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**About the effective Doppler sensitivity of certain
nonlinear chirp signals (NLFM)**

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I. Background

When talking about the Doppler sensitivity (or Doppler tolerance) of linear chirp signals (LFM) of duration T and bandwidth B one has to distinguish between two cases:

1. The Time-Bandwidth product ($T \cdot B$ product) is smaller than the inverse of the Doppler coefficient β which is defined by $\beta = 2 \cdot v/c$ (v = target speed, c = sound speed).
2. The $T \cdot B$ product is larger than $1/\beta$.

In the first case the Doppler effect on the received echo signal is defined by a mere shift of the centre frequency. The Doppler sensitivity as observed in the Ambiguity Function (ABF) is given by the overlap loss. This loss is caused by the spectra of the transmitted and the received signal not overlapping completely due to the Doppler shift so that the cross correlation function (CCF) has less bandwidth than the original signals.

In the second case the Doppler sensitivity is the result of the Doppler induced time compression (or stretching) of the echo signal. This time compression changes the duration of the received signal and also the slope of its linear time-frequency characteristic curve. A 3-dB correlation loss at the output of the matched filter occurs if the target moves through one range resolution cell during the signal duration T . For more details see [1,2].

Up to now conventional SONARs did not really exploit any kind of Doppler sensitivity of LFM-signals. In case the $B \cdot T$ product would exceed the limit mentioned above, a hyperbolic FM-signal (HFM) would be used. This type of nonlinear time-frequency dependence does not cause any correlation loss due to the time compression. HFM-signals therefore show (almost) no Doppler sensitivity [3].

During the development of Low Frequency Activated Towed Arrays (LFAS), however, the question came up again, whether the Doppler sensitivity of LFM-signals with sufficiently large $T \cdot B$ -product could and should be exploited. There were mainly two reasons for this consideration:

First, the $T \cdot B$ product of LFAS-systems seemed to settle around 512 (e.g. $T = 8$ s; $B = 64$ Hz) and would therefore in principle be large enough to allow for some kind of Doppler measurement.

Second, in view of the considerable difficulties with reverberation induced false targets Doppler information could be a useful classification clue for discriminating between true and false targets. It would, therefore, be of great help if Doppler information could be derived from the single ping echo analysis. Of course, the range rate information could also be derived from a multiping analysis. However, the very low ping rate together with the highly intermittent target contact due to the fluctuations of the propagation conditions would be time consuming. At the same time the computer load would be high because any detection would lead to a track initiation.

Consequently we had good reasons for trying to measure range and range rate (Doppler) simultaneously, i.e. on a 1-ping basis. During our experiments with an EXperimental Activated Towed Array SONar within the Project EXSON we tested LFM-signals of duration $T = 8$ s and bandwidth $B = 64$ Hz. Meaningfull results where only achieved when there was no time spread on the echo signal. The Doppler correlation filter output signals showed considerable level differences (about 5-6 dB within a Doppler range of ± 25 kn), however the maximum was not observed in the correct Doppler channel. These erroneous maxima were caused by interference effects in the ABF which were due to the unknown multipath configuration.

The simplest way to avoid the problem is to choose a different type of signal. The Pseudo Random Noise Code (PN-signal) has an ABF which rolls off fast in range as well as in Doppler. The decorrelation in Doppler direction occurs within $1/T$, i.e. much faster than the roll off along the Ambiguity Ridge of the LFM-ABF. In fact, the ABF of an PN-signal may roll off too fast for the practical SONAR case. A PN-signal of duration $T = 8$ s would exhibit a Doppler resolution capability corresponding to $1/8$ Hz which - around a center frequency of 1000 Hz (a typical LFAS frequency) - would lead to a Doppler resolution of about 0.2 kn. For practical (and financial) reasons this resolution may be too high. To cover a Doppler range of ± 25 kn at a center frequency of 1000 Hz would require 250 Doppler filters. Also from the signal processing point of view difficulties may arise: In order to keep the false alarm rate in the overall system down the detection probability would have to be decreased. Furthermore, due to the sidelobes in the ABF a large number of false alarms will be created by the target induced clutter.

These problems will, of course, arise in any processing concept where the dimension of the matched filter is extended into the Doppler domain. Therefore the number of matched filter channels should not be made larger than necessary from a practical SONAR point of view.

Modifying the originally linear time-frequency characteristic curve of a conventional chirp signal in a specific way will allow to control the Doppler sensitivity of a SONAR signal without being confined to the $1/T$ -limit mentioned above as demonstrated in [4]. Thus the number of required Doppler channnels to cover a given Doppler range could be reduced. This will be illustrated in the next section.

II. Description of a specific Nonlinear Chirp Signal (NLFM)

The basic idea behind the design of this signal was as follows: The hyperbolic chirp signal (HFM) is known to be (almost) Doppler insensitive [3]. The gradient of its time-frequency characteristic curve is concave. The LFM-signal has got already "a little bit" of Doppler sensitivity (due to the time compression effect), its time-frequency characteristic curve is a straight line. Argueing heuristically, one should get FM-signals with further increase in

Doppler sensitivity if one would go to convex time-frequency characteristic curves. One way to do so is to make the momentary frequency f proportional to some power of the time t , keeping the exponent α smaller than one. Such a nonlinear chirp signal (in complex notation) is described by the following expression

$$(1) \quad y(t) = \exp\left(2\pi j \left(f_c - \frac{B}{2} + \frac{B \cdot t^\alpha}{(\alpha - 1) \cdot T^\alpha} \right) \cdot t\right) \quad \text{Eq. 1}$$

with f_c = center frequency, T = signal duration, B = bandwidth, t = time, $j = \sqrt{-1}$. The Doppler sensitivity is controlled by the exponent α . For $\alpha = 1$ Eq.1 describes the well known LFM, for $\alpha = 0$ the CW-pulse follows. Without going into further detail the hyperbolic chirp (HFM) is approximated (in terms of minimum Doppler sensitivity) by choosing $\alpha = 1.2$.

In Fig.1 we show the spectra of the LFM-signals as described by Eq.1 for different values of α . With α decreasing the spectra loose their flatness, concentrating the signal energy in a slightly smaller bandwidth. The decrease in bandwidth causes an increase of the resolution cell thus deteriorating the processing gain against reverberation. This (among others) is the price one has to pay for being able to measure Doppler. The effect, however, is not very serious as long as the signal parameters are kept in reasonable ranges. As an example let us look at Fig.2, which displays the 6 dB-widths of the Autocorrelation Functions (ACF) as a function of α ($T = 8$ s; $B = 64$ Hz). The values have been normalized to the 6-dB-width of the LFM-signal in order to indicate the "loss" against reverberation. Down to a value of $\alpha = 0.3$ this loss will be less than 3 dB. During our EXSON experiments we have found the value $\alpha = 0.5$ to be a convenient choice. In this case the increase of the Doppler relevant resolution cell, as compared to the LFM signal occupying the same system bandwidth B , would be about 1.3 dB.

In order to discuss the Doppler sensitivity of the NLFM, we start from Eq.1 and replace the variable t by $t(1-2v/c)$ with v = target range rate and c = sound speed.

$$(2) \quad y(t, v) = \text{rect}\left[\frac{t \cdot \left(1 - \frac{2v}{c}\right)}{T}\right] \cdot \exp\left(2\pi j \left(f_c - \frac{B}{2} + \frac{B \cdot t^\alpha \cdot \left(1 - \frac{2v}{c}\right)^\alpha}{(\alpha - 1) \cdot T^\alpha} \right) \cdot t \cdot \left(1 - \frac{2v}{c}\right)\right) \quad \text{Eq. 2}$$

This is a twodimensional function of the variables t and v . Its ABF can be calculated as a set of crosscorrelation functions (CCF) in the time dimension keeping one signal fixed and stepping the parameter v for the second signal. This set of CCFs can be plotted in an intensity or color coded form from which the sidelobe structure of the ABF can be recognized. This has been done in Fig.3.

In the upper half of Fig.3 the ABF of a LFM-signal ($T = 8$ s, $B = 64$ Hz, $\alpha = 1.0$) is displayed. The central part of the ABF is presented in a distorted form for display purposes only. The mean inclination of the Ambiguity ridge has been increased in order to make better use of the plot format. One recognizes the typical ridge structure which due to the Doppler effect starts widening a little bit towards the higher range rate regimes. Looking into the data in more detail (which is not possible in this reproduction) one would find a 3 dB-width of about 10 kn. If we now change to $\alpha = 0.5$ we get a NLFM-signal whose ABF is displayed in the lower half of Fig.3. The maximum of the ABF (indicated by the bright area) is concentrated much more around the center. In fact, the 3 dB-width is 1 kn. The volume below the ABF which has to remain constant now starts spreading out in the

range-Doppler-plane. However the ridge shape is preserved to some extent. This actually means that the range sidelobes in the correct Doppler channel can be made very small. This is an advantage over PN-signals for which the range sidelobe levels are more or less limited to the ratio $1/B \cdot T$. The intensity modulation covers a dynamic range of 60 dB which has been evenly split up between 6 gray scales.

In order to show some details in the ABF structure Fig.4 shows a smaller section of the same ABF as displayed in the lower half of Fig.3. The dynamic range this time is 20 dB. One recognizes a relatively symmetric and simple structured pattern. The relevant sidelobe levels occur in a small fraction of the Ambiguity plane and would make it easy to solve the problem of false targets which are caused by the target induced clutter. Unfortunately the picture changes as soon as multipath induced time spread destroys the 1 point target situation. This is shown in the lower half of Fig.4. We have assumed a 2 point target extending over a little bit more than one radial resolution cell. The simple and regular pattern deteriorates more and more as the echo duration and the number of scatterers (or multipath components) increases.

We have made no attempt to derive the relationship between Doppler sensitivity and the exponent α in analytical form. This could be done following the way described in [1]. For reasons of simplicity we did some numerical analysis and determined the 3 dB-width of the ABF measured along the ridge as a function of α . This was done for various combinations of T and B (center frequency 960 Hz). As an example of the results we present Fig.5 which shows the roll off of the ABF as a function of Doppler speed with α being the parameter. The signal duration T was 8 s. Varying T between 4 s and 16 s resulted in a directly proportional change of the Doppler sensitivity. The dependence of the Doppler sensitivity on the bandwidth B is more complicated and is a function of α as well. For the signal parameters used in the EXSON experiments (32 Hz to 128 Hz, 4 s to 16 s, $\alpha = 0.5$) the Doppler sensitivity was almost not dependent on B . But this would for sure be different for $\alpha = 1$ (LFM).

III. Experimental results

In the last two figures we will present some experimental results. The first example (Fig.6) shows a comparison of two ABFs that have been calculated from echoes of oil drilling platforms in the Norway Sea using two different chirp signals. The upper left picture in Fig.6 displays the ABF using a NLFM with $T = 8$ s, $B = 32$ Hz and $\alpha = 0.5$. The ABF volume clearly concentrates around the center region of the range-Doppler-plane. Some target induced clutter can be observed and forms a regular pattern. The SONAR operator could easily separate the false targets. In the lower left of Fig.6 we depict the ABF calculated from the echo of the same target using, however, a conventional LFM-signal with the same values for B and T as before. In fact both signals were transmitted successively i.e. almost simultaneously. The ABF reveals the well known ridge shaped structure. Although this ridge decays towards the higher speed regime the maximum does not occur in the correct Doppler channel. This becomes more obvious if we look at the curves presented in the right half of Fig.6.

The upper right diagram is derived from the adjacent ABF: from each Doppler channel output signal the relative maximum has been extracted and plotted. The correct Doppler value (known from the experimental configuration) can be obtained from the Doppler channel (x-axis) corresponding to the absolute maximum. If we look at the corresponding curve for the LFM-signal (Fig.6, lower right) we see a plot with a much wider maximum which in addition appears in the wrong Doppler channel. Although both curves look rather noisy this is not caused by true noise signals and does not reveal the true signal/noise ratio. Rather it indicates the "clutter background" in the ABFs which is produced by the time spread in the echo together with the sidelobe structure of the theoretical ABFs.

In Fig. 7 two examples from the most recent experiment are presented. The echoes come from a submarine with a slightly positive range rate (negative radial speed). A NLFM-signal of duration $T = 4$ s with a bandwidth of 64 Hz and $\alpha = 0.5$ was used. The first echo (left half of Fig. 7) reveals a 3-component multipath situation which can be derived from the respective ABF. The ABF displays do not indicate clearly that a well pronounced absolute maximum in the correct Doppler channel can be detected. This becomes more obvious if one looks at the maxima-level-versus-Doppler-channel plot below the ABF plot. The right hand section displays the same type of information taken from an echo that was received 120 s (i.e. one ping period) later.

IV. Summary

We have presented a chirp signal with a nonlinear relationship between time and frequency. The momentary frequency was chosen to be proportional to the time taken to the power α , the value of α being smaller than 1. The case $\alpha = 1$ corresponds to the conventional linear chirp signal.

The Doppler sensitivity of this signal can be controlled by α and can thus be adjusted to the requirements with respect to Doppler resolution while at the same time preserving a sufficient range resolution capability.

It is obvious from the last two figures that the false targets produced by the "target clutter" in the Ambiguity Function can easily be recognized with the help of the pattern recognition capabilities of the human SONAR operator. It will still be a challenging task to develop computer based algorithms that would do the same job.

The experimental results are encouraging and make it worthwhile to think about including these signals into operational systems. An appropriate patent is pending.

References

- [1] Kramer, S.A.
Doppler and Acceleration Tolerances of High Gain, Wideband Linear FM Correlation SONARS
Proc. IEEE Vol. AES-26, Nr. 6, Nov. 1967, p. 627-636
- [2] Herman, J.P.; Roderick, W.I.
Delay-Doppler Resolution Performance of Large Time-Bandwidth Product Linear FM Signals in a Multipath Environment
JASA Vol. 84, Nr. 5, Nov. 88, p. 1709-1727
- [3] Rowland, R.O.
Detection of a Doppler-Invariant FM Signal by Means of a Tapped Delay Line
JASA Vol. 37, Nr. 4, April 1965, p. 608-615
- [4] Rosenbach, Kh.; Ziegenbein, J.
Ergebnisse der Activated Towed Array Versuche EXSON 90
FGAN/FHP Forschungsbericht Nr. 4-91, Lfd. Nr. 307, März 1991

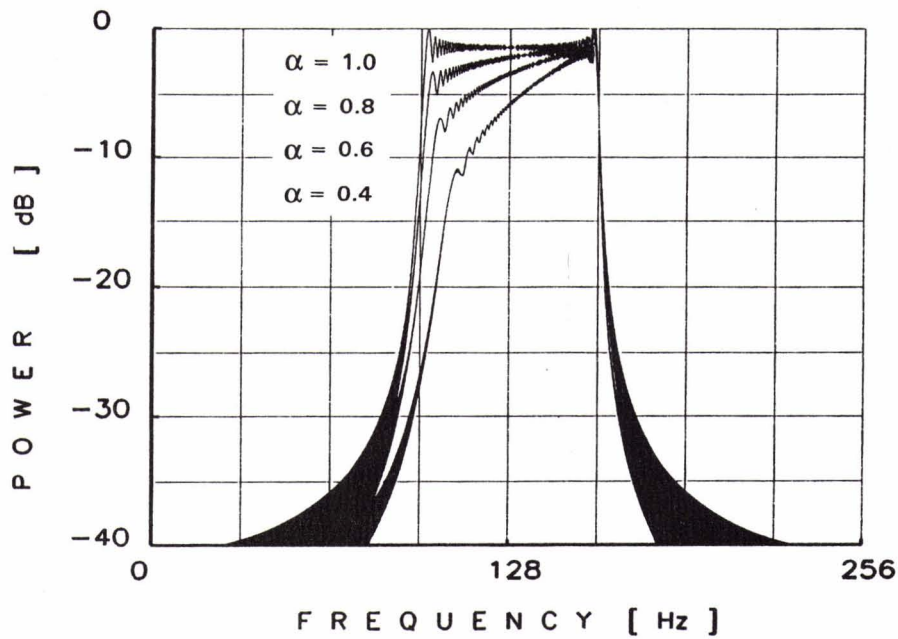


Fig. 1 Power spectra of chirp signals with instantaneous frequency $F \sim t^\alpha$. Signal duration $T = 8s$, bandwidth $B = 64$ Hz.

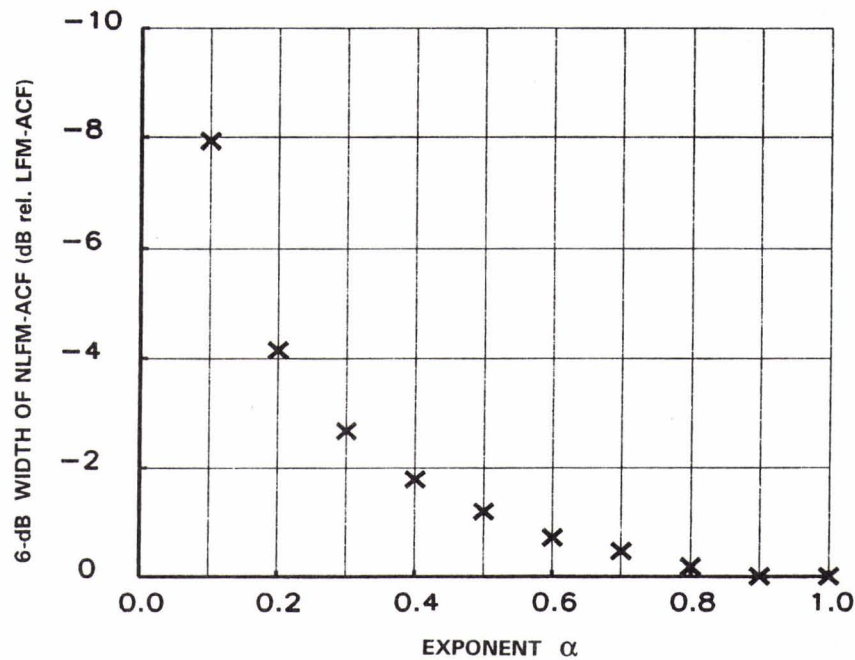


Fig. 2 6-dB width of the autocorrelation function (ACF) of a nonlinear chirp (NLFM) as function of exponent α . $T = 8s$, $B = 64$ Hz.

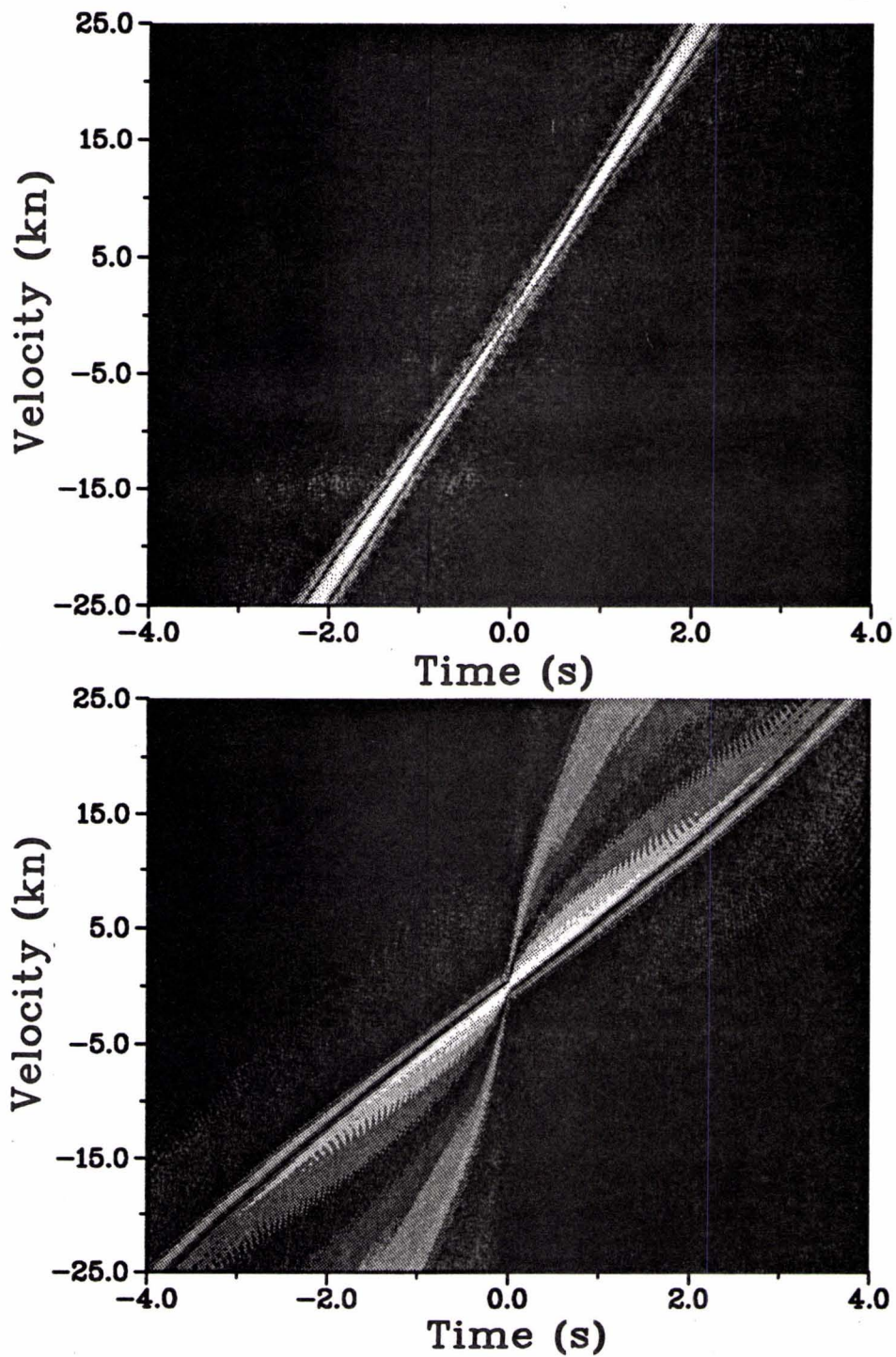


Fig. 3 Ambiguity functions of 1-point target echo using LFM (upper figure), $\alpha=1$, and NLFM (lower figure), $\alpha = 0.5$. Signal parameters : $T = 8\text{s}$, $f_0 = 960\text{ Hz}$, $B = 64\text{ Hz}$.

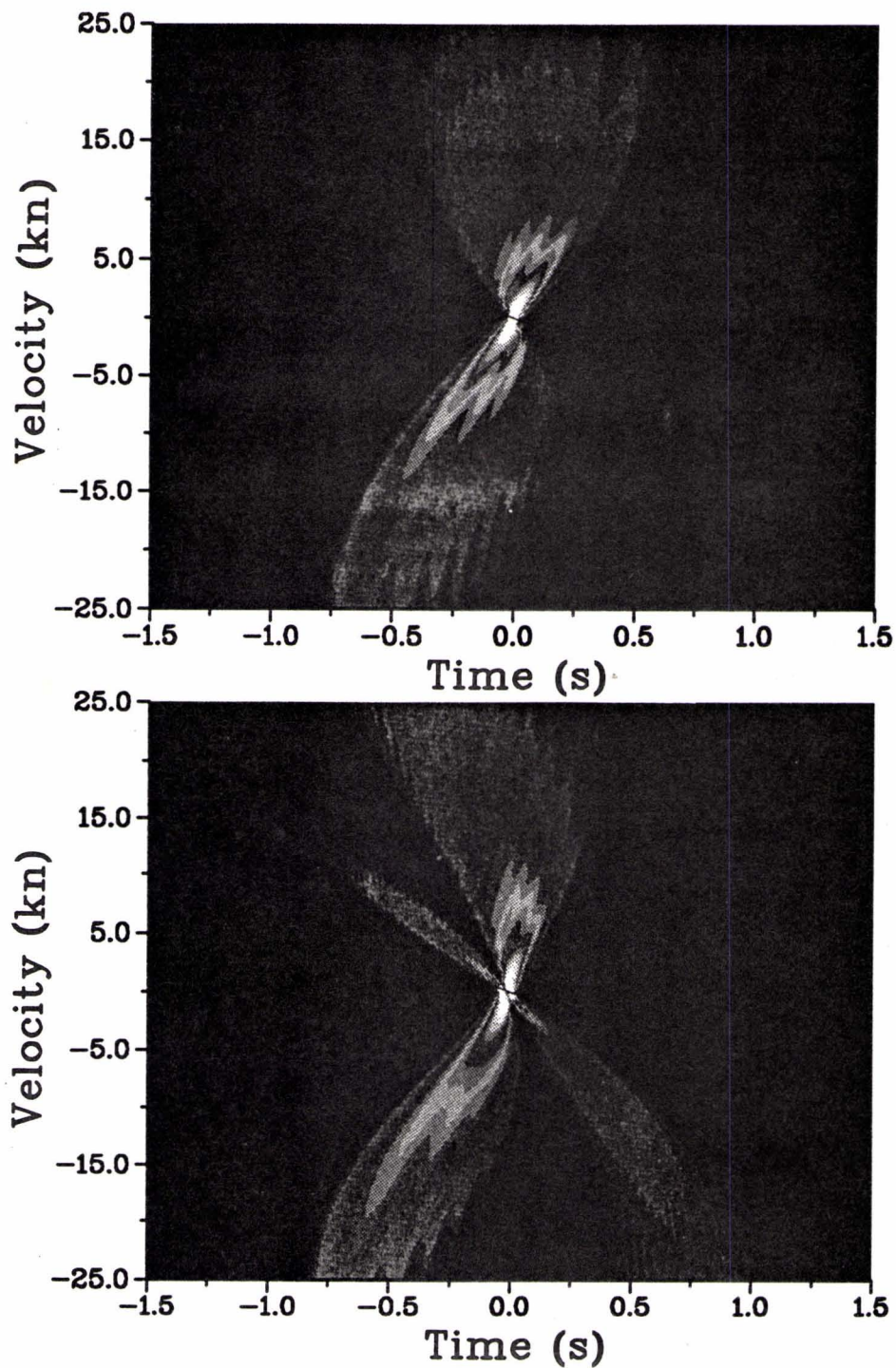


Fig. 4 Ambiguity functions of 2-point target echoes. Separation of scatterers about 1 resolution cell, $\Delta r = 15\text{m}$ (upper figure) and about 2 resolution cells, $\Delta r = 30\text{m}$ (lower figure).

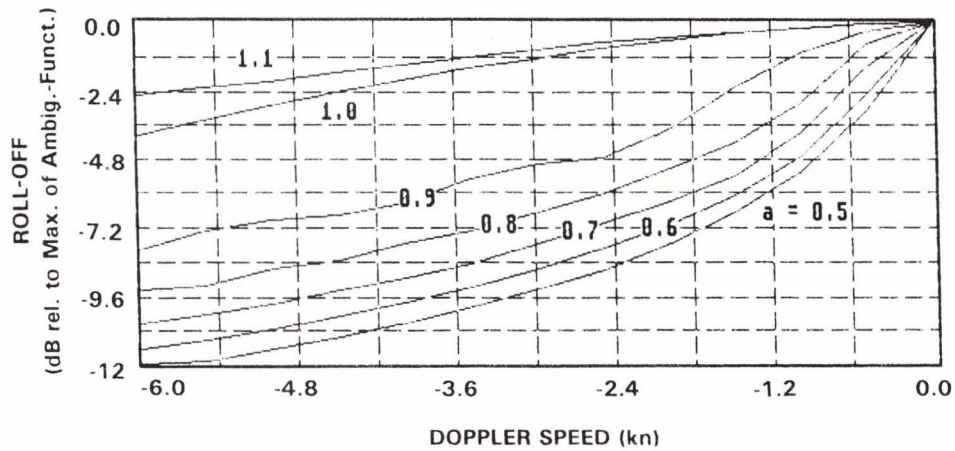


Fig. 5 Roll-off of the ambiguity function of a non-linear FM signal (Frequency $\sim t^\alpha$; signal duration is 8 s).

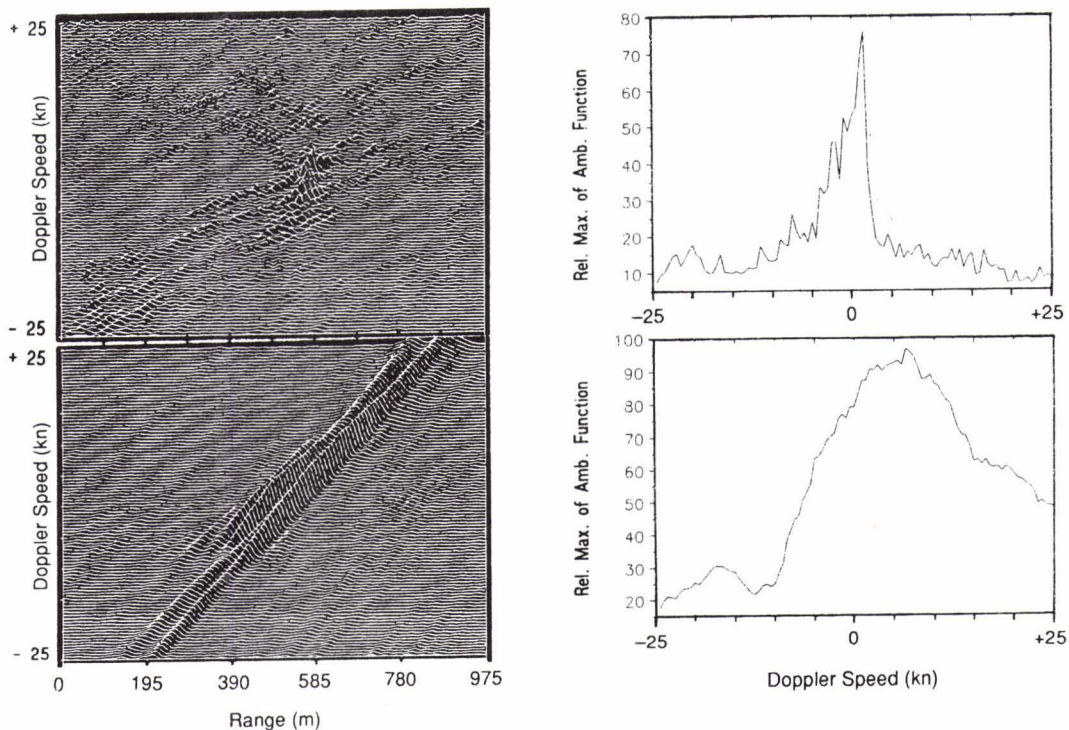


Fig. 6 Ambiguity functions of oil rig echoes using NLFM (upper left) and LFM (lower left). Right section shows the corresponding plots of the maxima for each doppler channel. Signal parameters:
 NLFM : $f_0 = 930$ Hz, $B = 32$ Hz, $T = 8$ s, $\alpha = 0.5$.
 LFM : $f_0 = 970$ Hz, $B = 32$ Hz, $T = 8$ s, $\alpha = 1.0$.

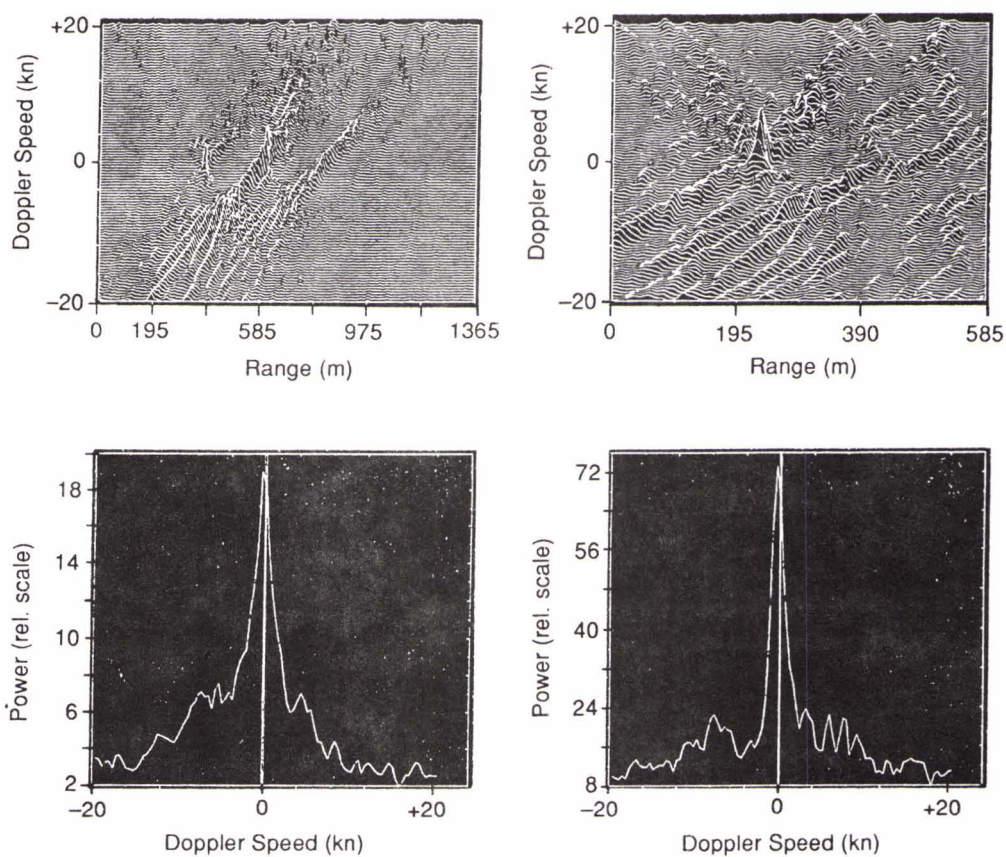


Fig. 7 Ambiguity functions of sub echoes using NLFM, range 17 km (upper figures) and "cut" through ambiguity functions combining maxima of doppler channels (lower figures). Signal parameters: $T = 4s$, $B = 64 \text{ Hz}$, $f_0 = 960 \text{ Hz}$, $\alpha = 0.5$.