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SACLANT ASW RESEARCH CENTRE MEMORANDUM

THE INFLUENCE OF FLUCTUATIONS IN ACOUSTIC TRAVEL TIME ON LOCATING THE POSITION OF AN UNDERWATER SOUND SOURCE

by

Tor KNUDSEN

Angelo LOMBARDI

OCTOBER 1985

NORTH ATLANTIC TREATY ORGANIZATION

SACLANTCEN LA SPEZIA, ITALY

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This memorandum has been prepared within the SACLANTCEN Systems Research Division as part of Project 22.

T.G. GOLDSBERRY
Division Chief

TABLE OF CONTENTS

	Pages
ABSTRACT	1
INTRODUCTION	1
1 MATHEMATICAL MODEL	2
2 OPTIMUM RELATIVE RECEIVER POSITION	3
3 SIMULATION	7
4 RANDOM PERTURBATIONS OF THE TRAVEL TIMES	8
5 RANDOM PERTURBATION OF SOUND SPEED	8
CONCLUSION	9
REFERENCES	9

List of Figures

1. Definition of coordinates.	2
2. Optimum receiver positions.	6
3. Receivers placed on a line.	11
4. Receivers placed at an angle.	13
5. Receivers placed in the apices of a triangle.	15

THE INFLUENCE OF FLUCTUATIONS IN ACOUSTIC TRAVEL TIME ON LOCALIZING THE POSITION OF AN UNDERWATER SOURCE

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ABSTRACT

This paper investigates the accuracy of an underwater acoustic positioning system in locating a sound source that transmits at unknown times in conditions in which the acoustic travel times between source and receivers fluctuate widely. The system uses three omnidirectional receivers. The accuracy of the system depends very much on the relative positions of these receivers. With well-chosen relative receiver dispositions, the position of a sound source at 20 km range can be located within ± 150 m, even with fluctuations in travel time of as high as ± 100 ms.

INTRODUCTION

The need to locate the position of a sound source in shallow water by means of three omnidirectional receivers led to an experimental investigation of the fluctuations in travel time between transmission and reception of the sound. This showed that, in severe multipath conditions, the travel time could vary by about ± 65 ms between transmissions. The fluctuations were so rapid that two transmissions 150 s apart could show a difference in travel time of about 130 ms [1]. If one knows when the transmission took place, this gives an error of only about 200 m in locating the position of the sound source. However, if it is not known when the transmission took place, the computation of the source's position must be based on the differences in the arrival times of the same signal at three points spaced some distance apart. The existence of fairly large fluctuations in the travel times themselves means that such fluctuations will have a large influence on the accuracy of locating the position of the source.

This paper investigates the effect of different magnitudes of travel-time fluctuation on the accuracy of the computed position of the sound source. It also shows how the effect of the fluctuations can be reduced by placing the receivers in optimum relative positions.

1 MATHEMATICAL MODEL

In the models the receivers are positioned as shown on Fig. 1 at (x_1,y_1), (x_2,y_2), (x_3,y_3). The source transmits a signal at time t, which is received at the three receivers at times t_1, t_2 and t_3. We therefore find:

$$(1/c)\sqrt{(x-x_1)^2 + (y-y_1)^2} = t_1-t$$
$$(1/c)\sqrt{(x-x_2)^2 + (y-y_2)^2} = t_2-t$$
$$(1/c)\sqrt{(x-x_3)^2 + (y-y_3)^2} = t_3-t \quad , \qquad \text{(Eq. 1)}$$

where c, the sound speed, is assumed to be constant and independent of depth, and the sound is assumed to travel in straight paths. In fact the sound rays are bent in response to the sound-speed profile; this can be compensated for by modifying the sound speed used in these calculations. From Eq. 1 we eliminate the unknown t and get

$$t_2-t_1 = (1/c)\left[\sqrt{(x-x_2)^2 + (y-y_2)^2} - \sqrt{(x-x_1)^2 + (y-y_1)^2}\right]$$
$$t_3-t_1 = (1/c)\left[\sqrt{(x-x_3)^2 + (y-y_3)^2} - \sqrt{(x-x_1)^2 + (y-y_1)^2}\right] \qquad \text{(Eq. 2)}$$

or, as matrix equation

$$Y = m(x,y) \quad , \qquad \text{(Eq. 3)}$$

where

$$Y = \begin{bmatrix} t_2-t_1 \\ t_3-t_1 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix}$$

and $m(x,y)$ is the right-hand side of Eq. 2.

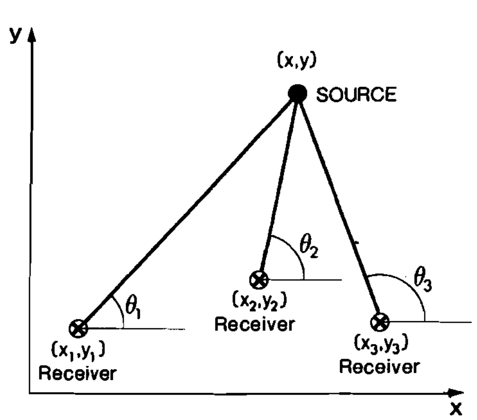

FIG. 1 DEFINITION OF COORDINATES

2 OPTIMUM RELATIVE RECEIVER POSITION [2]

We want to estimate the position of the sound source

$$X_k^T = [x,y]_{t=t_k}$$

from a series of observations of the differences in time between the acoustic transmission and the reception at the three receivers. The mathematical model for this is given in App. 7A of Jazwinsky [3]:

$$Y_i = M(i)\,\Phi(i,k)X_k + v_i \quad , \qquad i=1, \ldots, k \quad , \qquad \text{(Eq. 4)}$$

where $\{v_i\}$ is a white, but not necessarily gaussian, vector sequence with

$$E\{v_i\} = 0 \text{ and } E\{v_i\, v_j^T\} = R_i\, \delta_{ij} \quad , \qquad R_i > 0 \quad ,$$

and $\Phi(i,k)$ is the state transition matrix.

The parameter X_k may be viewed as the state at time t_k of the dynamical system

$$X_{i+1} = \Phi(i+1,\, i)\; X_i \qquad \text{(Eq. 5)}$$

$$Y_i = M(i)\; X_i + v_i \quad , \qquad \text{(Eq. 6)}$$

where Y_i is the measurement at time t_i.

We want to find the estimate $\hat{X}_k$ that minimizes the error variance

$$E\{(X_k-\hat{X}_k)^T\,(X_k-\hat{X}_k)\} = \text{tr}\; E\{(X_k-\hat{X}_k)(X_k-\hat{X}_k)^T\} \quad . \qquad \text{(Eq. 7)}$$

It can be shown [2] that the linear, unbiased minimum variance estimate $\hat{X}_k$ of X_k has a covariance matrix given by

$$E\{(X_k-\hat{X}_k)(X_k-\hat{X}_k)^T\} = \mathcal{J}^{-1}_{k,1} \quad , \qquad \text{(Eq. 8)}$$

where

$$\mathcal{J}_{k,1} = \sum_{i=1}^{k} \Phi^T(i,k)\; M^T(i)\; R_i^{-1}\; M(i)\; \Phi(i,k) \qquad \text{(Eq. 9)}$$

is the information matrix.

So minimizing the error variance is equivalent to minimizing $tr\,\mathcal{J}^{-1}_{k,1}$ or maximizing the determinant $|\mathcal{J}_{k,1}|$.

In our case the sound source does not move, so that

$$\Phi(i+1,i) = \Phi = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

and $\mathcal{J}_{k,1}$ reduces to

$$\mathcal{J}_{k,1} = \sum_{i=1}^{k} M^T(i)\, R_i^{-1}\, M(i) = \sum_{i=1}^{k} W_i \quad , \qquad \text{(Eq. 10)}$$

so that we shall maximize

$$|W_i| = |R_i^{-1}|\,|M(i)|^2 \quad . \qquad \text{(Eq. 11)}$$

Comparing Eqs. 3 and 6 we see that the actual observation describes a nonlinear relation between the source's position and the measured time difference. Equation 6 can be found from Eq. 3 by linearization.

$$M = \begin{bmatrix} \dfrac{\partial m_{11}(x,y)}{\partial x} & \dfrac{\partial m_{12}(x,y)}{\partial y} \\ \\ \dfrac{\partial m_{21}(x,y)}{\partial x} & \dfrac{\partial m_{22}(x,y)}{\partial y} \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \quad , \qquad \text{(Eq. 12)}$$

where

$$M_{11} = \frac{1}{c}\left\{ \frac{x-x_2}{\sqrt{(x-x_2)^2+(y-y_2)^2}} - \frac{x-x_1}{\sqrt{(x-x_1)^2+(y-y_1)^2}} \right\}$$

$$M_{12} = \frac{1}{c}\left\{ \frac{y-y_2}{\sqrt{(x-x_2)^2+(y-y_2)^2}} - \frac{y-y_1}{\sqrt{(x-x_1)^2+(y-y_1)^2}} \right\}$$

$$M_{21} = \frac{1}{c}\left\{ \frac{x-x_3}{\sqrt{(x-x_3)^2+(y-y_3)^2}} - \frac{x-x_1}{\sqrt{(x-x_1)^2+(y-y_1)^2}} \right\}$$

$$M_{22} = \frac{1}{c}\left\{ \frac{y-y_3}{\sqrt{(x-x_3)^2+(y-y_3)^2}} - \frac{y-y_1}{\sqrt{(x-x_1)^2+(y-y_1)^2}} \right\}$$

But referring to Fig, 1 we now find that M is also given by

$$M = \frac{1}{c}\begin{bmatrix} \cos\theta_2 - \cos\theta_1 & \sin\theta_2 - \sin\theta_1 \\ \cos\theta_3 - \cos\theta_1 & \sin\theta_3 - \sin\theta_1 \end{bmatrix}, \qquad \text{(Eq. 13)}$$

where θ_1 , θ_2 and θ_3 are the angles defined in Fig. 1.

If we measure the arrival times with a variance σ_t^2 we find that

$$R = \begin{bmatrix} 2\sigma_t^2 & \sigma_t^2 \\ \sigma_t^2 & 2\sigma_t^2 \end{bmatrix}$$

and thus

$$R^{-1} = \frac{1}{3\sigma_t^2}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} ,$$

and Eq. 11 then becomes

$$|W_i| = \frac{1}{3\sigma_t^4 c^4}\left\{\sin(\theta_1-\theta_3) - \sin(\theta_2-\theta_3) + \sin(\theta_2-\theta_1)\right\}^2 . \qquad \text{(Eq. 14)}$$

To maximize Eq. 14 we apply the Lagrangian multiplier method with the conditional function

$$\phi(\Delta) = \Delta_1 - \Delta_2 + \Delta_3 = 0 \quad , \qquad \text{(Eq. 15)}$$

where

$$\Delta_1 = (\theta_1 - \theta_3)$$

$$\Delta_2 = (\theta_2 - \theta_3)$$

$$\Delta_3 = (\theta_2 - \theta_1)$$

and the additional function

$$Z(\Delta) = \left| W_i \right| + \lambda\phi(\Delta) \quad . \qquad \text{(Eq. 16)}$$

Setting its partial derivatives to zero gives the following condition

$$\Delta_1 = \Delta_2$$

$$\Delta_2 = \Delta_3$$

$$\Delta_1 = \Delta_3 \quad .$$

Substituting these in Eq. 15 we find that

$\Delta_1 = \Delta_2$ gives $\Delta_3 = 0$

$\Delta_2 = \Delta_3$ gives $\Delta_1 = 0$

$\Delta_1 = \Delta_3$ gives $\Delta_2 = 2\Delta_1$,

where the last solution obviously is the maximum. Substituting $\Delta_1 = \Delta_3 = \Delta_2/2$ in Eq. 14 and maximizing with respect to Δ_2 gives

$$\Delta_2 = 240°$$

and

$$\Delta_1 = \Delta_3 = 120° \quad .$$

The optimum relative receiver positions are therefore in the shape of a triangle enclosing the source and such that the angle subtended by each leg of the triangle is 120°, as shown in Fig. 2.

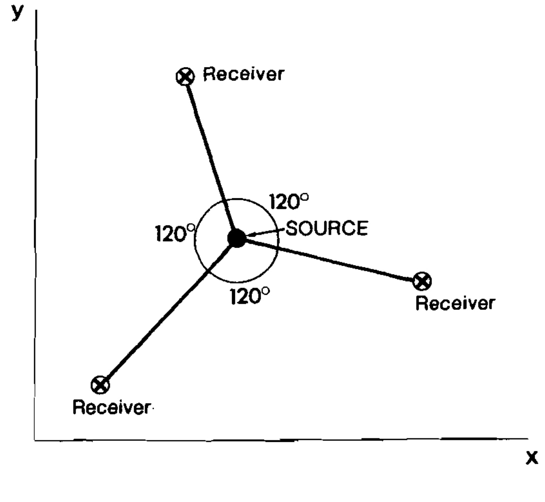

FIG. 2 OPTIMUM RECEIVER POSITIONS

3 SIMULATION

A simulation program was developed to determine how different receiver geometries would influence the accuracy in locating the position of a sound source. The program computes the theoretical travel time between the source and the receivers and perturbates these delays with random values drawn from uniform distributions. The position resulting from these perturbated travel times is then computed, and the process repeated with another set of random numbers in order to plot the error distribution in two dimensions.

Assuming that the real position of the source is (x_0,y_0), Eq. 1 is used to compute the theoretical travel times. The arrival times t_1, t_2 and t_3 are then perturbated according to

$$d_1 = t_1 + s\,U[-1,1]$$

$$d_2 = t_2 + s\,U[-1,1]$$

$$d_3 = t_3 + s\,U[-1,1] \quad , \qquad \text{(Eq. 17)}$$

where s is a constant used for scaling and $U[-1,1]$ means a number drawn from a uniform distribution of random numbers between -1 and 1.

Equation 2 describes the relationship between the position of the source and the measured travel-time differences. Since it is nonlinear, we start by linearizing around the actual position X_0, which gives travel-time differences of

$$Y_0 = \begin{bmatrix} t_2 - t_1 \\ t_3 - t_1 \end{bmatrix}$$

$$Y-Y_0 = M(X-X_0) \quad ,$$

so that

$$X-X_0 = M^{-1}(Y-Y_0) = M^{-1}\begin{bmatrix} d_2-d_1-(t_2-t_1) \\ d_3-d_1-(t_3-t_1) \end{bmatrix} \quad ,$$

where M is given by Eq. 12.

The estimated position due to the perturbated travel time is

$$X = M^{-1}(Y-Y_0)+X_0 \quad . \qquad \text{(Eq. 18)}$$

We then compute the new travel times using X in Eq. 1 and relinearize M around the new position estimate X . Using Eq. 18 we then compute a better estimate.

The iteration is ended when the new and the previously iterated positions are within a chosen limit, usually set to 5 m. The simulation is then repeated for another set of random perturbations by returning to Eq. 17.

4 RANDOM PERTURBATIONS OF THE TRAVEL TIMES

The computer program was run on several examples in order to investigate how the relative positions of the receivers influenced the error in estimating the sound source's position. The distance between the outermost receivers was 20 km and the sound source was moved to different positions within a square of 30 km × 30 km dimensions. Figure 3 shows the results for the three mean propagation delays of ± 25 ms, ± 50 ms and ± 100 ms when the receivers were placed along the y axis at 10 km intervals; because the errors are symmetric around a line that is parallel to the x axis and crosses the y axis at 10 km, only half the field is shown.

It is seen from the figure that the uncertainties in the position of each sound source are ellipses centred around its real position, which is marked with a cross. The error in the estimated position increases very much when the independent fluctuations in the time of arrival of the signal at the three receivers increase from ± 25 ms to ± 100 ms. At 20 km range (same distance as the receiver baseline) the maximum error has increased from about ± 1 km to about ± 3.5 km.

The situation is greatly improved when the receivers are positioned in a right angle, as shown in Fig. 4 for the same three mean propagation delays. The distance between the outermost receivers is again 20 km and it is seen that even with uniform random fluctuations of ± 100 ms in the time of arrival the maximum error is within ± 500 m for a sound source 20 km from the furthest receiver.

The optimum relative receiver disposition was found earlier to be as depicted in Fig. 2. The results of placing the receivers in this configuration are shown in Fig. 5 for the same three mean propagation delays. It is seen that the errors within the triangle formed by the receivers are very small (within ± 150 m), even with ± 100 ms time fluctuation. However, when the sound source is placed outside the triangle the errors start to increase considerably.

5 RANDOM PERTURBATION OF SOUND SPEED

Normally one may not know the sound speed accurately, or it may vary along the path from the sound source to the receiver and also between the paths to the different receivers. This effect can be interpreted as a range-dependent fluctuation in the acoustic travel times between source and receivers. To give a feeling for the problem we assume that there is an uncertainty in the actual sound speed, given by (1500 ± 5) m/s. This results in an uncertainty of ± 50 ms in travel time for distances between source and receivers that are greater than x , given by

$$x = \frac{0.05 \times 1500 \times 1505}{5} = 22 \text{ km} \quad .$$

Since the uncertainty in the sound speed can be interpreted as an uncertainty in acoustic travel times, the positioning error depends, as before, on the relative position of the receivers.

CONCLUSION

This paper shows that even with fluctuation in acoustic travel times of the order of ± 100 ms it is possible to locate the position of a sound source at 20 km range within ± 150 m by proper relative positioning of three omnidirectional receivers.

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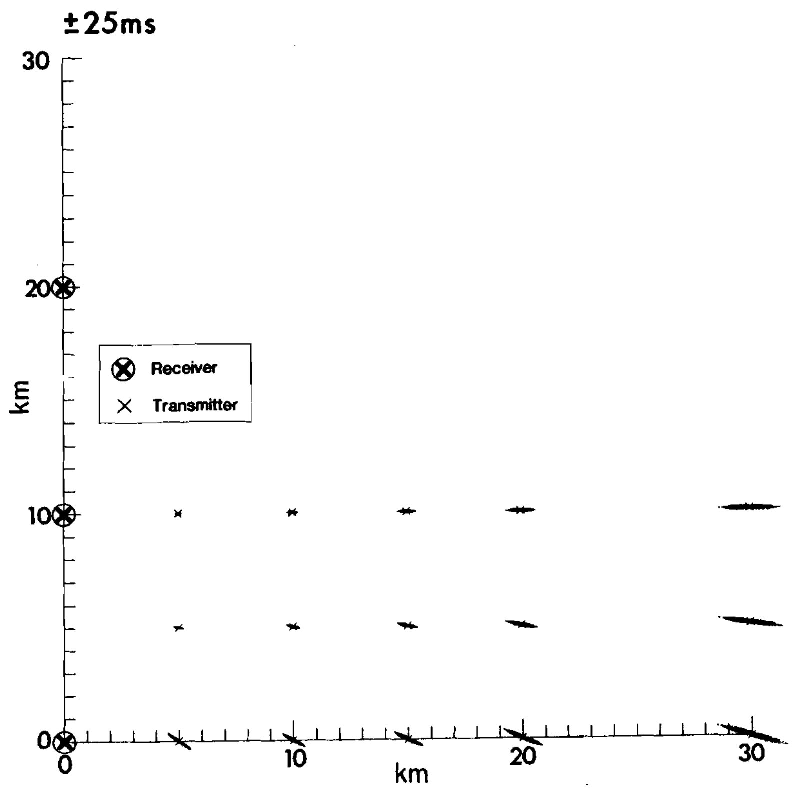
±25ms
30
20
10
0
km
Receiver
Transmitter
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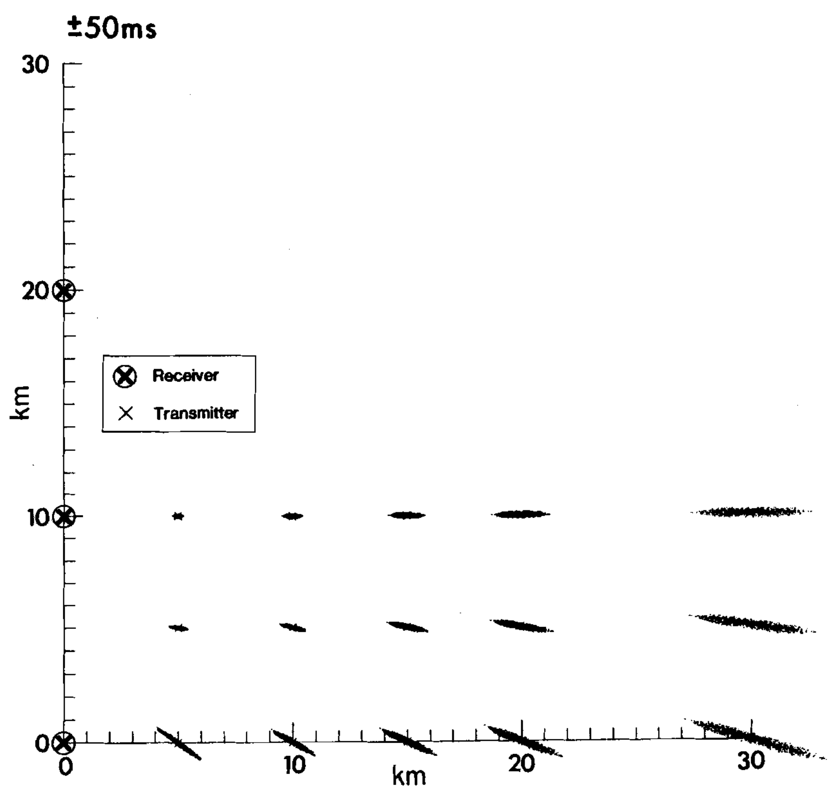
20
30
km

a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Receiver
Transmitter
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km

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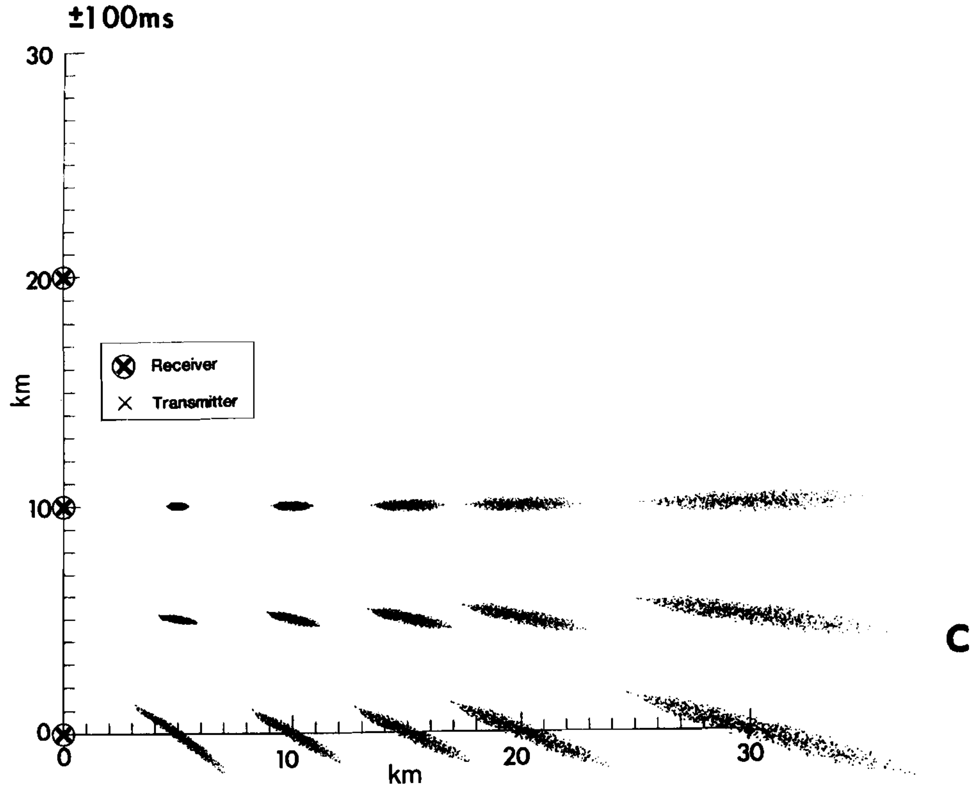
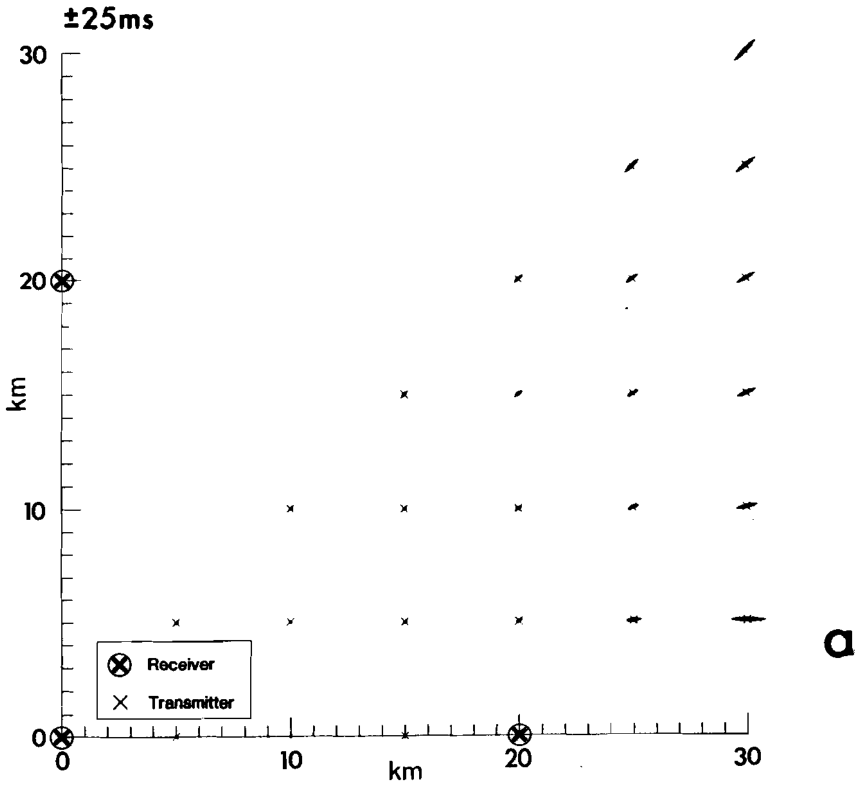


FIG. 3 RECEIVERS PLACED ON A LINE.
Uncertainty in position due to fluctuation in propagation delay of
a) ± 25 ms
b) ± 50 ms
c) ± 100 ms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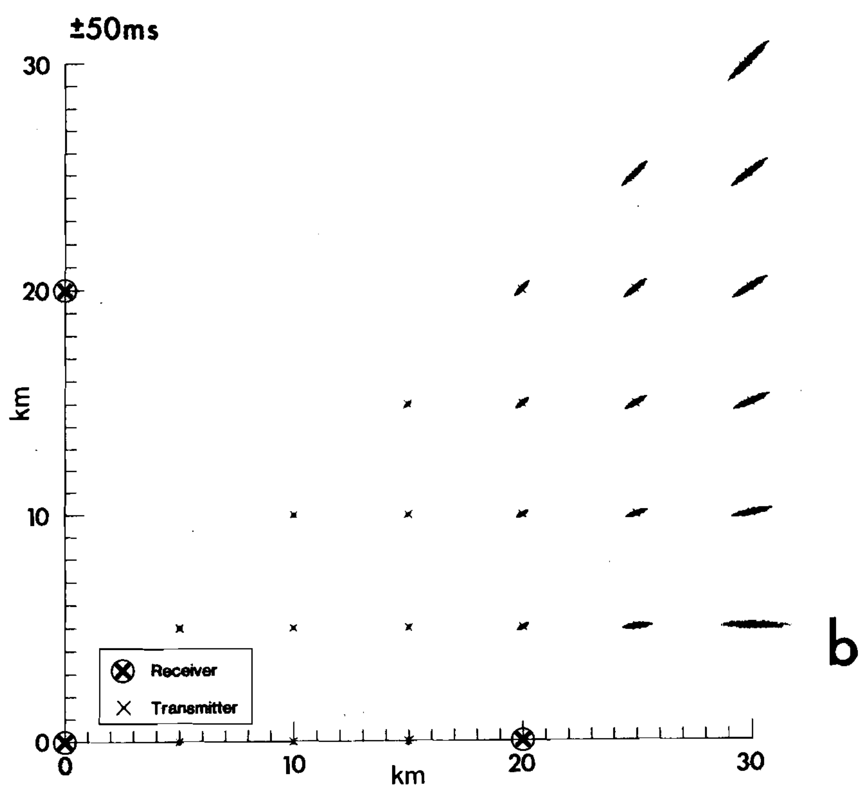
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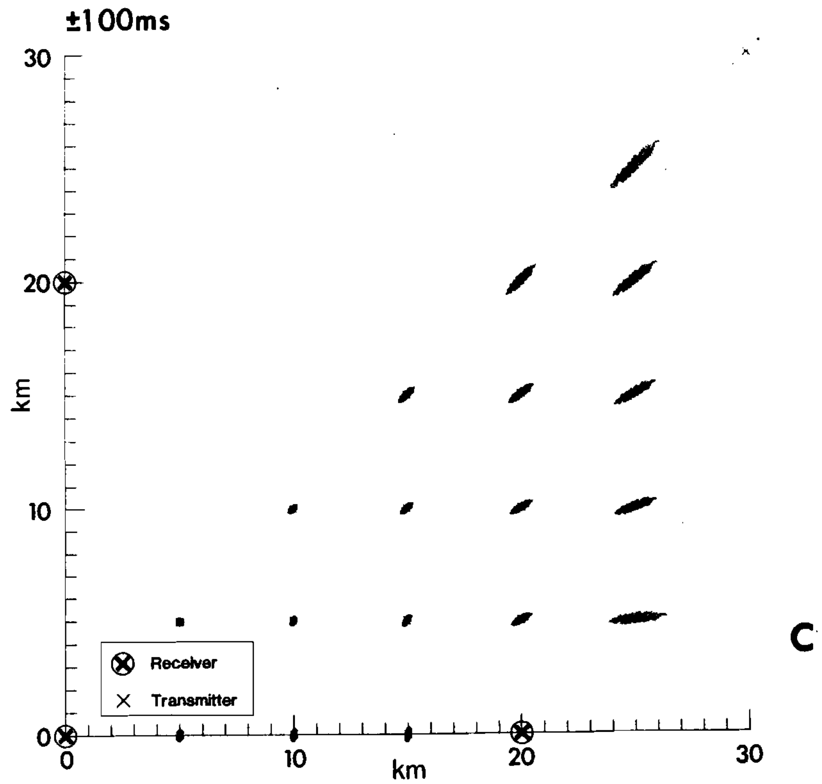
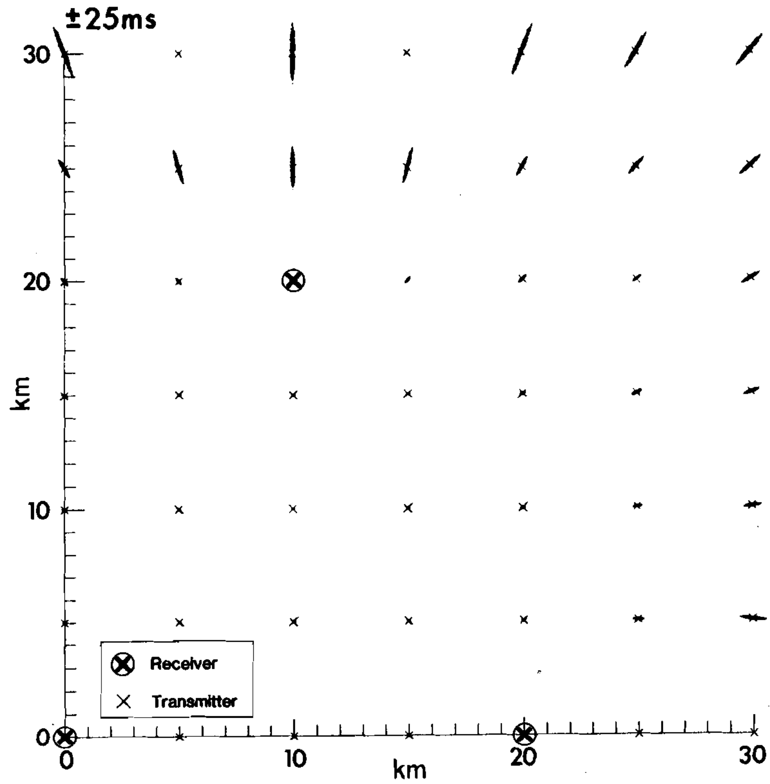


FIG. 4 *RECEIVERS PLACED AT AN ANGLE.*
Uncertainty in position due to fluctuation in propagation delay of
a) ± 25 ms
b) ± 50 ms
c) ± 100 ms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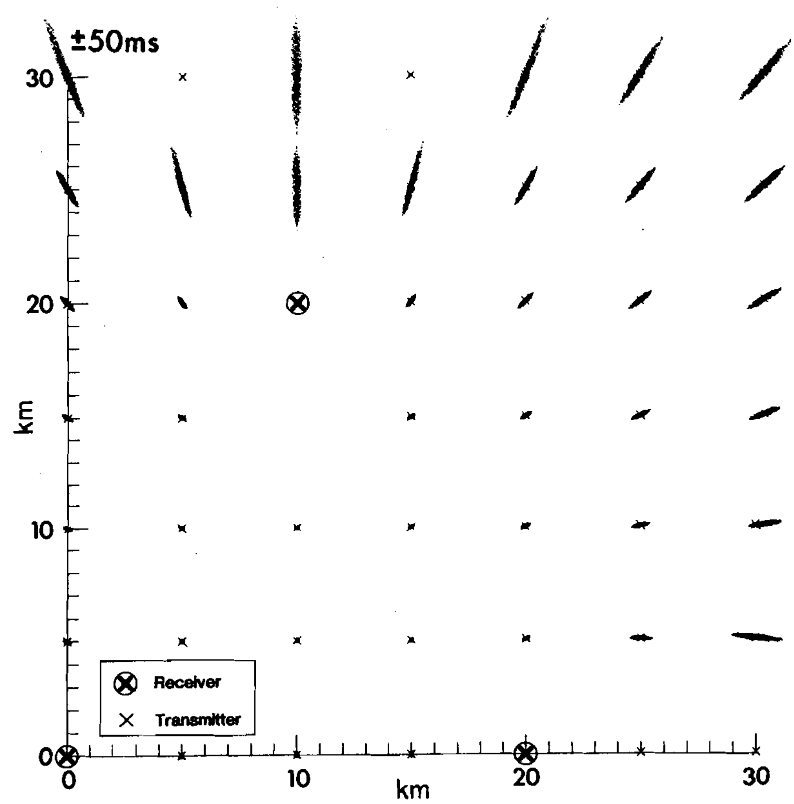
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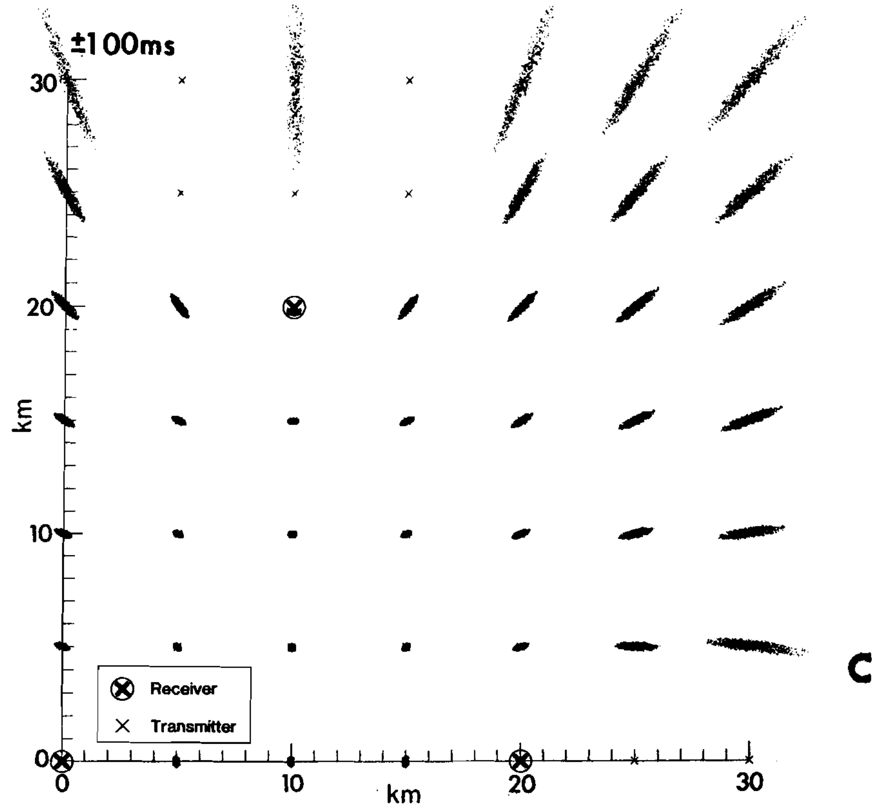

FIG. 5 RECEIVERS PLACED IN THE APICES OF A TRIANGLE.
Uncertainty in position due to fluctuation in propagation delay of
a) ± 25 ms
b) ± 50 ms
c) ± 100 ms

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KEYWORDS

ACOUSTIC POSITIONING SYSTEM
ACOUSTIC TRAVEL TIME
FLUCTUATION
MULTIPATH CONDITIONS
OMNIDIRECTIONAL RECEIVER
PERTURBATED TRAVEL TIME
PROPAGATION DELAY
SOUND SOURCE
SOUND-SPEED PROFILE

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or, as matrix equation

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where

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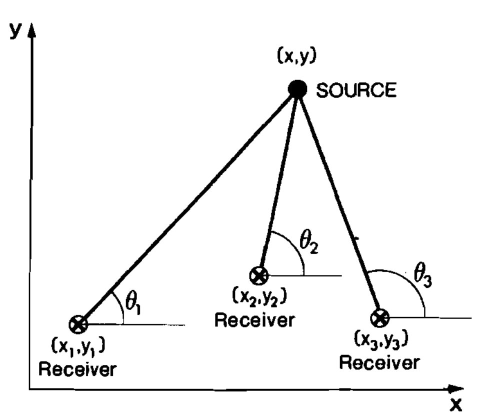

FIG. 1 DEFINITION OF COORDINATES

2 OPTIMUM RELATIVE RECEIVER POSITION [2]

We want to estimate the position of the sound source

$$X_k^T = [x,y]_{t=t_k}$$

from a series of observations of the differences in time between the acoustic transmission and the reception at the three receivers. The mathematical model for this is given in App. 7A of Jazwinsky [3]:

$$Y_i = M(i)\,\Phi(i,k)X_k + v_i \quad , \qquad i=1,\ \ldots,\ k \quad , \qquad \text{(Eq. 4)}$$

where $\{v_i\}$ is a white, but not necessarily gaussian, vector sequence with

$$E\{v_i\} = 0 \text{ and } E\{v_i\,v_j^T\} = R_i\,\delta_{ij} \quad , \qquad R_i > 0 \quad ,$$

and $\Phi(i,k)$ is the state transition matrix.

The parameter X_k may be viewed as the state at time t_k of the dynamical system

$$X_{i+1} = \Phi(i+1,\ i)\ X_i \qquad \text{(Eq. 5)}$$

$$Y_i = M(i)\ X_i + v_i \quad , \qquad \text{(Eq. 6)}$$

where Y_i is the measurement at time t_i .

We want to find the estimate $\hat{X}_k$ that minimizes the error variance

$$E\{(X_k-\hat{X}_k)^T\,(X_k-\hat{X}_k)\} = \mathrm{tr}\,E\{(X_k-\hat{X}_k)(X_k-\hat{X}_k)^T\} \quad . \qquad \text{(Eq. 7)}$$

It can be shown [2] that the linear, unbiased minimum variance estimate $\hat{X}_k$ of X_k has a covariance matrix given by

$$E\{(X_k-\hat{X}_k)(X_k-\hat{X}_k)^T\} = \mathcal{J}^{-1}_{k,1} \quad , \qquad \text{(Eq. 8)}$$

where

$$\mathcal{J}_{k,1} = \sum_{i=1}^{k} \Phi^T(i,k)\ M^T(i)\ R_i^{-1}\ M(i)\ \Phi(i,k) \qquad \text{(Eq. 9)}$$

is the information matrix.

So minimizing the error variance is equivalent to minimizing $\mathrm{tr}\,\mathcal{J}^{-1}_{k,1}$ or maximizing the determinant $|\mathcal{J}_{k,1}|$.

In our case the sound source does not move, so that

$$\Phi(i+1,i) = \Phi = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

and $\mathcal{J}_{k,1}$ reduces to

$$\mathcal{J}_{k,1} = \sum_{i=1}^{k} M^T(i)\, R_i^{-1}\, M(i) = \sum_{i=1}^{k} W_i \quad , \qquad \text{(Eq. 10)}$$

so that we shall maximize

$$|W_i| = |R_i^{-1}|\,|M(i)|^2 \quad . \qquad \text{(Eq. 11)}$$

Comparing Eqs. 3 and 6 we see that the actual observation describes a nonlinear relation between the source's position and the measured time difference. Equation 6 can be found from Eq. 3 by linearization.

$$M = \begin{bmatrix} \dfrac{\partial m_{11}(x,y)}{\partial x} & \dfrac{\partial m_{12}(x,y)}{\partial y} \\ \\ \dfrac{\partial m_{21}(x,y)}{\partial x} & \dfrac{\partial m_{22}(x,y)}{\partial y} \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} , \qquad \text{(Eq. 12)}$$

where

$$M_{11} = \frac{1}{c}\left\{\frac{x-x_2}{\sqrt{(x-x_2)^2+(y-y_2)^2}} - \frac{x-x_1}{\sqrt{(x-x_1)^2+(y-y_1)^2}}\right\}$$

$$M_{12} = \frac{1}{c}\left\{\frac{y-y_2}{\sqrt{(x-x_2)^2+(y-y_2)^2}} - \frac{y-y_1}{\sqrt{(x-x_1)^2+(y-y_1)^2}}\right\}$$

$$M_{21} = \frac{1}{c}\left\{ \frac{x-x_3}{\sqrt{(x-x_3)^2+(y-y_3)^2}} - \frac{x-x_1}{\sqrt{(x-x_1)^2+(y-y_1)^2}} \right\}$$

$$M_{22} = \frac{1}{c}\left\{ \frac{y-y_3}{\sqrt{(x-x_3)^2+(y-y_3)^2}} - \frac{y-y_1}{\sqrt{(x-x_1)^2+(y-y_1)^2}} \right\}$$

But referring to Fig, 1 we now find that M is also given by

$$M = \frac{1}{c}\begin{bmatrix} \cos\theta_2 - \cos\theta_1 & \sin\theta_2 - \sin\theta_1 \\ \cos\theta_3 - \cos\theta_1 & \sin\theta_3 - \sin\theta_1 \end{bmatrix}, \qquad \text{(Eq. 13)}$$

where θ_1 , θ_2 and θ_3 are the angles defined in Fig. 1.

If we measure the arrival times with a variance σ_t^2 we find that

$$R = \begin{bmatrix} 2\sigma_t^2 & \sigma_t^2 \\ \sigma_t^2 & 2\sigma_t^2 \end{bmatrix}$$

and thus

$$R^{-1} = \frac{1}{3\sigma_t^2}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} ,$$

and Eq. 11 then becomes

$$|W_i| = \frac{1}{3\sigma_t^4 c^4}\left\{\sin(\theta_1-\theta_3) - \sin(\theta_2-\theta_3) + \sin(\theta_2-\theta_1)\right\}^2 . \qquad \text{(Eq. 14)}$$

To maximize Eq. 14 we apply the Lagrangian multiplier method with the conditional function

$$\phi(\Delta) = \Delta_1 - \Delta_2 + \Delta_3 = 0 , \qquad \text{(Eq. 15)}$$

where

$$\Delta_1 = (\theta_1 - \theta_3)$$

$$\Delta_2 = (\theta_2 - \theta_3)$$

$$\Delta_3 = (\theta_2 - \theta_1)$$

and the additional function

$$Z(\Delta) = \left| W_i \right| + \lambda\phi(\Delta) \quad . \qquad \text{(Eq. 16)}$$

Setting its partial derivatives to zero gives the following condition

$$\Delta_1 = \Delta_2$$

$$\Delta_2 = \Delta_3$$

$$\Delta_1 = \Delta_3 \quad .$$

Substituting these in Eq. 15 we find that

$$\Delta_1 = \Delta_2 \quad \text{gives} \quad \Delta_3 = 0$$

$$\Delta_2 = \Delta_3 \quad \text{gives} \quad \Delta_1 = 0$$

$$\Delta_1 = \Delta_3 \quad \text{gives} \quad \Delta_2 = 2\Delta_1 \quad ,$$

where the last solution obviously is the maximum. Substituting $\Delta_1 = \Delta_3 = \Delta_2/2$ in Eq. 14 and maximizing with respect to Δ_2 gives

$$\Delta_2 = 240°$$

and

$$\Delta_1 = \Delta_3 = 120° \quad .$$

The optimum relative receiver positions are therefore in the shape of a triangle enclosing the source and such that the angle subtended by each leg of the triangle is 120°, as shown in Fig. 2.

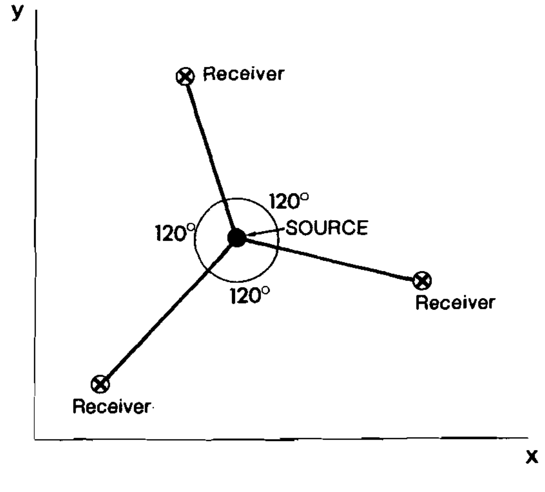

FIG. 2 OPTIMUM RECEIVER POSITIONS

3 SIMULATION

A simulation program was developed to determine how different receiver geometries would influence the accuracy in locating the position of a sound source. The program computes the theoretical travel time between the source and the receivers and perturbates these delays with random values drawn from uniform distributions. The position resulting from these perturbated travel times is then computed, and the process repeated with another set of random numbers in order to plot the error distribution in two dimensions.

Assuming that the real position of the source is (x_0,y_0) , Eq. 1 is used to compute the theoretical travel times. The arrival times t_1, t_2 and t_3 are then perturbated according to

$$d_1 = t_1 + s\ U[-1,1]$$

$$d_2 = t_2 + s\ U[-1,1]$$

$$d_3 = t_3 + s\ U[-1,1] \quad , \qquad \text{(Eq. 17)}$$

where s is a constant used for scaling and $U[-1,1]$ means a number drawn from a uniform distribution of random numbers between -1 and 1.

Equation 2 describes the relationship between the position of the source and the measured travel-time differences. Since it is nonlinear, we start by linearizing around the actual position X_0 , which gives travel-time differences of

$$Y_0 = \begin{bmatrix} t_2 - t_1 \\ t_3 - t_1 \end{bmatrix}$$

$$Y-Y_0 = M(X-X_0) \quad ,$$

so that

$$X-X_0 = M^{-1}(Y-Y_0) = M^{-1}\begin{bmatrix} d_2-d_1-(t_2-t_1) \\ d_3-d_1-(t_3-t_1) \end{bmatrix} \quad ,$$

where M is given by Eq. 12.

The estimated position due to the perturbated travel time is

$$X = M^{-1}(Y-Y_0)+X_0 \quad . \qquad \text{(Eq. 18)}$$

We then compute the new travel times using X in Eq. 1 and relinearize M around the new position estimate X . Using Eq. 18 we then compute a better estimate.

The iteration is ended when the new and the previously iterated positions are within a chosen limit, usually set to 5 m. The simulation is then repeated for another set of random perturbations by returning to Eq. 17.

4 RANDOM PERTURBATIONS OF THE TRAVEL TIMES

The computer program was run on several examples in order to investigate how the relative positions of the receivers influenced the error in estimating the sound source's position. The distance between the outermost receivers was 20 km and the sound source was moved to different positions within a square of 30 km × 30 km dimensions. Figure 3 shows the results for the three mean propagation delays of ± 25 ms, ± 50 ms and ± 100 ms when the receivers were placed along the y axis at 10 km intervals; because the errors are symmetric around a line that is parallel to the x axis and crosses the y axis at 10 km, only half the field is shown.

It is seen from the figure that the uncertainties in the position of each sound source are ellipses centred around its real position, which is marked with a cross. The error in the estimated position increases very much when the independent fluctuations in the time of arrival of the signal at the three receivers increase from ± 25 ms to ± 100 ms. At 20 km range (same distance as the receiver baseline) the maximum error has increased from about ± 1 km to about ± 3.5 km.

The situation is greatly improved when the receivers are positioned in a right angle, as shown in Fig. 4 for the same three mean propagation delays. The distance between the outermost receivers is again 20 km and it is seen that even with uniform random fluctuations of ± 100 ms in the time of arrival the maximum error is within ± 500 m for a sound source 20 km from the furthest receiver.

The optimum relative receiver disposition was found earlier to be as depicted in Fig. 2. The results of placing the receivers in this configuration are shown in Fig. 5 for the same three mean propagation delays. It is seen that the errors within the triangle formed by the receivers are very small (within ± 150 m), even with ± 100 ms time fluctuation. However, when the sound source is placed outside the triangle the errors start to increase considerably.

5 RANDOM PERTURBATION OF SOUND SPEED

Normally one may not know the sound speed accurately, or it may vary along the path from the sound source to the receiver and also between the paths to the different receivers. This effect can be interpreted as a range-dependent fluctuation in the acoustic travel times between source and receivers. To give a feeling for the problem we assume that there is an uncertainty in the actual sound speed, given by (1500 ± 5) m/s. This results in an uncertainty of ± 50 ms in travel time for distances between source and receivers that are greater than x , given by

$$x = \frac{0.05 \times 1500 \times 1505}{5} = 22\ \text{km} \quad .$$

Since the uncertainty in the sound speed can be interpreted as an uncertainty in acoustic travel times, the positioning error depends, as before, on the relative position of the receivers.

CONCLUSION

This paper shows that even with fluctuation in acoustic travel times of the order of ± 100 ms it is possible to locate the position of a sound source at 20 km range within ± 150 m by proper relative positioning of three omnidirectional receivers.

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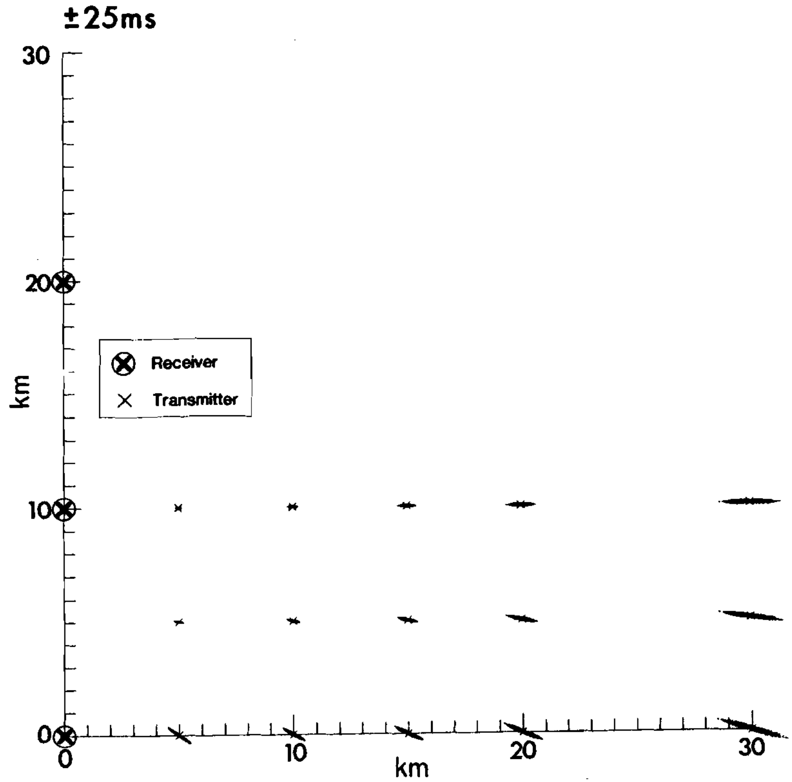
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km
Receiver
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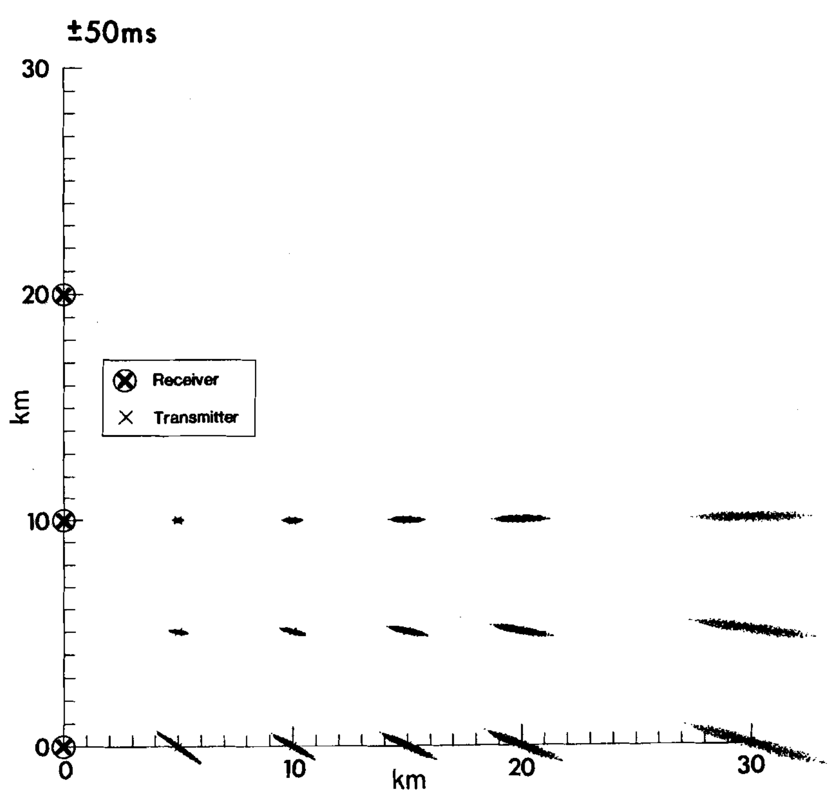
km

a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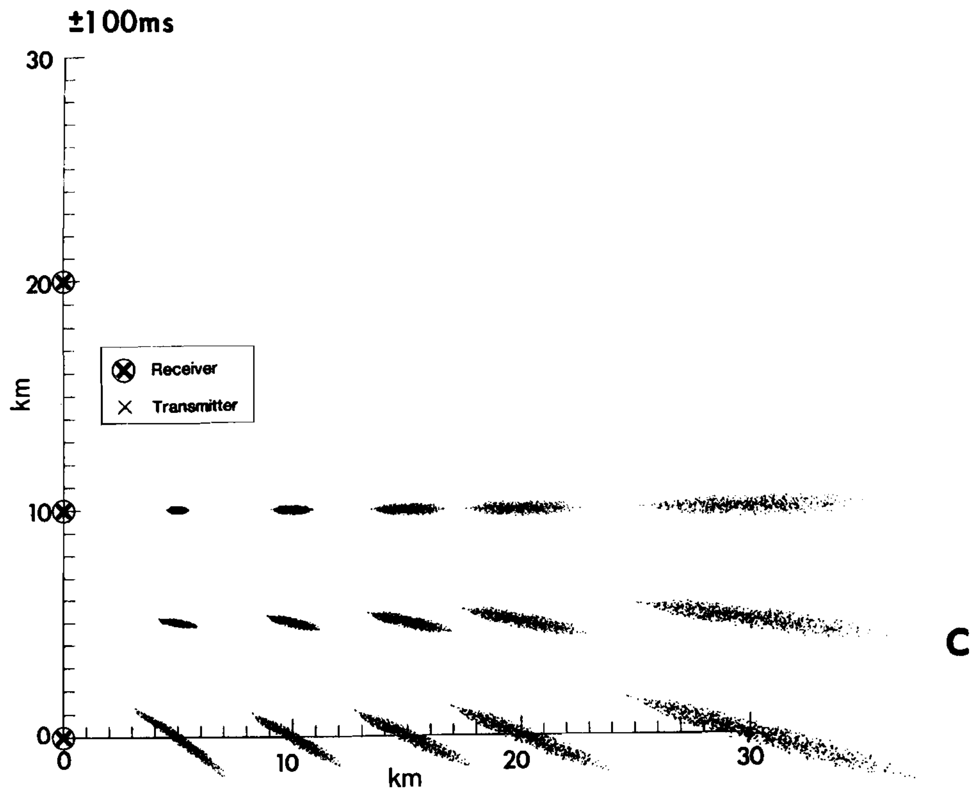
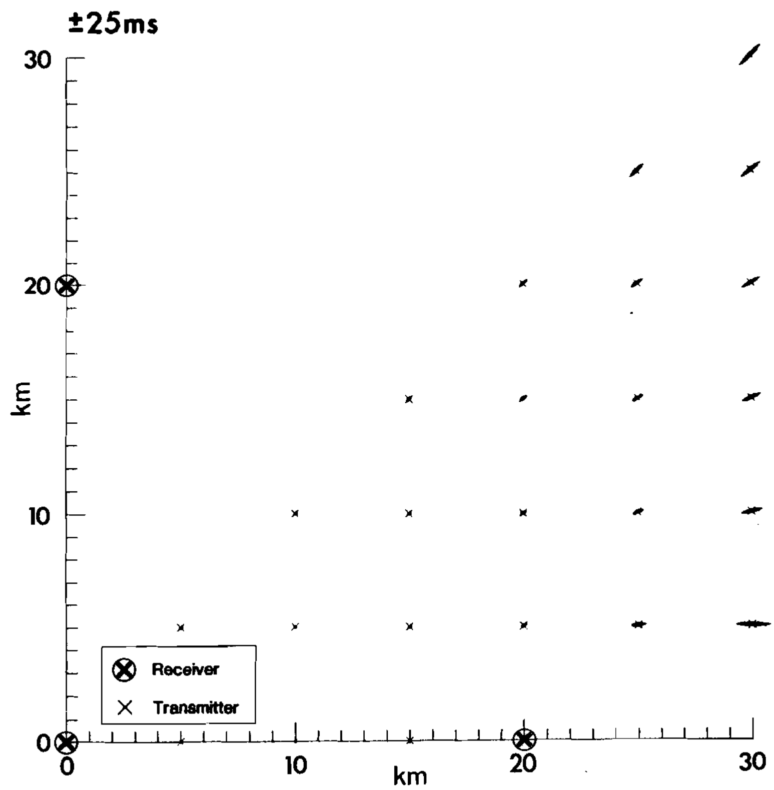


FIG. 3 *RECEIVERS PLACED ON A LINE.*
Uncertainty in position due to fluctuation in propagation delay of
a) ± 25 ms
b) ± 50 ms
c) ± 100 ms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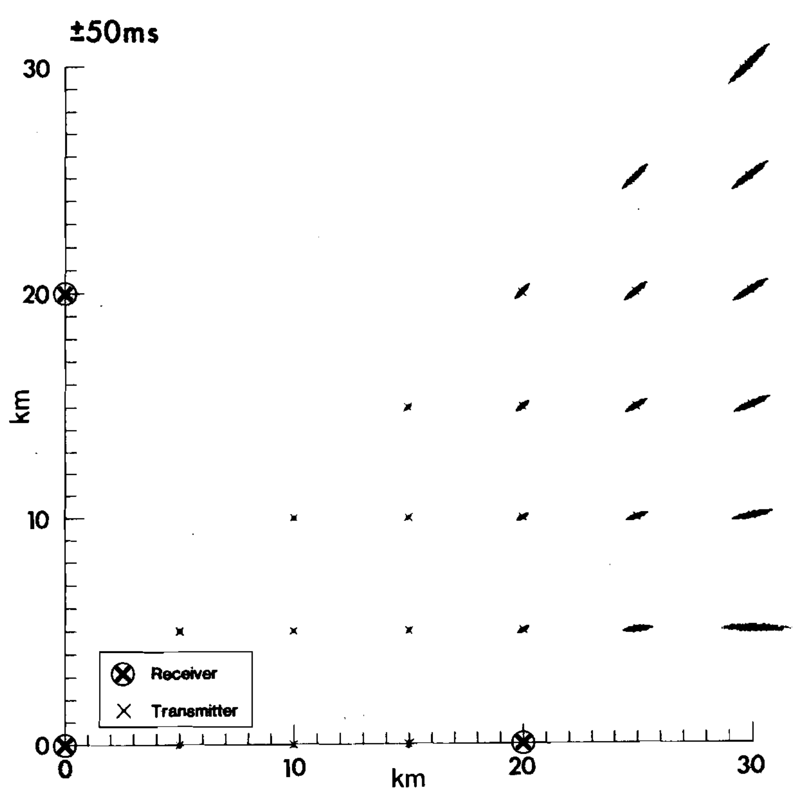
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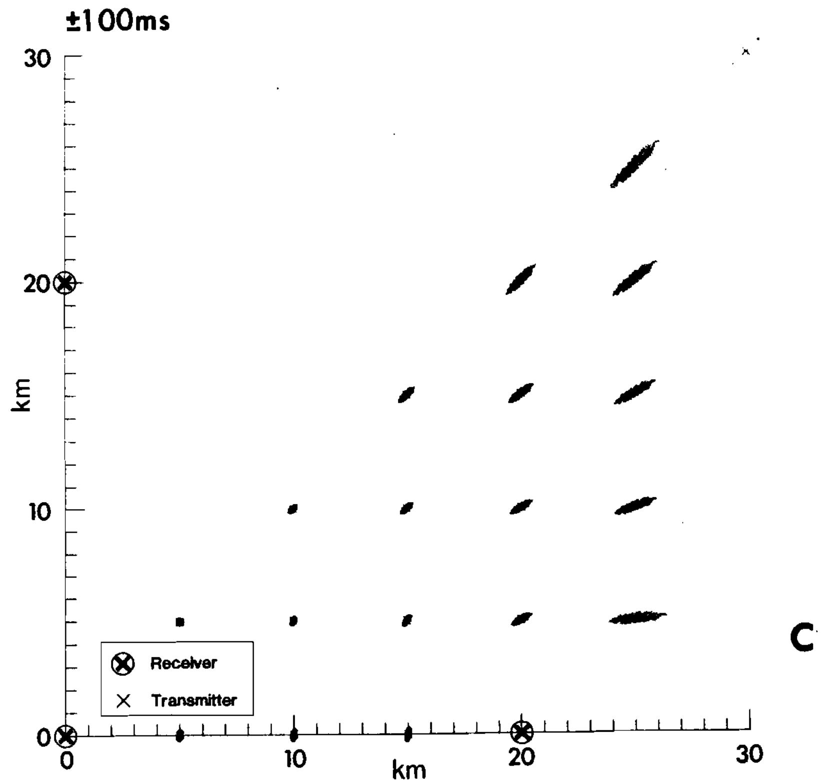
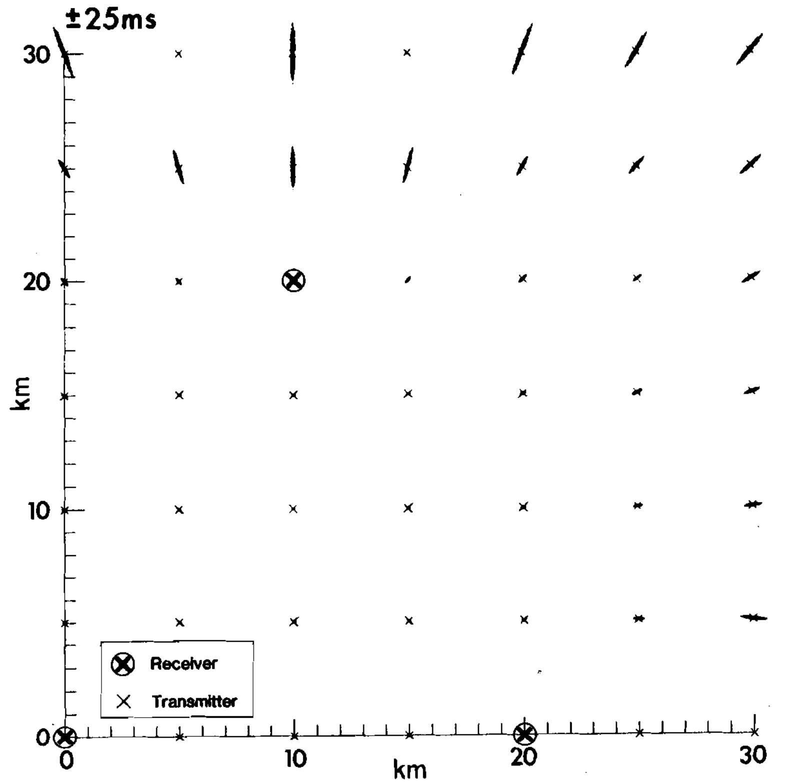


FIG. 4 RECEIVERS PLACED AT AN ANGLE.
Uncertainty in position due to fluctuation in propagation delay of
a) ± 25 ms
b) ± 50 ms
c) ± 100 ms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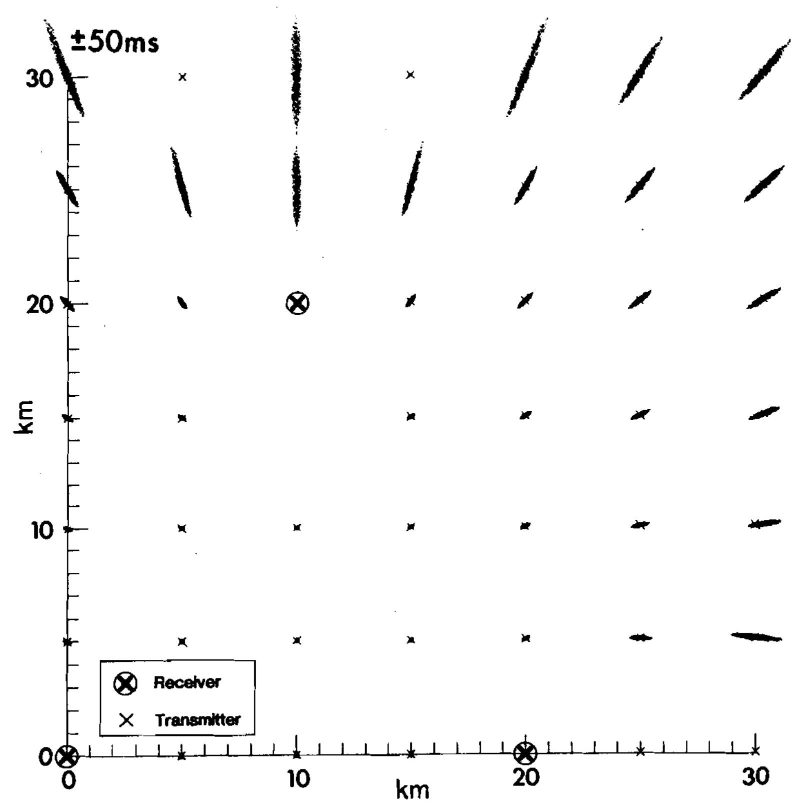
Transmitter

a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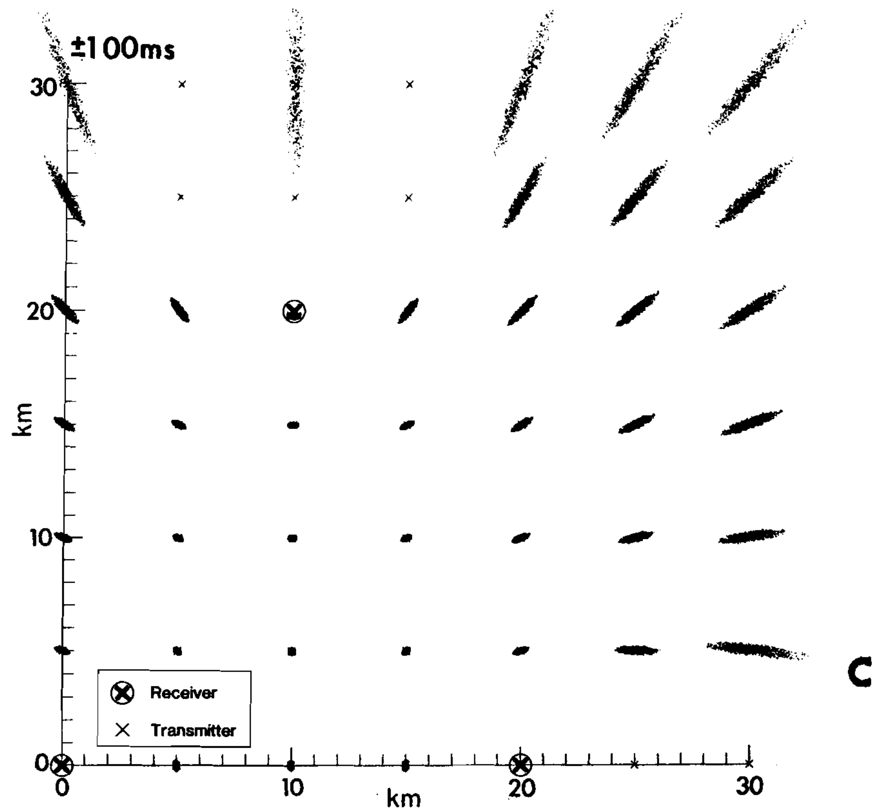

FIG. 5 RECEIVERS PLACED IN THE APICES OF A TRIANGLE.
Uncertainty in position due to fluctuation in propagation delay of
a) ± 25 ms
b) ± 50 ms
c) ± 100 ms

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KEYWORDS

ACOUSTIC POSITIONING SYSTEM
ACOUSTIC TRAVEL TIME
FLUCTUATION
MULTIPATH CONDITIONS
OMNIDIRECTIONAL RECEIVER
PERTURBATED TRAVEL TIME
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SOUND SOURCE
SOUND-SPEED PROFILE

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