INTENSITY AT CAUSTICS

by

C.W. Spofford
Bell Telephone Laboratories
Whippany, New Jersey, U.S.

The classical divergence expressions predict an infinite intensity at caustics. This infinite intensity is a fundamental limitation of geometrical acoustics (ray theory). The intensity at a caustic is of primary concern since the caustic is assumed to be the point of highest intensity within a convergence zone. Several approaches to estimating the field at a caustic have been made using stationary phase and more refined asymptotic methods.

The recent uniform asymptotics work of Ludwig and Kravtsov may be used to obtain both the intensity at the caustic and the transition from the caustic to the point where ray theory is applicable. Whereas in the context of uniform asymptotics, coherent ray theory* is the zeroth order solution (in wavenumber) to the wave equation, Ludwig shows that the first order solution is given (excluding a phase factor) by

$$\psi(\vec{r}) \sim \pi^{1/2} \{ (A_1 + A_2)\, x^{1/4}\, Ai(-x) + i(A_2 - A_1)\, \bar{x}^{1/4}\, Ai'(-x) \} \qquad \text{[Eq. 1]}$$

where

$$x = [\tfrac{3}{4}\, \omega (T_2 - T_1)]^{2/3} \qquad \text{[Eq. 2]}$$

---

* Coherent ray theory estimates intensity by adding all amplitudes on a phased basis.

and A_1, A_2, T_1, and T_2 are the amplitudes and travel times of the two rays intersecting the point of interest (R) on the illuminated side of the caustic. ω is the angular frequency, Ai and Ai' are the Airy function and its first derivative.

This result may be used to study the validity of coherent ray theory near a caustic. As one moves away from the caustic (on the illuminated side) ψ goes uniformly, with wavenumber, to the result obtained from coherent ray theory. In fact, if ψ is compared with the coherent sum of the paths (adding a $-\pi/2$ phase shift to Ray 2 after tangency to the caustic), one finds good agreement up to the last point (first point in range) of constructive interference. After this point, the coherent sum tends to infinity, whereas the uniform asymptotics result experiences an exponential decay into the shadow zone.

The intensity at the last peak is 3.5 dB greater than that at the caustic and the peak is frequently displaced from the caustic by many wavelengths. Also the position of this peak corresponds to the last point of constructive interference as given by ray theory, and the amplitude is given to within 0.5 dB by the in-phase sum of the geometric amplitudes.

The field at the caustic proper is given by

$$\psi_C \sim 2^{2/3} \pi^{1/2} \, Ai(0) \left[\frac{c_R \cos \theta_S}{c_S \sin \theta_R R}\right]^{1/2} \frac{[\omega(T''' R'' - T''R''')]^{1/6}}{(R'')^{2/3}} \qquad \text{[Eq. 3]}$$

where R'', R''', T'' and T''' are the second and third derivatives of range and travel time with respect to θ_S at fixed y_R, and θ_R is the angle at the receiver. These derivatives may be evaluated directly from the ray trace. In fact, for vertically stratified media, for which Snell's law holds, ψ_C may be written in terms of R''. The results of Fig. 1, however, suggest that it may not be necessary to evaluate Eq. 3 for the intensity at a caustic but merely calculate the coherent sum of the geometrical-acoustics amplitudes at the last point of constructive interference.

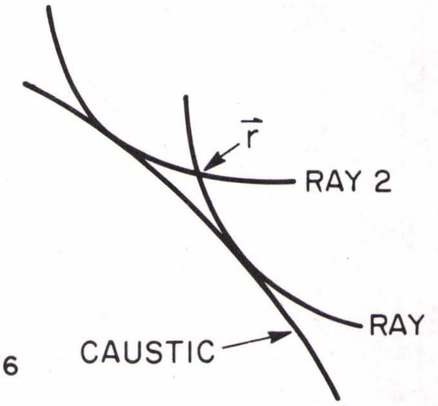
ACOUSTIC FIELD AT CAUSTICS

UNIFORM ASYMPTOTICS

$$\psi(\vec{r}) \sim \pi^{1/2}(A_1 + A_2)\, X^{1/4}\, Ai(-X) + \ldots$$

$$X = \left[\frac{3}{4}\,\omega\,(T_2 - T_1)\right]^{2/3}$$

$$\psi(R_C) = k(\theta_S, \theta_R, R_C)\,\frac{\left[\omega(T'''R'' - T''R''')\right]^{1/6}}{(R'')^{2/3}}$$



COHERENT RAY THEORY (---) VS UNIFORM ASYMPTOTICS (-)

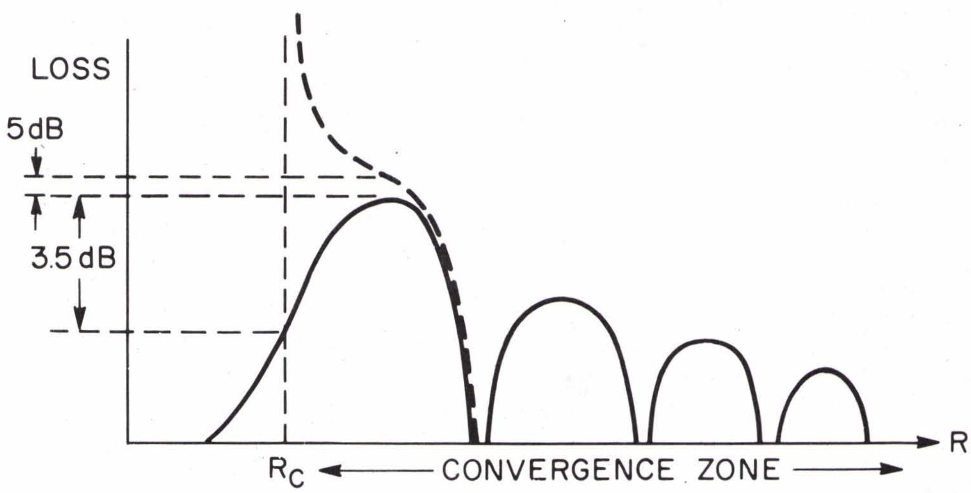


FIG. 1