy is assumed and an example of $P_{\mathrm{d|poss(e)}}$ is worked out for illustration.

7.4. $P(G|E,F)$

The probability that the sensor emits while S is within range, given that S encounters the sensor, is given by the area of the detection circle divided by the area swept during the duty cycle not subject to evasion. This may be visualized in Fig. 7 as the area of the circle divided by the area of the rectangle less the shaded area:

$$P(G|E,F) = (\pi R^2)/[2Rwc - R^2(\pi/6 + \sqrt{3}\,)] = (\pi R)/[2wc - R(\pi/6 + \sqrt{3}\,)]. \tag{15}$$

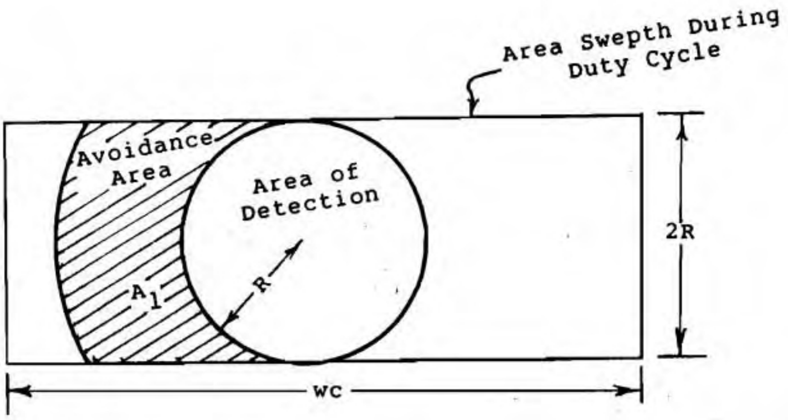

Fig. 7. Representation of detection area, given target passes through avoidance area without warning.

7.5. PROBABILITY OF DETECTION IN AN AREA

Substituting Eqs. (11), (13) and (15) in Eq. (8) and allowing for multiple encounters,

$$P_d = 1 - \exp(-\pi R^2 t/Ac). \tag{16}$$

The choice of c, the duty cycle length, is crucial. The detection range R is very much fixed by physics and the speed u cannot be varied enough to have as great an effect. But c is not confined and a best c should be investigated.

From Eq. (13), the smaller the c, the more likely the target will be warned and avoid the detection circle. However, in contrast, from Eq. (15) the smaller the c, the more likely the target will be detected, given he does not avoid. If Eqs. (13) and (15) are jointly evaluated as a function of c, the intersection where the two probabilities are equal yields a value of c optimum in the minimax sense. Setting Eq. (13) = Eq. (15) yields a quadratic equation in c. Solving for c and substituting constants yields the approximate solution

$$c = 3.22R/w. \tag{17}$$

Thus, c varies directly as R and inversely as w. For example, if $R = 0.2$ (400 yd) and $w = 3.82$ (both S and AUSV moving at 3 kn – see Appendix B), then $c = 0.169$ (10 min).

If the optimum c as in Eq. (17) is substituted into Eq. (16), P_d becomes

$$\begin{aligned} P_d &= 1 - \exp(-\pi Rwt/3.22A) \\ &= 1 - \exp(-0.9756\, Rwt/A). \end{aligned} \tag{18}$$

For the case of optimum c, MAD AUSV is of interest to be compared with sonar AUSV. The ratio of MAD P_d to sonar P_{d}, i.e. Eq.(7) divided by Eq. (18), provides a measure of relative effectiveness.

8. Effectiveness in barrier search using MAD

The AUSV moves back and forth in a random fashion along the barrier so that its position at the time S crosses the barrier is random. If the barrier is looked at in lateral view, as if from the view of an approaching S, Fig. 3 may be used again, however with the AUSV moving from side to side rather than toward the viewer. It follows that

$$P_{\mathrm{d}} = 1 - \exp(-2\alpha R/L). \tag{19}$$

9. Effectiveness in barrier search using active sonar

For high-frequency active sonar, the AUSV moves back and forth at random along the barrier, but emitting active sonar signals at randomly chosen intervals. The development of P_d is a slight variation on Eqs. (8)–(16).

$P(E)$ is the length along the barrier covered by the AUSV sonar divided by the length of the barrier, or

$$P(E) = 2R/L. \tag{20}$$

In the ensuing terms, the speed appears. As the path of S is perpendicular to the barrier, and as the calculations are not altered by slanting the path of S across the barrier to adjust for AUSV speed, v may be used instead of w. The avoidance terms cancel in the same way as in 13×15, i.e. $P(F|E)\ P(G|F,E) = P(F,G|E) = P(G|E)$ so that $P(G|E)$ may be calculated directly. $P(G|E)$ is the area of detection divided by the area swept by S during the duty cycle, or

$$P(G|E) = \pi R^2/2Rvc = \pi R/2vc. \tag{21}$$

Multiplying,

$$P_d = P(E,G) = \pi R^2/Lvc. \tag{22}$$

The best c is again of interest and follows the same development. Equation (17) again gives the approximate c as depending on R and the speed, using v in place of w.

10. Examples of operational effectiveness

Values for the parameters appearing in the formulae must be given for examples. In order to keep this paper unclassified, actual values of the parameters will not be used; the values chosen will be arbitrary for illustration.

10.1. AREAS

■ *10.1.1. MAD*

The area patrol time t has already been taken as 48 h. Two ranges at which the MAD detects its target are taken as Pessimistic $R = 250\,\mathrm{m} = 0.135$ n.mi and Optimistic $R = 1000\,\mathrm{m} = 0.54$ n.mi. The usual maximum operating depth of S is taken as $D_\mathrm{T} = 500\,\mathrm{m} = 0.27$ n.mi. The AUSV operating depth D_A is half D_T. Substituting in Eq. (6) yields the adjustment factors

$$\alpha = \begin{cases} 0.785 & \text{(pessimistic)} \\ 0.989 & \text{(optimistic)}. \end{cases} \tag{23}$$

Now w is needed. For v, 3 kn is taken as the usual minimum operating speed of a submerged submarine, barring exceptional circumstances, and typical of submarines on patrol. Our AUSV is a small, low-powered vehicle, but must move to be effective and move fast enough to maintain way and navigational accuracy; $u = 3$ kn is reasonable. From the Appendix, $v = u = 3$ gives $w = 3.82$. Substituting in Eq. (7) for various areas yields Table 2.

The rate of area coverage in the above example is

$$2\alpha R w/\mathrm{h} = \begin{cases} 0.81\ \mathrm{n.mi^2/h} & \text{(pessimistic } R\text{)} \\ 4.08\ \mathrm{n.mi^2/h} & \text{(optimistic } R\text{)}. \end{cases} \tag{24}$$

■ *10.1.2. Active sonar*

Consider Fig. 4. The operating depth of the AUSV will be taken as 900 m with the beams range-gated to 25 m below the surface, so that $D_\mathrm{A} = 875$ m. The largest usual operating depth of potential targets will be taken as $D_\mathrm{T} = 500$ m. The effective range of the sonar will be taken as $R_s = 1000$ m. Then Eq. (9) becomes

$$R = 346.4\ \mathrm{m} = 0.187\ \mathrm{n.mi} \tag{25}$$

Table 2

Probabilities of detection of a patrolling submarine by the AUSV using MAD over a 48 h period as depending on area

Area	P_d		Example area
(n.mi^2)	(pessimistic)	(optimistic)	
100	0.322	0.859	
200	0.177	0.624	Golfo di Napoli
300	0.122	0.478	
400	0.093	0.387	
500	0.075	0.324	Folla–Frohavet Gap
750	0.050	0.230	
1000	0.038	0.178	SWAP

Table 3

Probabilities of detection of a patrolling submarine by the AUSV using short-range active sonar over a 48 h period as a function of patrol area

Area	P_d		Example area
(n.mi^2)	($R = 0.187$)	($R = 0.54$)	
100	0.285	0.620	
200	0.155	0.383	Golfo di Napoli
300	0.106	0.275	
400	0.081	0.215	
500	0.065	0.176	Folla–Frohavet Gap
750	0.044	0.121	
1000	0.033	0.092	SWAP

Substituting in Eq. (17)

$$c = 3.22R/w = 0.157 \text{ h or } 9.45 \text{ min.} \tag{26}$$

Substituting the above parameters in Eq. (16), we can generate Table 3. An $R = 0.54$ is added for comparison with optimistic MAD range results in Table 2.

Evasive target Let us now consider a sample evasion policy: If S detects the sonic emission before reaching the detection circle, he turns and flees at speed $2v$ (enough to escape without making excessive noise) to the edge of the patrol area, where he begins again his random patrol. (While S has his own patrol area which may be different when not in confined waters, its border may be closer or farther, so we assume that on the average it is the same distance.) On the average, the starting point will be in the centre of the patrol area. On average, the distance to the border, say b, of a square area A will be about $1.08\sqrt{A}$ (distance for angle median between angles giving smallest and largest distances).

Under this evasion policy, $P_{\mathrm{d}|e}$ depends on the relative positions of S and AUSV as a function of time after flight. After a very short time, the position of AUSV in A is approximately bivariate normal with mean on the position of the AUSV at onset of evasion.

However, the bivariate normal is in error until a great deal of time has passed, substantially affecting the time lost from the surveillance patrol due to evasiveness by the target. This error from the normal results from the possibility space of a normal being assumed infinite, whereas the true possibility space of the AUSV position at some time after onset of evasion is a finite circle with radius equal to $ut' = r_0$, say. The theory for converting a normal curve defined on the correct (finite) possibility space appeared in Riffenburgh (1969). Because of symmetry in the (circular) possibility space, no generality will be lost by using the theory for the univariate case.

If $n(x)$ is the standard normal density and $N(r_0)$ is the standard normal integral from 0 to r_0, the transformation to finite possibility space is given by

$$g(x) = \frac{n(x) - n(r_0)}{2[N(r_0) - r_0 n(r_0)]}, \quad -r_0 \leq x \leq r_0. \tag{27}$$

In notation analogous to common normal expression, let us denote the area under the transformed probability curve as

$$G(x_0) = \int_0^{x_0} g(x)\,\mathrm{d}x. \tag{28}$$

As Eqs. (27) and (28) are somewhat involved to work out numerically, this paper used tables of $g(x)$ and $G(x_0)$ appearing in the referenced publication.

By the search policy of the AUSV, when it reaches a border, it is reflected back into the patrol area. Because of this reflection, as the standard deviation of $g(x)$ grows, the probability density over the minor segments of the possibility circle fold over upon the major segments and the resulting density approaches a bivariate uniform. This process can be visualized by the right half of a cross section of the distributions shown in Fig. 8. (The distribution has been drawn as standardized, so the standard deviation is $s = 1$.)

As t', the time after onset of evasion, grows larger, the approximation to the original uniform probability distribution grows better, but the time lost to searching grows larger. It was necessary to find t', s, and the error in approximating the uniform by a finite-space-transformed normal simultaneously. These values were calculated iteratively using converging trial-and-error methods until the error grew small. t' was expressed implicitly in r_0. The value of q is the point at which the sum of folds of the transformed normal equals the height of the uniform, i.e. the point to the left of which the uniform is lesser and to the right of which the uniform is greater. Then with q, r_0, and b defined, the error can be written as the sum of differences of integrals over the probability densities, or, standardized for s,

$$\begin{aligned}
\text{error} &= 2\int_0^{q/s} g(x)\,dx + 2\int_{2(b-q)/s}^{2b/s} g(x)\,\mathrm{d}x - q/b \\
&\quad + \frac{b-q}{b} - 2\int_{q/s}^{b/s} g(x)\,\mathrm{d}x - 2\int_{b/s}^{(2b-q)/s} g(x)\,dx \\
&= 1 - 2q/b + 2[G(q/s) + G(2b/s) - G((2b-q)/s) - G(b/s) \\
&\quad + G(q/s) - G((2b-q)/s) + G(b/s)] \\
&= 1 - 2q/b + 4[G(q/s) - G((2b-q)/s)] + 2G(2b/s). \qquad (29)
\end{aligned}$$

For minimum error, r_0 came out to be approximately $2b$, so $G(2b/s) = 0.5$ and the error became

$$2 - 2q/b + 4\big[G(q/s) - G((2b-q)/s)\big]. \qquad (30)$$

The values r_0, s, b, q, and error were calculated iteratively until the error fell below 1%. The values for this error were

$$\begin{cases} r_0 & = 2\mathrm{b}/s \\ s & = 0.91\ \mathrm{b} \\ b/s & = 1.1 \\ q/s & = 0.86 \\ \text{error} & = 0.006\ . \end{cases} \qquad (31)$$

It may be of passing interest that the normal approximation to the uniform for the same variable values, not using the finite space transformation, gave an error double in size.

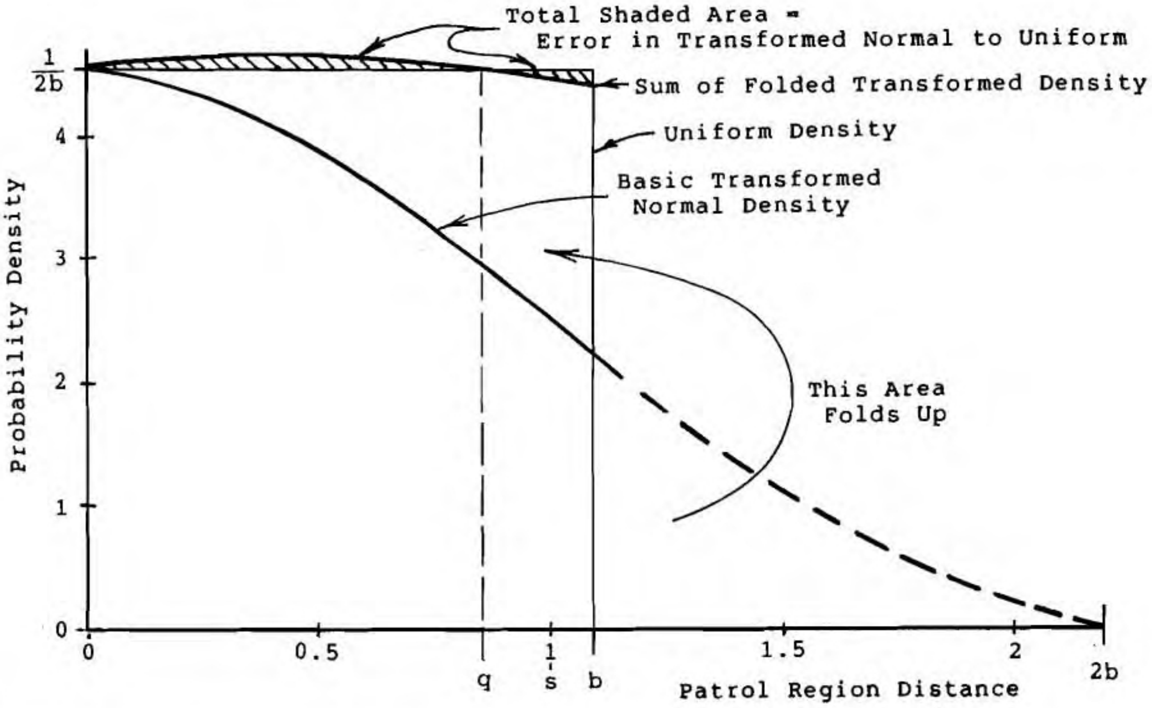

Fig. 8. Cross-section view of uniform and transformed normal (adjusted to have finite possibility space) representing transition from approximately normal to uniform probability of position of AUSV as time after target starts evasion. 'b' is bound of patrol region.

The important figure to be had from these foregoing paragraphs is (since standard $s = 1$) $r_0 = 2b = 2.16\sqrt{A}$ and, with our example of $u = 3$

$$t' = 2b/u = 2.16\sqrt{A}/u = 0.72\sqrt{A}. \tag{32}$$

Thus for $A = 100$ n.mi^2, for example, the time lost due to the target evading is 7.2 h, after which the search has returned to its initial state and probabilities.

Since from Eq. (16) P_d is proportional to t (the total search time), we can derive a $P_{d|e}$ as P_d adjusted for search time loss, or

$$P_{d|e} = P_d(t - t')/t. \tag{33}$$

Substituting Eq. (33) into Eq. (14), we obtain Table 4.

How serious is this loss in probability? A useful measure of debility would be the expected, or 'average', loss in detection probability per patrol, given by the proba-

Table 4

Probability of detection reduced by the possibility of a single evasion by S and the proportion of such probability lost by this evasion[1]

Area (n.mi^2)	t' (h)	$P_{d\|e}$	P_d	$P_{d\|poss(e)}$	Propagation loss
100	07.2	0.285	0.335	0.317	0.05
200	10.2	0.132	0.167	0.155	0.08
300	12.5	0.083	0.112	0.102	0.10
400	14.4	0.059	0.084	0.075	0.11
500	16.1	0.045	0.067	0.059	0.12
750	19.7	0.027	0.045	0.039	0.15
1000	22.8	0.017	0.033	0.027	0.18

[1] Also given are the search time lost (h), the probability of detection remaining given an evasion does occur ($P_{d|e}$), and the probability of detection given no evasion.

bility that an evasion occurs multiplied by the proportion loss in detection probability if it does occur. The probability that an evasion occurs may be calculated by $P(E)[1-P(F|E)]$. The expected loss in detection probability due to target evasiveness per patrol comes out to be 2% for $A = 100$ and diminishes to less than 1% for $A = 1000$, not a worrisome amount.

What would be the chance of and effect of multiple evasions? The probabilities of successive evasions may be calculated from $P(E)[1 - P(F|E)]$, with declining t remaining for the patrol entering into successive evasions. Since the probabilities return to their original state after t', the probability of two evasions is the product of probabilities of the first and the second. Table 5 results.

It can be seen that the probability of repeated evasions is negligible except for very small areas. For the smallest area, 100 n.mi^2, the probability that two evasions occur times the expected loss in detection probability is of the order of a tenth of a per cent. It is much smaller for larger areas.

Table 5

Probability that a first evasion occurs, probability that a second occurs, and the probability that both occur

Area ($n.mi^2$)	Probability (one evasion)	(second evasion)	(both)
100	0.240	0.204	0.049
200	0.120	0.094	0.011
300	0.080	0.059	0.005
400	0.060	0.042	0.003
500	0.048	0.032	0.002
750	0.032	0.019	0.001
1000	0.024	0.013	0.000

10.2. BARRIERS

▪ *10.2.1. MAD*

From before, pessimistic R was given as 0.135 with associated α of 0.785 and optimistic R was 0.54 with α of 0.989. For L, let us consider barriers of 2 to 100 n.mi in length. To picture the lengths involved (even though the depths of the examples may not match the model assumptions), we shall annotate the table with the Gibraltar Strait, the Skagerrak, and the Sicily Strait. Substitution in Eq. (19) yields Table 6.

▪ *10.2.2. Active sonar*

The variables R, v, c, and L appear in Eq. (22). However, if we optimize c by Eq. (17), v falls out and P_d becomes $0.9756R/L$. R was given in Eq. (22) independent of search type, so again $R = 0.187$. Values of L will be taken as in Subsect. 10.1.

Substituting the above parameters in $0.9756R/L$, we can generate Table 7. Again, let us introduce $R = 0.54$ for comparison with optimistic MAD results from Table 6.

Table 6

Probabilities of detection by a MAD AUSV of a submarine transitting across a barrier of length L (n.mi)

Length	P_d		Example area
(n.mi)	(pessimistic)	(optimistic)	
2	0.101	0.414	
4	0.052	0.234	
6	0.035	0.163	
8	0.026	0.125	Gibraltar
10	0.021	0.101	
15	0.014	0.069	
20	0.011	0.052	
30	0.007	0.035	
50	0.004	0.021	Skagerrak
75	0.003	0.014	Sicily Strait
100	0.002	0.011	

Table 7

Probabilities of detection by the AUSV using short-range active sonar of a submarine transitting across a barrier of length L (n.mi)

Length	P_d		Example area
(n.mi)	($R = 0.187$)	($R = 0.54$)	
2	0.087	0.232	
4	0.045	0.123	
6	0.030	0.084	
8	0.022	0.064	Gibraltar
10	0.018	0.051	
15	0.012	0.035	
20	0.009	0.026	
30	0.006	0.018	
50	0.004	0.011	Skagerrak
75	0.002	0.007	Sicily Strait
100	0.002	0.005	

10.3. RELATIVE EFFECTIVENESS MAD AUSV TO SONAR AUSV

The ratio of detection probabilities for the MAD *vs* active-sonar AUSVs used in area search gave a measure of their relative effectiveness. This ratio can be calculated for area search by dividing a Table 2 entry by its corresponding Table 3 entry and for barrier search using Tables 6 and 7 similarly. The pessimistic MAD has a detection probability of 13% to 15% better than short-range sonar. The optimistics MAD performs 300% to more than 500% better than short-range sonar. Also, since the sonar P_d did not allow for evasion of a warned target, the true MAD advantage would be even greater.

The results imply that a MAD AUSV outperforms a short-range active-sonar AUSV. Even with pessimistic MAD ranges, a MAD AUSV is about 20% more likely to make a detection. However, considering modelling error, we must say that they are close enough to competitive as to require more detailed study. If, on the other hand, the optimistic MAD range should be achievable, a MAD AUSV is clearly far superior.

11. Comparison with other ASW assets

Comparison with other assets is dependent on assumed scenarios, tactics, and capabilities of both own and threat forces, which would include quoting classified data. It is therefore beyond the limitations and intended scope of this feasibility exploration. However, to set the mental stage for the reader, a magnitude comparison with one example will be attempted. Willems and Moore calculate detection rates per 24 h patrol period in the Folla–Frohavet Gap off the coast of Norway for various ASW assets. They define the boundaries of the Gap to include 600 $\mathrm{n.mi}^2$. If the Gap is patrolled by four AUSVs in four equal areas and a helicopter frigate acting as AUSV mother keeps station at the intersection of the four areas, Eq. (7), for $t = 24$ and for our assumptions yields a detection probability during the patrol of 0.13. This probability is better than the performance of some assets and at least competitive with others. The point may be taken that predicted AUSV performance is good enough in comparison with other assets to justify further development of the concept.

12. Uses for an AUSV-type platform other than surveillance

The AUSV miniunit described in this paper could easily be reconfigured to accomplish other missions, either instead or in addition. Among these missions are:

- Attack (already mentioned).
- EW/jamming.
- Acting as deceptive drone.
- Target acquisition and OTHT.
- Tattletailing/marking.
- Communications relay.
- MCM harbour survey and NOMBO updatin.
- Distantly launched mobile mining.
- Distantly launched running to a harbour, then seeking/homing as a torpedo.

13. Summary, conclusions, and recommendations

Summary A conceptualized specific autonomous undersea vehicle dedicated to surveillance (AUSV) and its operation are described and its operational effectiveness is investigated. It seems to be a 'win-win' asset, since either it deters or discovers an encroaching submarine or its low cost and lack of personnel aboard allow it to be sacrificed to serve as a flaming datum, at once detecting, classifying, and locating the target.

The AUSV would be a low-cost, simple platform, about the size of a heavy-weight torpedo. It would have programmable self-control, allowing enough randomness of movement as to be unpredictable by a target. It would have either MAD or short-range, high-frequency active sonar. Sonar 'pings' would occur at random times. When detecting a target, or when destroyed itself, it would notify a fast-attack 'mother' platform.

Operational effectiveness theory is developed and examples produced. Parameters are arbitrarily assumed for the examples to avoid security classification. Both AUSV and target are assumed to travel at 3 kn, submarines are assumed to operated to 500 m depths, and area patrols are assumed to last 48 h. MAD range is taken as slightly better than WWII ranges, viz. 250 m (pessimistic MAD), and 1000 m (optimistic MAD) as less than ranges claimed by some contemporary industrial developers.

The upward-looking sonar range is assumed to be 1000 m, giving a horizontal effective range of 350 m.

Area search was exemplified for a number of areas, including one the size of the Golfo di Napoli and one the size of the SouthWestern Approaches (SWAP). For MAD, the detection probabilities, pessimistic and optimistic, respectively, were 0.177 and 0.624 in the smaller area and 0.038 and 0.138 in the larger over a 48 h patrol. For sonar, detection probabilities were 0.167 in the smaller area and 0.033 in the larger over 48 h.

While evasiveness by the target lost 10 patrol hours in the Napoli-sized area and 23 in the SWAP-sized area, the expected loss in detection probability was not large: 2% and 1%, respectively, of the original probability.

Barrier search was exemplified for a number of choke points, including one the length of a Gibraltar Strait barrier and one the length of a Sicily Strait barrier. Barrier performance was much poorer than area performance. MAD detection probabilities, pessimistic and optimistic, were 0.026 and 0.125 across the shorter barrier and 0.003

and 0.014 across the longer. Sonar detection probabilities were 0.022 across the shorter barrier and 0.002 across the longer.

The relative effectiveness of MAD to sonar was 13% to 15% better for pessimistic MAD and 300% to more than 500% better for optimistic MAD, growing even better for MAD considering target evasiveness due to warning by the active sonar.

Conclusions We can conclude that the AUSV is a very much better tool for area search than for barrier search.

For small areas, an AUSV appears to be a useful tool alone, i.e. not augmented by other assets. If the optimistic MAD range should be attainable, although it does not seem too likely, it would be remarkably good in small area patrols. While less effective in larger areas, it appears to be at least competitive with a number of currently used assets. If used in conjunction with such assets, it could increase detection probabilities significantly. In particular, it could be an alternative to marine mines in certain areas, since it poses no threat to own units and does not have to be swept later.

For barriers, it is inadequate alone. If several lines of AUSVs could compose the barrier, it could be a useful adjunct to other assets. (For example, three MAD AUSVs across Gibraltar would have a 1-in-3 chance of detecting a transitting target.)

The very encouraging probabilities obtained from the detection models would in reality be reduced by reliability and maintainability (support) problems, which have not entered the models. There is also the problem of transport for the AUSV and the ever present degradation of trying to operate systems at sea. Furthermore, if a MOE of P_{kill} were used instead of P_{det}, false targets and time late in prosecution would make further reductions. The amounts of such reductions have not been estimated, but they could be mitigated by various devices, e.g. the capability to clamp on the target hull. The conclusion remains: the AUSV offers enough promise to be worth further investigation.

Recommendations It is recommended that the AUSV be investigated in somewhat further detail, including engineering and costing considerations, for use as an antisubmarine warfare asset. Use in area search should receive primary concentration. It is further recommended that an up-to-date and careful comparison be made between detection ranges of MAD and high-frequency active sonar.

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Appendix A

Numerous small *vs* few large combatants

A.1. INTRODUCTION

In recent years throughout NATO and many national ministries (or departments) of defence a controversy has arisen periodically: What is the comparative efficacy of numerous small low-cost combatant units versus few greater units. Recently the question has been formalized by appearing in the MO 2005 study's conclusions as to the anticipated forms of future naval warfare (NATO, 1987). The question is far more than academic; it is critical to the future direction of defence programs. Also, the question is not simple; measures of effectiveness (MOEs) are complex and generally tied to specific scenarios.

When this question arises, frequently among rotating staffs containing members who have not previously encountered it, the arguments emerge as sometimes naive, sometimes irrelevant, and sometimes specious. There is a need for an exposition of fundamental ideas, sort of a simply stated primer, that can be used to clarify concepts and give a more realistic base to arguments. In the absence of such a primer, this paper is offered in a 'better-than-nothing' vein as a first collection of such basics to serve until a complete primer should appear.

A.2. TACTICAL LIMITATION ON MEGAUNITS

Consider a large, costly, manned, self-defended, unexpendable platform capable of many tactical functions and produced one-by-one. Let us denote such a platform as a megaunit. (Mega: very large, complex, multifaceted.) Examples of megaunits could be aircraft carriers, large nuclear submarines, and the B-1 bomber. Similarly, let us denote small, simple units at the other end of the cost spectrum as miniunits.

Suppose the relationship of number of units (N) and cost per unit (C) is constrained by a fixed total cost:

$$NC = \text{constant}. \tag{A.1}$$

As the cost per unit rises, the number of units that can be had diminishes. At some point, the number of units becomes so small as to be tactically ineffective, a state now being approached by aircraft carrier battle groups (CVBGs). This tactical limitation places an upper bound on the number of megaunits, resulting in a maximum cost and consequently an upper bound on the size/capability/sophistication of the megaunit. This relationship is shown in Fig. A1.

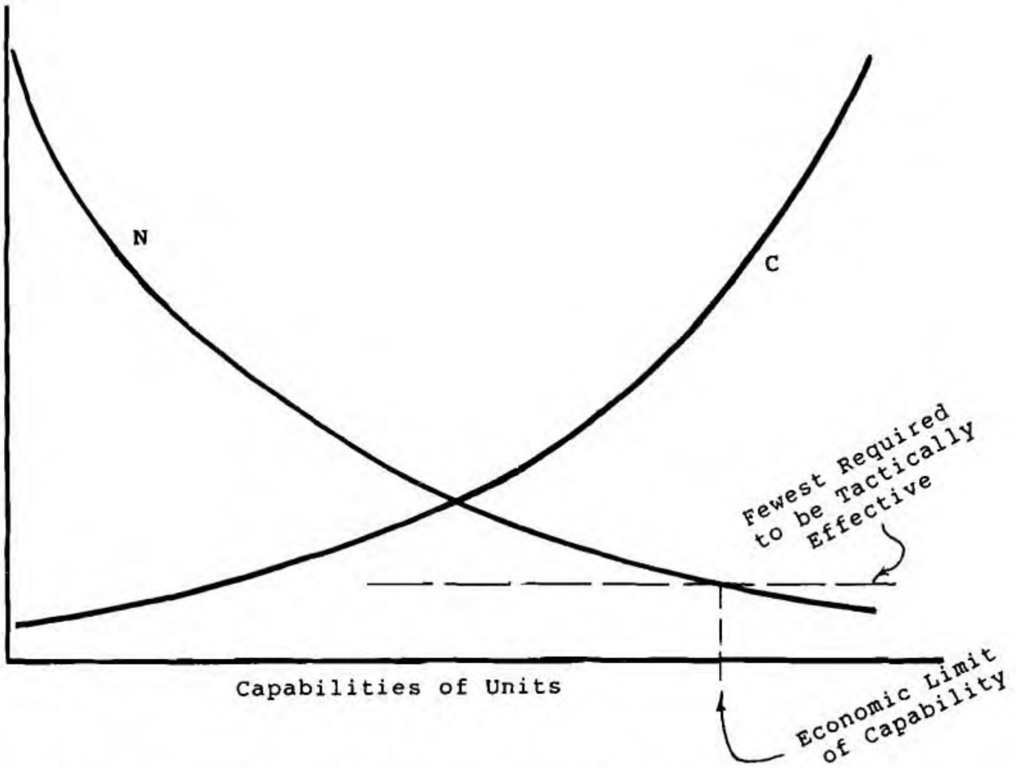

Fig. A1. Effectiveness of a force as related to number (N) of units in the force and cost (C) per unit, subject to the constraint that $NC =$ constant.

A.3. ASSET NUMBERS AND MISSION SUCCESS

It has been shown repeatedly that megaunits have worrisome probabilities, far from negligible, of being put out of action. In Olsen et al. (1985), for example, CVBG survival against air attack is shown to be improbable and in Riffenburgh (1986) it is shown that a single CVBG is many times as likely to receive missile hits as a pair of CVBGs and that the survival chance of both of two CVBGs under a month's intermittent submarine attack is very small.

From the reasoning of Fig. A1, cost limits the number and capabilities of units and prevents replacement of units. This limit coupled with a non-trivial susceptibility poses a risk of mission failure. If the megaunit is lost, the mission fails, so that, as example, an 80% survival probability implies a 20% chance of mission failure. In contrast, an 80% survival probability per miniunit implies 80% of the units will still be functioning to carry out the mission.

In summary, with megaunits, the chance of mission failure is tied to amount of expenditure at risk; this is not so with a multiplicity of miniunits.

A.4. THE SAVINGS SPIRAL

What are the technical and economic forces which would allow a megaunit expenditure to be converted into multiple miniunits capable of accomplishing a mission?

There exists a relevant joint technological and economic phenomenon which is widely recognized, particularly in relation to unmanned space probes, but which I have not found documented. Thus, I will try to document it here. For want of a better name, I call it the savings spiral.

Consider a sophisticated undersea vehicle such as a submarine. If it could accomplish its mission without people aboard, it could avoid the requirement for living space and all the many 'hotel' requirements, e.g. kitchens and the complication of toilet function under high pressure, as well as air replenishers, etc. Indeed, the pressure hull requirements could be reduced to negligible. The elimination of all this space and equipment would reduce the size and power requirement; the reduced size and power would reduce the fuel requirement; the reduced fuel would further reduce the size, further reducing the power, further reducing the fuel, and so on until an equilibrium is reached. This converging series is what I call the savings spiral. The result is thought provoking: if the cost of human support were, say for illustration, half of the total cost, then the elimination of humans might lead to, not double the number of units, but three or four times the number of units due to the spiral effect.

There is more. With many more units and no people aboard, we might be able to eliminate self-defense capabilities. This could lead to another savings spiral. With still more units, we might be able to allow unifunction units, having different miniunits for each desired capability, leading to still another spiral. And for weapon-type units, we might make them expendable. One-way units would allow elimination of weapon release, separate weapon guidance capability, recovery or landing capability, and half the fuel. Another savings spiral results. And, finally, the more units built, the greater the mass production savings.

In summary, going from a megaunit to an unmanned, undefended, unifunction, expendable, mass-produced miniunit initiates a series of savings spirals leading to a small, cheap, low-powered platform.

A.5. PROCUREMENT RESPONSE TIME ADVANTAGE

Megaunits built in a one-of-a-kind fashion take a disturbing length of time to procure and deploy. From legislative request for funds through building, testing, and training to actual deployment is a matter of several years. To fight any but the most protracted war with megaunits is to fight the war with the assets available at outbreak. In contrast, replacement of lost miniunits could well be weeks or months and the flexibility of redeploying existing stocks could reduce the time without assets to days.

A.6. ECONOMIC AND TACTICAL WIN-WIN

One strikingly appealing aspect of cheap enough miniunits is that we place the opponent in a position in which we gain whatever he does – a 'win-win' situation.

In a tactical sense, if he attacks our miniunits, we reduce his stock of sophisticated weapons to use against our more conventional platforms; if he does not attack, our units accomplish their mission.

In an economic sense, it may cost the enemy more to destroy our miniunit than it costs us to employ it. When this is true and he attacks our miniunits, we step ahead in the economic attrition front; if he does not attack, our units accomplish their mission.

An example is given in the main body of this report, in which an area-patrolling autonomous undersea surveillance vehicle (AUSV) is postulated and examined. In this situation, the win-win property was quite strong. Submarines carry a very limited stock of weapons. Not only are these weapons costly, but replenishment may cost the opponent weeks out of action and risks of loss to transit to base for rearming and return. A submarine ignoring an AUSV will be detected with a very high probability and the AUSV's mission is accomplished. To avoid the AUSV, a submarine must stay out of the area of its targets and the AUSV's mission is accomplished. If a submarine attacks the AUSV, the best of options for us, the AUSV acts as a flaming datum, signalling the detection, location, and classification of the submarine; even should the submarine survive, it costs the opponent an expensive weapon and reduces the stock of threatening weapons.

A.7. SOME USES OF NUMEROUS SMALL COMBATANTS

Numerous miniunits can be configured to accomplish many types of missions, among which are:

- Surveillance.
- Attack, local reactive.
- Attack, distant preplanned.
- EW/jamming.
- Acting as deceptive drone.
- Target acquisition and OTHT.
- Communications relay.
- Distantly launched mobile mining.
- Environmental data gathering.
- Convoy or surface group escort roles (maritime).
- Tattletailing/marking (maritime).
- MCM harbour survey & NOMBO updating (maritime).
- Distant towed array source or receiver (maritime).

A.8. A SUMMARY OF THE BASIC IDEAS

The basic ideas set forth here are summarized below:

(1) As we increase our self-defensive capability, the enemy increases his offensive capability, requiring an ever expanding size and cost of megaunits; eventually a limit is reached where too few megaunits can be afforded to accomplish the (strategic) mission. Miniunits are not so limited.

(2) With megaunits, the chance of mission failure due to loss of the unit is proportional to the amount of expenditure at risk. Miniunits are not subject to this proportionality.

(3) Every element eliminated from a megaunit saves that element plus (a spiraling sequence of) other supporting elements. Thus N miniunits of cost C each (total cost NC) will have much greater capability than a megaunit of cost NC.

(4) Megaunits may not be replaced during early phases (perhaps years) of a war. Miniunits may be replaced quickly and in the meantime may be redeployed more flexibly.

(5) At present, it costs the enemy more both in available weapon stocks during a mission and in money units to attack miniunits than it costs us. If he does not attack, the miniunits may operate with impunity. Thus, miniunits win either way.

Appendix B
Dynamic enhancement table

Table B1
Dynamic enhancement table – relative speed (kn) between two units U, V moving randomly

V	\U: 1	2	3	4	5	6	7	8	9	10
1	1.27	2.13	3.08	4.06	5.05	6.04	7.03	8.03	9.02	10.02
2	2.13	2.55	3.35	4.26	5.20	6.17	7.14	8.12	9.11	10.10
3	3.08	3.35	3.82	4.59	5.46	6.38	7.33	8.28	9.25	10.23
4	4.06	4.26	4.59	5.09	5.85	6.69	7.59	8.51	9.45	10.40
5	5.05	5.20	5.46	5.85	6.37	7.10	7.93	8.81	9.71	10.64
6	6.04	6.17	6.38	6.69	7.10	7.64	8.37	9.18	10.04	10.93
7	7.03	7.14	7.33	7.59	7.93	8.37	8.91	9.64	10.44	11.27
8	8.03	8.12	8.28	8.51	8.81	9.18	9.64	10.19	10.92	11.69
9	9.02	9.11	9.25	9.45	9.71	10.04	10.44	10.92	11.46	12.19
10	10.02	10.10	10.23	10.40	10.64	10.93	11.27	11.69	12.19	12.73
11	11.02	11.09	11.21	11.37	11.58	11.84	12.15	12.52	12.95	13.46
12	12.02	12.08	12.19	12.34	12.53	12.77	13.05	13.39	13.77	14.20
13	13.02	13.07	13.17	13.31	13.49	13.71	13.97	14.27	14.62	15.03
14	14.02	14.07	14.16	14.29	14.45	14.65	14.89	15.17	15.50	15.86
15	15.01	15.07	15.15	15.27	15.42	15.61	15.83	16.10	16.39	16.73
16	16.01	16.06	16.14	16.25	16.39	16.57	16.78	17.02	17.30	17.62
17	17.01	17.05	17.13	17.23	17.37	17.54	17.73	17.96	18.22	18.51
18	18.01	18.05	18.12	18.22	18.35	18.50	18.69	18.91	19.15	19.43
19	19.01	19.05	19.11	19.21	19.33	19.48	19.65	19.85	20.09	20.35
20	20.01	20.05	20.11	20.20	20.31	20.45	20.62	20.81	21.03	21.28
21	21.01	21.04	21.10	21.19	21.30	21.43	21.59	21.77	21.98	22.21
22	22.01	22.04	22.10	22.18	22.28	22.41	22.56	22.74	22.93	23.16
23	23.00	23.04	23.10	23.17	23.27	23.39	23.54	23.70	23.89	24.11
24	24.00	24.03	24.09	24.16	24.26	24.37	24.51	24.67	24.85	25.06
25	25.00	25.03	25.08	25.15	25.25	25.36	25.49	25.64	25.82	26.01
26	26.00	26.03	26.08	26.15	26.23	26.34	26.47	26.62	26.79	26.97
27	27.00	27.03	27.07	27.14	27.23	27.33	27.45	27.60	27.76	27.94
28	28.00	28.04	28.07	28.14	28.22	28.32	28.44	28.57	28.73	28.90
29	29.00	29.03	29.07	29.13	29.21	29.31	29.42	29.55	29.70	29.87
30	30.00	30.03	30.07	30.13	30.20	30.29	30.41	30.54	30.68	30.84

Table B1 (cont'd)

Dynamic enhancement table – relative speed (kn) between two units U, V moving randomly

V \U:	11	12	13	14	15	16	17	18	19	20
1	11.02	12.02	13.02	14.02	15.01	16.01	17.01	18.01	19.01	20.01
2	11.09	12.08	13.07	14.07	15.07	16.06	17.05	18.05	19.05	20.05
3	11.21	12.19	13.17	14.16	15.15	16.14	17.13	18.12	19.11	20.11
4	11.37	12.34	13.31	14.29	15.27	16.25	17.23	18.22	19.21	20.20
5	11.58	12.53	13.49	14.45	15.42	16.39	17.37	18.35	19.33	20.31
6	11.84	12.77	13.71	14.65	15.61	16.57	17.54	18.50	19.48	20.45
7	12.15	13.05	13.97	14.89	15.83	16.78	17.73	18.69	19.65	20.62
8	12.52	13.39	14.27	15.17	16.10	17.02	17.96	18.91	19.85	20.81
9	12.95	13.77	14.62	15.50	16.39	17.30	18.22	19.15	20.09	21.03
10	13.46	14.20	15.03	15.86	16.73	17.62	18.51	19.43	20.35	21.28
11	14.01	14.73	15.46	16.28	17.11	17.96	18.85	19.73	20.63	21.55
12	14.73	15.28	16.01	16.74	17.54	18.37	19.20	20.08	20.96	21.85
13	15.46	16.01	16.55	17.28	18.01	18.79	19.62	20.45	21.31	22.19
14	16.28	16.74	17.28	17.83	18.55	19.28	20.05	20.87	21.70	22.54
15	17.11	17.54	18.01	18.55	19.10	19.83	20.56	21.30	22.13	22.96
16	17.96	18.37	18.79	19.28	19.83	20.37	21.10	21.83	22.56	23.38
17	18.85	19.20	19.62	20.05	20.56	21.10	21.65	22.37	23.10	23.83
18	19.73	20.08	20.45	20.87	21.30	21.83	22.37	22.92	23.65	24.38
19	20.63	20.96	21.31	21.70	22.13	22.56	23.10	23.65	24.19	24.92
20	21.55	21.85	22.19	22.54	22.96	23.38	23.83	24.38	24.92	25.46
21	22.48	22.76	23.08	23.42	23.79	24.21	24.64	25.11	25.65	26.19
22	23.40	23.68	23.97	24.31	24.65	25.04	25.47	25.89	26.38	26.92
23	24.34	24.60	24.89	25.20	25.54	25.89	26.29	26.72	27.15	27.65
24	25.28	25.53	25.81	26.09	26.43	26.77	27.12	27.55	27.97	28.40
25	26.23	26.47	26.73	27.01	27.31	27.66	28.00	28.38	28.80	29.23
26	27.18	27.41	27.66	27.94	28.22	28.54	28.89	29.23	29.63	30.06
27	28.13	28.36	28.59	28.86	29.14	29.43	29.77	30.12	30.46	30.89
28	29.09	29.31	29.54	29.79	30.06	30.35	30.66	31.00	31.35	31.72
29	30.06	30.26	30.48	30.72	30.99	31.27	31.55	31.89	32.23	32.58
30	31.02	31.21	31.43	31.66	31.91	32.19	32.47	32.78	33.12	33.46

Table B1 (cont'd)

Dynamic enhancement table – relative speed (kn) between two units U, V moving randomly

V \U:	21	22	23	24	25	26	27	28	29	30
1	21.01	22.01	23.00	24.00	25.00	26.00	27.00	28.00	29.00	30.00
2	21.04	22.04	23.04	24.03	25.03	26.03	27.03	28.04	29.03	30.03
3	21.10	22.10	23.10	24.09	25.08	26.08	27.07	28.07	29.07	30.07
4	21.19	22.18	23.17	24.16	25.15	26.15	27.14	28.14	29.13	30.13
5	21.30	22.28	23.27	24.26	25.25	26.23	27.23	28.22	29.21	30.20
6	21.43	22.41	23.39	24.37	25.36	26.34	27.33	28.32	29.31	30.29
7	21.59	22.56	23.54	24.51	25.49	26.47	27.45	28.44	29.42	30.41
8	21.77	22.74	23.70	24.67	25.64	26.62	27.60	28.57	29.55	30.54
9	21.98	22.93	23.89	24.85	25.82	26.79	27.76	28.73	29.70	30.68
10	22.21	23.16	24.11	25.06	26.01	26.97	27.94	28.90	29.87	30.84
11	22.48	23.40	24.34	25.28	26.23	27.18	28.13	29.09	30.06	31.02
12	22.76	23.68	24.60	25.53	26.47	27.41	28.36	29.31	30.26	31.21
13	23.08	23.97	24.89	25.81	26.73	27.66	28.59	29.54	30.48	31.43
14	23.42	24.31	25.20	26.09	27.01	27.94	28.86	29.79	30.72	31.66
15	23.79	24.65	25.54	26.43	27.31	28.22	29.14	30.06	30.99	31.91
16	24.21	25.04	25.89	26.77	27.66	28.54	29.43	30.35	31.27	32.19
17	24.64	25.47	26.29	27.12	28.00	28.89	29.77	30.66	31.55	32.47
18	25.11	25.89	26.72	27.55	28.38	29.23	30.12	31.00	31.89	32.78
19	25.65	26.38	27.15	27.97	28.80	29.63	30.46	31.35	32.23	33.12
20	26.19	26.92	27.65	28.40	29.23	30.06	30.89	31.72	32.58	33.46
21	26.74	27.47	28.20	28.93	29.66	30.48	31.31	32.14	32.97	33.81
22	27.47	28.01	28.74	29.47	30.20	30.93	31.74	32.57	33.39	34.22
23	28.20	28.74	29.28	30.01	30.74	31.47	32.20	32.99	33.82	34.65
24	28.93	29.47	30.01	30.56	31.29	32.02	32.75	33.48	34.25	35.07
25	29.66	30.20	30.74	31.29	31.83	32.56	33.29	34.02	34.75	35.50
26	30.48	30.93	31.47	32.02	32.56	33.10	33.83	34.56	35.29	36.02
27	31.31	31.74	32.20	32.75	33.29	33.83	34.38	35.11	35.84	36.57
28	32.14	32.57	32.99	33.48	34.02	34.56	35.11	35.65	36.38	37.11
29	32.97	33.39	33.82	34.25	34.75	35.29	35.84	36.38	36.92	37.65
30	33.81	34.22	34.65	35.07	35.50	36.02	36.57	37.11	37.65	38.20

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