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SACLANT ASW RESEARCH CENTRE MEMORANDUM

MULTIPLE BEAMFORMING

by

Walter M.X. ZIMMER

1 JULY 1985

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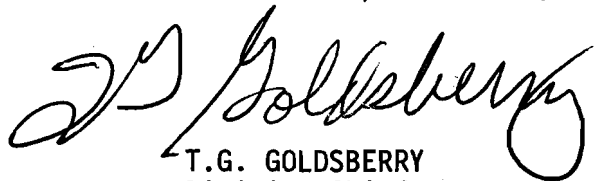
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T.G. GOLDSBERRY
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ABSTRACT

Beamforming techniques are generally based on the assumption that the received signal arrives from a single source. In a realistic situation, however, there is more than one source present and these sources can interact with each other in the beamformer. This paper describes a multiple beamformer in which the interaction of different sources is included in the algorithm. The maximum likelihood principle is used to find the different source amplitude and bearing estimates. The performance of the algorithm is compared for varying conditions with the Cramer-Rao lower bound. Two pre-processors necessary and suitable to ensure globally optimized results are derived from the ideas of Pisarenko and Prony. Finally, a post-processor is proposed to evaluate the success of the multiple beamformer.

INTRODUCTION

In general beamforming is understood as a procedure for combining spatially distributed measurements, either in a predefined or in an adaptive way, to increase the sensibility of the receiving system in the directions of interest.

The usual assumption made in the implementation of a beamformer algorithm is that the signal arrives from a single direction and the interaction of sources from other directions can be neglected. However, in realistic situations, we have to modify this assumption since we know either that there is more than one source or that we have multipath arrivals, therefore we should include the interaction between the sources or the multipath arrivals.

In this paper we describe a method that allows us to form multiple beams in order to steer the receiver toward two or more sound sources simultaneously. The main advantage of this procedure is that it includes the full interactions of the different sources in the beamformer.

We expect to gain the resolution of closely spaced sources without losing too much of the robust behaviour of the conventional beamformer and without biasing the estimate of the quantities we want to know.

Statistics suggest that the Cramer-Rao lower bound (CRLB) is a good measure of the robustness of a signal processor. If there exists a method to

achieve this bound then it will be the maximum likelihood parameter estimation technique. Therefore it is sensible to base the multiple beamformer on this technique [1 to 5].

Due to the explicit modelling of reality, parameter estimation techniques are easy to understand and to modify. This explains the widespread use of such techniques.

Why has this rather simple approach not received more attention in the literature? The answer can be found in the fact that the problems we are dealing with generally do not have unimodal solutions and require either an exhaustive search [5] or appropriate preprocessing. However, with the development of so-called high resolution methods suitable techniques are available to achieve optimized results.

The complete algorithm presented in this paper is composed of four parts. The essential components are the multiple beamformer that estimates the amplitudes of the signals in closed form and the fine bearing estimator necessary to calculate the correct phase relationships between the different sound sources. For the fine bearing estimator we use an iterative technique to find the solution. We need a preprocessor to begin the iteration. The fourth component estimates the background noise level.

This paper first presents the concept and algorithm of the multiple beamformer including the fine bearing estimator. Secondly it analyses the performance of this kernel, with respect to the detection and resolution capabilities. The necessary preprocessor will only be sketched and some possible implementations discussed. The paper concludes with a proposal for a postprocessor.

1   FORMULATION OF THE BEAMFORMING PROBLEM

The formulation will be restricted to passive sonar applications, where the receiver is assumed to be a line array with equidistantly spaced hydrophones. Further, we assume that our data are preprocessed with a narrowband filter bank.

Therefore we have measurements with complex values as a function of space and time:

$$y_{n,t} \, , \tag{1}$$

where

    n is the hydrophone index $n = 0, \ldots , M-1$ and
    t is the sample index in time $t = 0, \ldots , T-1$.

We wish to estimate the sound field producing these hydrophone measurements:

$$(a_{k,t}, \, \omega_{k,t}) \, , \tag{2}$$

where

$a_{k,t}$ is the sound amplitude,

$\omega_{k,t}$ is the source bearing angle and

$k = 1, \ldots , K$ is the source index.

From the principle of superposition we know that the sound fields from different sources add up linearly in space. Assuming also linear transfer characteristics for the hydrophones and for the narrowband filter bank we find a linear relation between the sound field and the measurements. This means that we can model our measurements as follows

$$y_{n,t} = \psi_{n,t}(\omega_t) \cdot a_t + w_{n,t} \tag{3}$$

where

$$\psi_{n,t}(\omega_t) = (\psi_{n,t}(\omega_{1t}), \ldots , \psi_{n,t}(\omega_{Kt}))$$
is the soundfield-measurement transfer vector

$$a_t = (a_{1t}, \ldots , a_{Kt})$$
is the soundfield vector and

$w_{n,t}$ is the measurement error

* denotes the complex conjugate transpose.

Clearly there is no unique solution to the beamforming problem as defined by Eq. 3. On the one hand we have uncertain measurements due to noise effects; on the other hand the number of unknown parameters we wish to estimate generally will not coincide with the number of measurements we have available.

Thus, the problem formulation must be restricted by introducing some sort of a priori information such as known features of the solution, optimality criteria, or constraints.

This information is usually introduced to reduce the beamforming problem so that a useful solution can be achieved.

2   DEFINITION OF THE SCENARIO

It is now necessary to define the scenario for which the solution should be valid. The scenario is based on the following assumptions:

- The received sound field consists of only a small number of point sources. This excludes the presence of coloured background noise. (This assumption is made for convenience but in principle the model can be extended with terms describing coloured background noise.)

- The point sources are in the extreme far field so that they can be treated as stationary planewave signals. (This assumption is normally fullfilled in the case of distant shipping. The

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case of moving targets can be included in the model but then the computational workload will increase. Multipath arrivals are treated as different but correlated sources.)

- The array speed is known. The array speed is included in the algorithm to allow coherent processing not only in space but also in time.

- The measurement errors and the signal amplitudes are random variables which can be approximated by gaussian distributions:

$$a_{k,t} = N(a_k, \sigma_k)^2 \tag{4}$$

$$w_{n,t} = N(0, \sigma_n^2)$$

where

$N(m, \sigma^2)$ represents a complex valued gaussian variable with mean m and variance σ^2.

For a line array with equidistantly spaced hydrophones we can now specify the k-the element of the soundfield-measurement transfer vector

$$\psi_{n,t}(\omega_{k,t}) = e^{-i\, u_{k,t}\cdot(n+\gamma t)} \tag{5}$$

with

$$u_{k,t} = \pi\, \frac{2d}{\lambda}\, \cos(\omega_{k,t})$$

$$\gamma = \frac{v\cdot T_s}{d}$$

where
    λ is the signal wavelength
    d is the hydrophone spacing
    v is the array speed
    T_s is the narrowband sampling interval

Because of these assumptions, one must be careful when applying the results to real-world situations.

3   THE OPTIMIZATION CRITERION

Due to measurement errors it is impossible to build up a set of consistent equations to solve the beamforming problem; but an optimal solution can be found.

Which optimization criterion should we use? The natural way to answer this question seems to be that the optimal set of parameters θ is found when the average cost function,

$$C(\hat{\theta}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} C(\theta - \hat{\theta}) \; p(y, \theta) \; dy \; d\theta \; , \qquad (6)$$

becomes minimal [6]. Here $C(\theta - \hat{\theta})$ describes the cost as a function of the estimation error $\theta - \hat{\theta}$ and $p(y, \theta)$ is the joint probability density of measurement y and parameter $\hat{\theta}$.

Having no additional information about the cost function $C(\theta - \hat{\theta})$ it seems proper to select constant cost for undesired deviation of the parameter θ from the optimal value $\hat{\theta}$ thus,

$$C(\theta - \hat{\theta}) = \rho \left(\; \left| \; \theta - \hat{\theta} \; \right| > \Delta \right) \qquad (7)$$

where

$$\rho \; (\text{expression}) = \begin{cases} 1 \; (\text{if expression is true}) \\ 0 \; (\text{if expression is false}) \end{cases}$$

is the generalized Kroneker symbol.

Minimizing the average cost yields the maximum <u>a posteriori</u> (MAP) estimate defined by the following equation [6]:

$$\frac{\partial}{\partial \theta} \log p(\theta \mid y) \Big|_{\theta \; = \; \hat{\theta}_{MAP}} = 0 \qquad (8)$$

Using the relation

$$p(\theta, y) = p(\theta \mid y) \; p(y) = p(y \mid \theta) \; p(\theta)$$

the MAP equation reads

$$\frac{\partial}{\partial \theta} \log p(y \mid \theta) \Big|_{\theta \; = \; \hat{\theta}_{MAP}} + \frac{\partial}{\partial \theta} \log p(\theta) \Big|_{\theta \; = \; \hat{\theta}_{MAP}} = 0 \qquad (9)$$

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This equation shows how the a priori density $p(\theta)$ has to be used to obtain an optimal estimate of the parameter θ. In the case of a constant a priori density $p(\theta)$ the second term disappears and the optimal estimate is then called the maximum likelihood estimate (MLE)

$$\frac{\partial}{\partial \theta} \log p(y \mid \theta) \bigg|_{\theta = \hat{\theta}_{MLE}} = 0 \qquad (10)$$

In this paper we assume that the a priori density $p(\theta)$ is constant so that it is sufficient to work only with Eq. 10, simplified to

$$\frac{\partial}{\partial \theta} L = 0 \, , \qquad (11)$$

where

$L = \log p(y \mid \theta)$ is the log likelihood function.

4   MULTIPLE BEAMFORMER

To define the multiple beamformer we must first recall the basic relation between the sound field and the measurements, as given in Eq. 3,

$$y_{n,t} = \psi^{*}_{n,t}(\omega) \cdot a + w_{n,t} \, , \qquad (12)$$

where the signal bearing, ω_t, and amplitude vector, a_t, are now assumed to be time independent (indicated by eliminating the subscript, t).

The measurement errors, $w_{n,t}$, are assumed to be independent gaussian variables in space and time; therefore we can write for the log likelihood function, t,:

$$L = \text{const} - 2 \log \sigma - \sum_{n,t} \left(\frac{y_{n,t} - \psi^{*}_{n,t} a}{\sigma} \right)^{2} \, . \qquad (13)$$

where

$$\sum_{n,t} = \frac{1}{MT} \sum_{n=0}^{M-1} \sum_{t=0}^{T-1}$$

This symbol, $\sum_{n,t}$ will be used throughout this paper.

With the definitions

$$b = \sum_{n,t} y_{n,t} \, \psi_{n,t}$$

$$\Phi = \sum_{n,t} \psi_{n,t} \, \psi^*_{n,t}$$

$$\text{Tr}(R) = M \cdot \sum_{n,t} \left| y_{n,t} \right|^2$$

and after some algebraic manipulations the log likelihood function, L, can be rewritten as

$$L = \text{const} - 2 \log \sigma - \sigma^{-2} \{ (\Phi a - b)^* \, \Phi^{-1} (\Phi a - b) + \text{Tr}(R) - b^* \, \Phi^{-1} b \} \quad . \tag{14}$$

It is obvious that the Eq. 14 is at maximum when

$$\Phi a - b = 0 \quad . \tag{15}$$

This matrix equation can be solved algebraically thus,

$$a = \Phi^{-1} b \quad , \tag{16}$$

defining the multiple beamformer.

This definition is well-posed because the matrix Φ includes all interactions between the independent source directions.

From both Eq. 16 and the earlier definition of b we can see that this multiple beamformer processes the data coherently in space and in time.

The estimation of the variance σ^2 can be found using the equation

$$\frac{\partial}{\partial \sigma} L \, \bigg|_{\sigma = \hat{\sigma}_{MLE}} = 0 \tag{17}$$

which yields

$$\sigma^2 = \sum_{n,t} \left| y_{n,t} - \psi^*_{n,t} \cdot a \right|^2 \tag{18}$$

or, with Eq. 15,

$$\sigma^2 = \frac{1}{M} Tr(R) - b^* \phi^{-1} b \quad .$$ (19)

Summarizing, we can write for the multiple beamformer,

$$a = \phi^{-1} b,$$

and for the variance of the estimation error,

$$\sigma^2 = \frac{1}{M} Tr(R) - b^* \phi^{-1} b .$$ (20)

The residual log likelihood function is given by

$$L_r = const - \log \frac{1}{M} \{ Tr(R) - b^* \phi^{-1} b \} \quad .$$ (21)

The multiple beamformer as defined by Eqs. 16 or 20 allows us to estimate amplitude and phase for a finite number of sources if the source directions are known in advance. In the next section we will weaken this condition by requiring that the source directions be known only roughly. Then a fine bearing estimator will be used to find the optimal bearing estimates.

5   FINE BEARING ESTIMATOR

To estimate the fine bearing of the different sources the residual log likelihood function, L_r, as given in Eq. 21, can be used as an optimization measure since it does not depend explicitly on the amplitudes.

As usual we find the desired bearing estimates by maximizing this residual log likelihood, which is also equivalent to the minimization of the variance of the amplitude estimation error σ^2. In this sense the maximum likelihood equation can be written as

$$\frac{\partial}{\partial u} L_r \bigg|_{u=\hat{u}_{MLE}} = \frac{\partial}{\partial u} \sigma^2 \bigg|_{u=\hat{u}_{MLE}} = 0$$ (22)

where u is the parameter vector we wish to estimate, as defined in Eq. 6:

$$u_k = \pi \frac{2d}{\lambda} \cos \omega_k$$

Unfortunately Eq. 22 has no simple solution so that we are forced to use some iterative techniques.

Although more efficient techniques exist, for reasons of simplicity we have chosen the Newton procedure to solve Eq. 22.

As generally known, Eq. 22 is replaced in the Newton procedure by a first-order Taylor approximation of the solution, which yields:

$$\frac{\partial}{\partial u_i} \sigma^2 + \sum_j \left(\frac{\partial^2 \sigma^2}{\partial u_j \, \partial u_i} \right) (u_j - \hat{u}_j) = 0 \tag{23}$$

With the definitions

$$\Delta u_j = u_j - \hat{u}_j$$

$$g_i = \frac{\partial \sigma^2}{\partial u_i} \tag{24}$$

$$H_{ij} = \frac{\partial^2 \sigma^2}{\partial u_j \, \partial u_i}$$

equation 23 can be written as

$$g = - H \, \Delta u \quad , \tag{25}$$

having the solution

$$\Delta u = - H^{-1} g \ . \tag{26}$$

Being close to the MLE solution of Eq. 22, this equation can be used to iterate towards the optimal bearing estimate.

We now develop the expressions for the gradient, g, and the Hessian, H. Starting with Eq. 15

$$\Phi a = b$$

we obtain

$$\frac{\partial b}{\partial u_i} = \left(\frac{\partial \Phi}{\partial u_i} \right) a + \Phi \left(\frac{\partial a}{\partial u_i} \right) \tag{27}$$

and

$$\frac{\partial^2 b}{\partial u_j \, \partial u_i} = \left(\frac{\partial^2 \Phi}{\partial u_j \, \partial u_i} \right) a + \left(\frac{\partial \Phi}{\partial u_i} \right) \left(\frac{\partial a}{\partial u_j} \right) + \left(\frac{\partial \Phi}{\partial u_j} \right) \left(\frac{\partial a}{\partial u_i} \right) + \Phi \left(\frac{\partial^2 a}{\partial u_j \, \partial u_i} \right) \tag{28}$$

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combining Eqs. 15 and 19

$$\sigma^2 = \frac{1}{M} \, \mathrm{Tr}(R) - b^* a$$

we find

$$g_i = - \frac{\partial b^*}{\partial u_i} \, a - b^* \, \frac{\partial a}{\partial u_i} \tag{29}$$

and

$$H_{ij} = a^* \left(\frac{\partial^2 \Phi}{\partial u_j \, \partial u_i} \right) a - 2 \left(\frac{\partial a^*}{\partial u_j} \right) \Phi \left(\frac{\partial a}{\partial u_i} \right) - 2 \, \mathrm{Real} \; a^* \left(\frac{\partial^2 b}{\partial u_j \, \partial u_i} \right) \tag{30}$$

where

$$\Phi \, \frac{\partial a}{\partial u_i} = \frac{\partial b}{\partial u_i} - \frac{\partial \Phi}{\partial u_i} \, a \; . \tag{31}$$

This completes the general derivation of the terms necessary for the Newton iteration.

Next we have to determine under which conditions the Newton iteration will converge to the desired solution. The simplest way to sketch the problems that can occur, is to investigate the graphic representation of a simple case.

Figure 1 shows the log likelihood function, the gradient, and the Hessian of a single source scenario.

As indicated, we identify three areas:

Area I   The global maximum can be found by use of some maximum gradient search techniques.

Area II   The global maximum can be found via the Newton iteration.

Area III   The Hessian is negative definite, which is necessary for the solution to be a maximum.

Within Area II we see that the iteration approaches the solution in an alternating way. Outside Area II but within Area I the Newton iteration diverges away from the true solution. There is also a singular behaviour of the Hessian at the border of Area III.

The following technique allows us to combine the advantages of the Newton algorithm within Area II with a simple gradient technique. This ensures convergence to the desired maximum inside Area I.
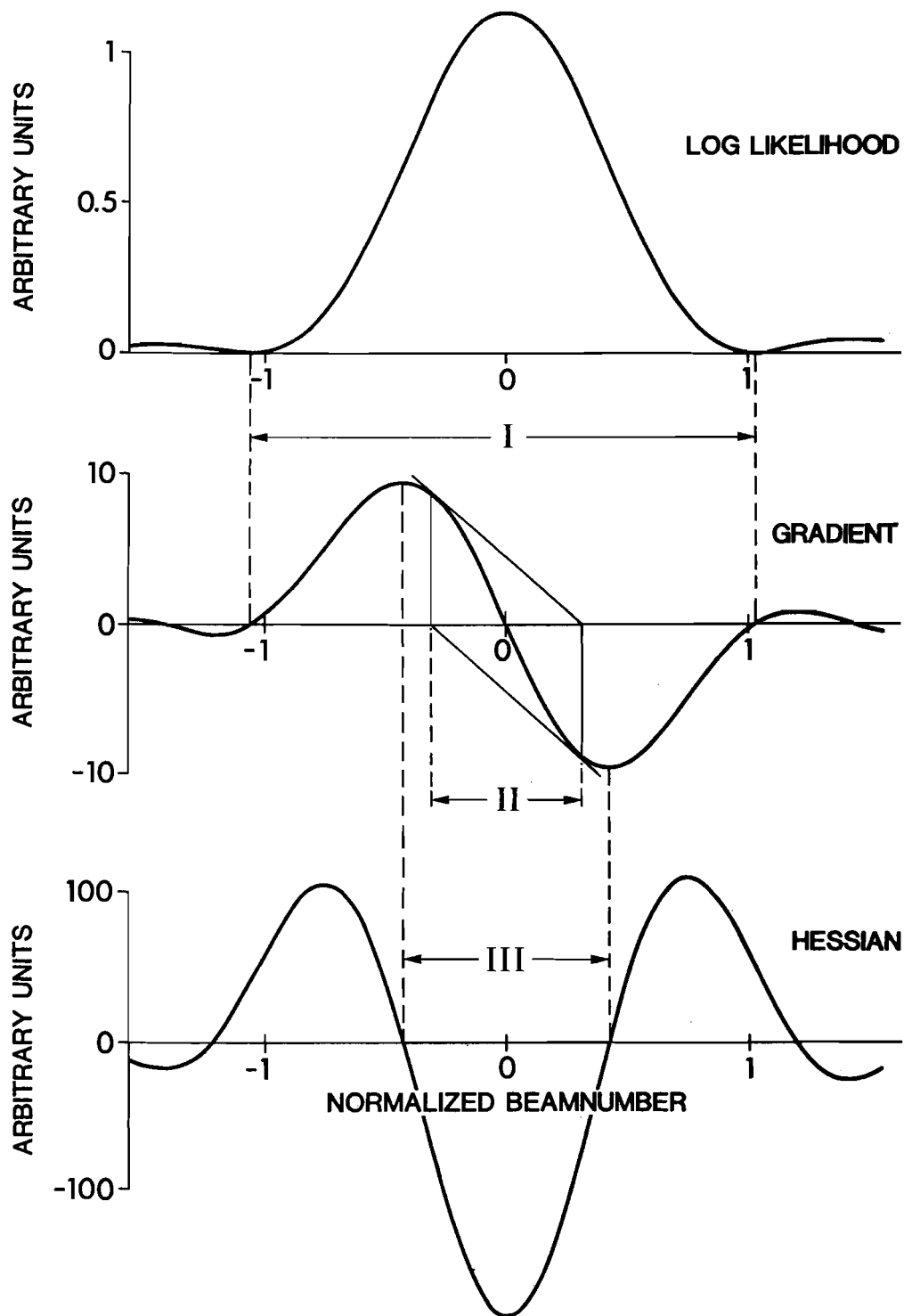
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*FIG. 1   LOG LIKELIHOOD, GRADIENT AND HESSIAN
OF A SINGLE SOURCE SCENARIO.*

Considering the general formula for the Newton algorithm

$$\Delta u = - H^{-1} g \tag{32}$$

we first replace the regular inverse by the generalized inverse, i.e.,

$$H^{-1} \rightarrow H^{-} \tag{33}$$

This is done in order to avoid problems with the singularities of the Hessian. Remember that the generalized inverse H^{-} is any matrix satisfying the relation

$$H \, H^{-} \, H = H \tag{34}$$

In the case of a non-singular Hessian the generalized inverse is equal to the regular inverse. More details are given by Rao-Mitra [7] and Bjerhammar [8], among others.

Next we limit the maximum step size for every component of Δu :

$$\left| \Delta u_i \right| < d_{max} \tag{35}$$

Finally we reverse the resulting step for every component, when it has the same direction as the gradient:

$$\Delta u_i \rightarrow \delta_i \, \Delta u_i$$

$$\delta_i = \rho(g_i \, \Delta u_i < 0) - \rho(g_i \, \Delta u_i > 0) \tag{36}$$

Combining the individual steps, the following algorithm is proposed:

1. Calculate $d = H^{-} g$, (37a)
   where H^{-} is the generalized inverse of the Hessian H

2. Clip the vector d so that

   $$d^2 < d^2_{max} \tag{37b}$$

3. Reverse the sign of d_i if

   $$g_i \, d_i > 0 \tag{37c}$$

4. Equate

   $$\Delta u_i = d_i \tag{37d}$$

The actual value of the maximum step size depends on the number of hydrophones. As a rule of thumb, one quarter of the classical resolution limit has been found to be appropriate for d_{max}.

6   PERFORMANCE ANALYSIS

It is known from statistics that a good measure for the performance of a signal-processing algorithm is given by the Cramer-Rao lower bound (CRLB).

Defining Σ as the parameter estimation covariance matrix

$$\Sigma = E\{(\theta(y)-\alpha)(\theta(y)-\alpha)^*\} \tag{38}$$

and J as the Fisher information matrix

$$J = - E\{\nabla_\alpha^2 \log p(y \mid \alpha)\} \tag{39}$$

where $E\{...\}$ denotes the expectation operator

α is the vector to be estimated

∇_α^2 is the matrix of second derivatives

it can be shown [6] that for an unbiased estimator, the following Cramer-Rao bound holds:

$$\Sigma \geqslant J^{-1} \quad . \tag{40}$$

For the general case of multiple sources the inverse of the Fisher information matrix is estimated numerically.

In the single source case, however, the performance bound can be given in closed form [1].

Let α be defined by

$$\alpha = (s,\mu,\nu)^T \tag{41}$$

where

$$a =: s\, e^{i\nu}$$

$$\psi_{n,t} =: e^{i\pi\frac{2d}{\lambda}(n+\gamma t)(1-2\mu/N)}$$

s   is the signal (amplitude) level
ν is the signal (amplitude) phase
μ is the beam number connected with the source bearing angle ω via
     $\cos \omega = (1-2\mu/N)$
N   is the total number of beams.

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then from Eq. 44

$$\text{var } \{b\} \geqslant \sigma_n^2 \cdot \frac{1}{1280}$$

$$\text{var } \{\mu\} \geqslant \frac{\sigma_n^2}{s^2} \cdot \frac{1}{4.108}$$

(45)

$$\text{var } \{\nu\} \geqslant \frac{\sigma_n^2}{s^2} \cdot \frac{1}{335.238}$$

The variance of the signal phase will be ignored in the detailed analysis.

The performance of the multiple beamformer is presented in two steps. First, the single source case is treated. Here the standard deviation of the bearing and amplitude estimation is tabulated as a function of signal level to noise ratio (SLNR) and signal variance to noise ratio (SVNR) where

$$\text{SLNR} = 20 \log (s/\sigma_n)$$

$$\text{SVNR} = 20 \log (\sigma_a/\sigma_n)$$

The purpose of this part of the analysis is to obtain an indication of the detection capabilities of the multiple beamformer.

Second, the case of two closely spaced sources with equal amplitude levels is used to study the resolution capabilities of the processor. In this case the signal level to noise ratio (SLNR) and the signal variance to noise ratio (SVNR) are kept fixed.

Tables 1 and 2 give the standard deviation of the bearing and amplitude estimation together with the calculated Cramer-Rao Lower Bound CRLB. In these tables the values for the SVNR are selected as follows:

   i) SVNR = - ∞ perfect correlation in time
  ii) SVNR = - 10 dB
 iii) SVNR = - 5 dB

The reasons for this choice are that

- in case (i) the simulated data fit the model assumptions

- in case (ii) the incoherent part the signal alone is just detec-table with a conventional beamformer having M hydrophones (for M = 32 we have 10 log M ≈ 15 dB)

TABLE 1

STANDARD DEVIATION OF THE BEARING ESTIMATION

SLNR	CRLB	SVNR		
		$-\infty$	-10	-5
10	0.156	0.181	0.195	0.197
5	0.277	0.306	0.313	0.334
0	0.493	0.466	0.550	0.521
-5	1.140	0.818	0.758	0.845
-10	2.028	1.586	1.511	1.395
-15	3.606	2.899	2.664	3.022
-20	6.412	6.122	6.769	7.515
-25	11.402	11.456	9.843	9.369
-30	20.277	17.096	13.528	14.789

$$95\% \text{ confidence } \begin{pmatrix} +1.0 \\ -0.8 \end{pmatrix} \text{ dB}$$

TABLE 2

STANDARD DEVIATION OF THE AMPLITUDE ESTIMATION

SLNR	CRLB	SVNR		
		$-\infty$	-10	-5
10	0.0280	0.0244	0.0343	0.0533
5	0.0280	0.0281	0.0268	0.0595
0	0.0280	0.0278	0.0335	0.0620
-5	0.0280	0.0307	0.0312	0.0551
-10	0.0280	0.0273	0.0305	0.0615
-15	0.0280	0.0311	0.0286	0.0500
-20	0.0280	0.0258	0.0352	0.0523
-25	0.0280	0.0224	0.0285	0.0386
-30	0.0280	0.0231	0.0261	0.0403

$$95\% \text{ confidence } \begin{pmatrix} +1.0 \\ -0.8 \end{pmatrix} \text{ dB}$$

- in case (iii) the signal can easily be detected with a conventional beamformer even when the mean amplitude vanishes.

Considering the bearing estimation (Table 1) one sees that for all SLNR and for all three SVNR values the standard deviation follows quite well the CRLB.

Table 2 presents the performance of the amplitude estimator. Here also the standard deviation is found to be very close to the CRLB, at least for small SVNR values. For a signal variance to noise ratio of -5 dB the performance in the amplitude estimation is 4 to 7 dB worse than the CRLB.

This means that a SVNR of -5 dB is sufficient to violate the coherence assumption within the model and therefore to degrade the accuracy of the amplitude estimation.

Next we consider the resolution capabilities of the multiple beamformer. Figures 2 and 3 plot the standard deviation of the bearing and amplitude estimate as a function of the separation of the two signals. For this part of the analysis a signal level to noise ratio (SLNR) = 0 dB and a signal variance to noise ratio (SVNR) = -10 dB were selected.

The two figures show nearly optimal performance for the multiple beamformer for separations of more than 0.5 separation units.

$$1 \text{ unit } = \frac{2d}{\lambda} \frac{M}{N}$$

where
$\quad$ d = Sensor spacing
$\quad$ λ = Signal wavelength
$\quad$ M = Number of sensors
$\quad$ N = Total number of beams

The direct comparison with the calculated CRLB for two sources shows a remarkable discrepancy around 1 separation unit (half the width of the main lobe), where the CRLB is significantly worse than the simulated performance. These figures suggest that at these close separations the Cramer-Rao lower bound cannot be used in the simplified form of Eq. 40.

At very small separations the simulation shows a degraded performance with respect to the CRLB. This is because the appearance of outlayers increases the variance drastically.
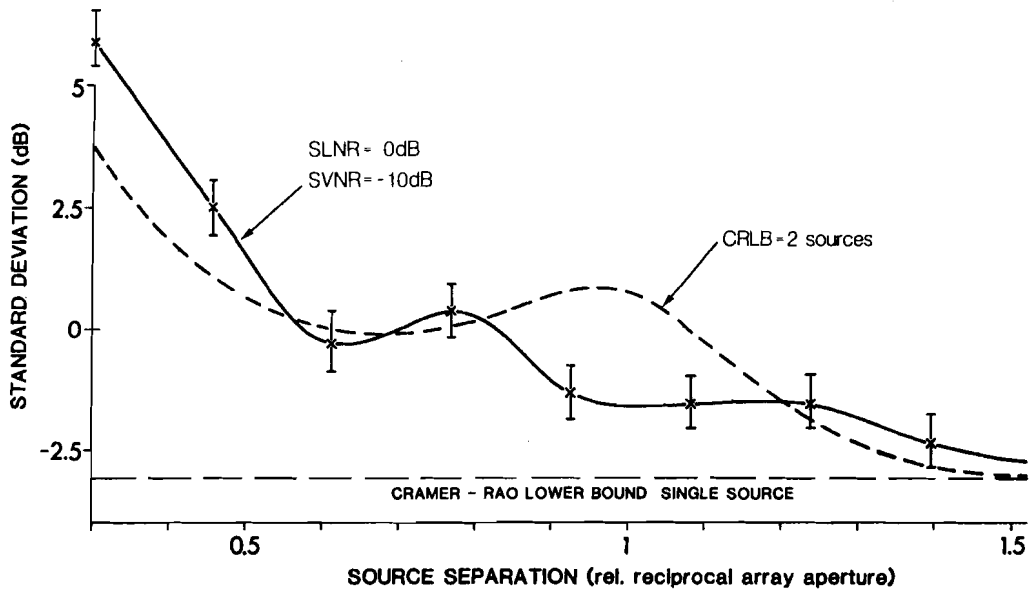
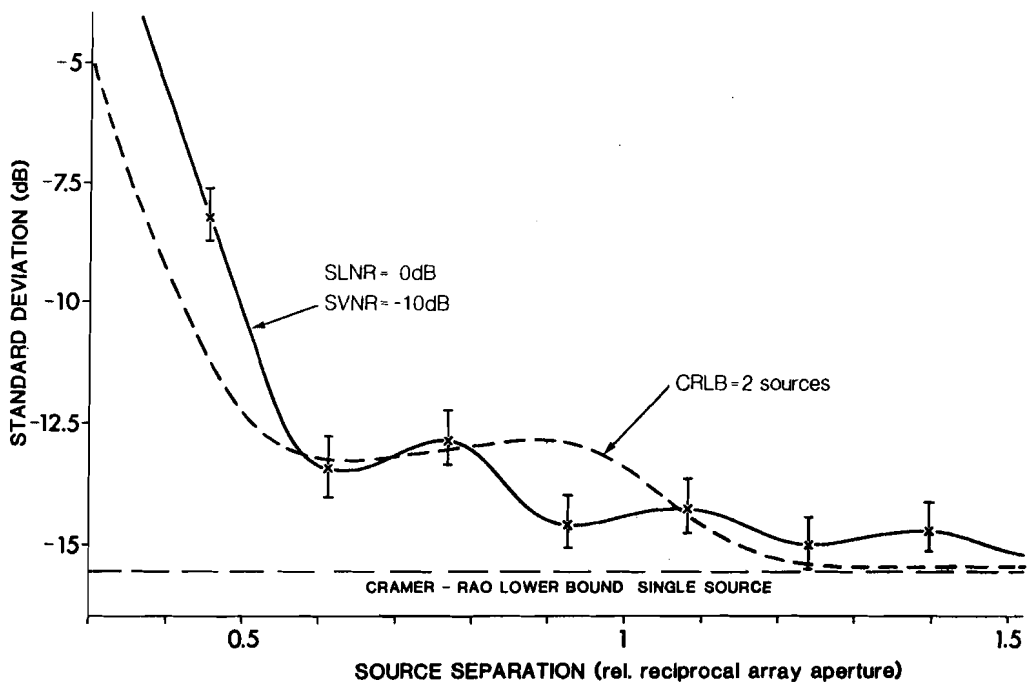FIG. 2 PERFORMANCE ANALYSIS: BEARING ESTIMATION.

FIG. 3 PERFORMANCE ANALYSIS: AMPLITUDE ESTIMATION.

Then the Fisher information matrix is given by

$$\sigma^2 \; J \; = \; \sum_{n,t} \begin{pmatrix} 1 & 0 & 0 \\[2mm] 0 & s^2\left(\frac{2\pi}{N} \cdot \frac{2d}{\lambda}\right)^2 (n+\gamma t)^2 & s^2\left(\frac{2\pi}{N} \cdot \frac{2d}{\lambda}\right)(n+\gamma t) \\[4mm] 0 & s^2\left(\frac{2\pi}{N} \cdot \frac{2d}{\lambda}\right)(n+\gamma t) & s^2 \end{pmatrix} \tag{42}$$

It is easy to see that the performance bounds can be written as follows:

$$\mathrm{var}\,\{s\} \; \geqslant \; \sigma^2$$

$$\mathrm{var}\,\{\mu\} \; \geqslant \; \frac{\sigma^2}{s^2}\left(\frac{N}{2\pi} \cdot \frac{\lambda}{2d}\right)^2 \frac{12}{(M^2-1)+\gamma^2(T^2-1)} \tag{43}$$

$$\mathrm{var}\,\{\nu\} \; \geqslant \; \frac{\sigma^2}{s^2}\; 2\; \frac{(M-1)(2M-1)+3\gamma(M-1)(T-1)+\gamma^2(T-1)(2T-1)}{(M^2-1)+\gamma^2(T^2-1)}$$

Next the error σ^2 has to be estimated. From Eq. 20 we find that:

$$\sigma^2 \; = \; \frac{\sigma_n^2}{MT} \tag{44}$$

where σ_n^2 is the noise variance as defined in Eq. 4.

For the present analysis the following values are selected

$$\begin{aligned} M &= 32 \\ T &= 40 \\ N &= 1024 \\ 2d &= \lambda \\ v &= 0 \end{aligned}$$

7 PRE-PROCESSOR

During the derivation of the multiple beamformer algorithm it was always assumed that the number of sources and their rough bearings are known. In the following we will drop this unjustified assumption and sketch some possibilities by which we can obtain this information.

Let us first deal with the question of the model order or number of sources. Here the answer seems to be very simple. Especially after the recent work of Bienvenu and Kopp [9, 10] the favourite should be the eigenvalue detector.

This method uses the eigenvalue λ_i $i = 1, \ldots, M$ of the estimated correlation matrix

$$R_{n,m} = \frac{1}{T} \sum_{t=0}^{T-1} y_{n,t} \, y_{m,t}^{*} \tag{46}$$

to find the model order K. In detail, the number of sources is estimated by the value of K, which minimizes the following quantity

$$MDL(K) = T \, F(K) + \frac{1}{2} K \, (2M-K) \, \log T$$

with

$$F(K) = (M-K) \, \log\left(\frac{1}{M-K} \sum_{i=1}^{M-K} \lambda_i\right) - \sum_{i=1}^{M-K} \log \lambda_i \tag{47}$$

where the Minimum Description Length (MDL) after Rissanen is used instead of the Akaike Criterion [10].

Having now estimated the model order we have to find the coarse bearing of the K sources. For this the use of some sort of high-resolution technique is needed to ensure that every source has an appropriate bearing value to start the fine bearing iteration.

Using the ideas behind the Pisarenko Harmonic Decomposition and the Prony method, two candidates are derived [11].

The basic point is to realize that the signal model given in Eq. 2.

$$z_{n,t} = \sum_{k=1}^{K} a_k \, \psi_k^{n+\gamma t} \tag{48}$$

can be understood as the general solution of the linear difference equation

$$\sum_{m=0}^{K} c_m \, z_{n-m,t} = 0 \qquad\qquad (49)$$

where

$$c_0 = 1$$

$$K \leqslant n \leqslant M-1$$

or in Matrix notation

$$Z_t \, c = 0 \qquad\qquad (50)$$

where

$$Z_t = \begin{pmatrix} z_{K,t} & \cdots & z_{0,t} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ z_{M-1,t} & \cdots & z_{M-1-K,t} \end{pmatrix} \quad .$$

Expressing now the model in terms of the measurement data and error

$$Z_t = Y_t - W_t \qquad\qquad (51)$$

where

Y_t is the measurement data matrix

W_t is the measurement error matrix

Eq. 50 becomes an Autoregressive-Moving Average (ARMA) model formulation

$$Y_t \, c = W_t \, c \quad . \qquad\qquad (52)$$

Starting with the ARMA model, two methods of solving for the coefficient vector c are presented.

The first method, which is connected with Pisarenko Harmonic Decomposition, assumes the coefficient vector c to be constant in time so that we can get an average ARMA model

$$(R_{YY} - \sigma_w^2 I) c = 0 \tag{53}$$

where

$$R_{YY} = \langle Y_t^* Y_t \rangle$$

$$\sigma_w^2 I = \langle W_t^* W_t \rangle = \langle Y_t^* W_t \rangle$$

$\langle \cdot \rangle$ is the time average.

But this is an eigenvalue equation, where we find the vector c as the eigenvector to the eigenvalue σ_w^2, which turns out to be the smallest eigenvalue of the R matrix.

The second approach takes Eq. 52 and replaces the MA term by a single noise vector

$$u_t = W_t c \quad . \tag{54}$$

This means that the ARMA model is modified to an autoregressive (AR) model and the usual technique to minimize the prediction error can be used to obtain the vector c

$$\langle |W_t|^2 \rangle \rightarrow min \tag{55}$$

yields to

$$c^* R_{YY} c \rightarrow min \tag{56}$$

with the constraint

$$c_0 = 1 \quad .$$

This approach corresponds to the Prony method.

The next step is to solve the characteristic polynomial of the linear difference Eq. 49

$$\sum_{m=0}^{K} c_m \, \psi^{K-m} = 0 \tag{57}$$

to find a basic set of solutions $\{\psi_k\}$ $k = 1, \ldots, K$, which we need to build our source model.

This basic set of solutions is now used for the coarse bearing estimates necessary for the fine bearing iterations.

However, before we complete this section we suggest a simple improvement of these methods.

Spurious harmonic solutions of Eq. 57 are suppressed by replacing the R_{yy} matrix in Eqs. 53 and 56 by a forward/backward averaged matrix. This method of additional spatial averaging also gives increased stability to the solution [12].

8   POST-PROCESSOR

The post-processor serves two purposes. After having estimated the signal components of the received soundfield the post-processor can evaluate the success of this estimation procedure. Also it can show the spatial distribution of the signal-free ambient noise power.

The idea behind the post-processor is to subtract the estimated signal from the data and to apply a conventional beamformer to the residual.

This means that when we subtract the signal from the data

$$w_{n,t} = y_{n,t} - \sum_{k=1}^{K} a_k \, e^{iu_k(n+\gamma t)} \tag{58}$$

and then estimate the signal-free covariance matrix

$$Q_{n,m} = \frac{1}{T} \sum_{t=0}^{T-1} w_{n,t} \, w_{m,t}^* = R_{n,m} - \sum_{k=1}^{K} a_k^* \, e^{-iu_k m} \frac{1}{T} \sum_{t=0}^{T=1} y_{n,t} \, e^{-iu_k \gamma t} +$$

$$+ \sum_{k=1}^{K} a_k \, e^{iu_k n} \sum_{t=0}^{T-1} (y_{m,t}^* - z_{m,t}^*) \, e^{iu_k \gamma t} \tag{59}$$

we will get with

$$\frac{1}{T} \sum_{t=0}^{T-1} (y_{m,t}^* - z_{m,t}^*) \, e^{iu_k \Upsilon t} = 0 \qquad (60)$$

the relation

$$Q = R - S \qquad (61)$$

where

$$S_{n,m} = \frac{1}{T} \sum_{t=0}^{T-1} z_{n,t} \, z_{m,t}^*$$

so that the ambient noise power estimate P_{an} can be written as

$$P_{an} = v^* \, Q \, v = v^* \, R \, v - v^* \, S \, v \quad . \qquad (62)$$

where v is the usual plane wave steering vector.

The first term in Eq. 62 is thereby nothing else than the conventional beamformer applied to the estimated measurement cross-correlation matrix; the second term is simply the signal-alone beam pattern.

SUMMARY

A multiple beamformer has been presented and analyzed. The performance in amplitude and bearing estimation is found to follow quite well the Cramer-Rao Lower Bound for varying conditions. The increased detection and resolution capabilities are a consequence of defining a coherent processor in space and time. However, it has also been shown that small incoherent components in time do not degrade the optimal performance.

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KEYWORDS

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