ACHIEVABLE SIGNAL GAINS FOR LINE ARRAYS IN SHALLOW WATER

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Abstract The well-documented conformance of signal gain to a 20 log n relationship for deep water, refracted paths, does not apply to shallow water where angular spreading effects from boundary roughness limit the signal gains that can be achieved. In depths less than 200 fathoms, the central issue is how long can a line array be and still contribute significantly to signal gain? The methodology employed considers spatial coherence in a direction transverse to propagation and then relates spatial coherence to signal gain.. The literature from 1970 to 1992 was searched for measurements of coherence in shallow water. This information on coherence was used to infer coherence lengths and to calculate its reciprocal parameter, angular uncertainty, as the means for generating bounds on the array gain which can be achieved with linear arrays as a function of range, frequency, area, and season. In anticipation that specific applications may demand firm signal gain requirements, evidence is introduced which suggests that multiple, vertically stacked or volumetric arrays offer a means for off-setting expected degradations with linear arrays.

Introduction

Both active and passive sonar systems are increasingly exploiting the advantages of variable depth towed arrays. In order to meet system performance requirements, designers are pursuing longer towed arrays to enable passive systems to cope with greatly reduced radiated noise levels from threat submarines and active systems to neutralize reverberation.

The objectives of this paper are to provide answers to, or at least guidance on, the two following questions:

(1) How long can a towed array be and still contribute significantly to array gain?

(2) Is the answer to question (1) above dependent upon water depth?

Methods

The analysis begins with the standard equations for signal gain and noise gain which are:

$$\text{Array Signal (or Noise) Gain} = 10 \log_{10} k^{-1} \sum_{n=1}^{k} \sum_{m=1}^{k} \rho_{nm} \qquad (1)$$

where ρ_{nm} is correlation coefficient between n^{th} and m^{th} hydrophone
k is the total number of hydrophones

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The difference between the signal and noise gains is the quantity of interest, namely, array gain.

Array Gain = Array Signal Gain - Array Noise Gain (2)

When nearly identical signals from two hydrophones are cross-correlated, the correlation coefficient approaches unity. Conversely, a coefficient of 0 is obtained when two signals are dissimilar. Array gain benefits from high correlation coefficients for signal and low values for noise.

A special case arises in the situation where the signals are correlated at each hydrophone but the noise is uncorrelated across the hydrophone array. The array gain then reduces to

$$10 \log_{10} k^{-1} (k^2) - 10 \log_{10} k^{-1} (k) = 10 \log_{10} k \qquad (3)$$

This is the well-known result that array gain increases as 10 times the logarithm to the base 10 of the number of hydrophones.

The term spatial coherence refers to the variation in correlation coefficient with separation between hydrophones. The practice is to portray coherence as a function of the dimensionless quantity shown below.

$$2\pi \text{ (separation)/wavelength} \qquad (4)$$

The coherence to be examined herein is transverse spatial coherence which is measured perpendicular to the direction of propagation. This parameter is the one that impacts array gain. In some of the reports examined, measurements along the direction of propagation of the longitudinal spatial coherence are also reported but it was of no use in this study.

It is convenient in spatial coherence studies to represent the spatial coherence function in a Gaussian expression of the form

$$\rho(\Delta x) = \exp\langle -\{[(2\pi/\lambda)\, \Delta x\, \sigma]^2\}/2 \rangle \qquad (5)$$

where Δx is separation between hydrophones
σ is angular uncertainty
λ is acoustic wavelength

The spatial coherence function varies in a negatively exponential manner, depending upon the separation distance between hydrophones, the angular uncertainty, and acoustic wavelength.

When the correlation or coherence has a value of 60%, then

$$(2\pi/\lambda)\, \Delta x = 1/\sigma \qquad (6)$$

The expression $(2\pi/\lambda)\, \Delta x$ is called coherence length. Its reciprocal is termed the angular uncertainty. Note that coherence length is a dimensionless quantity--it denotes different separations for different frequencies for a specified coherence length.

The importance of angular uncertainty should not be missed because it represents the angular spreading in the medium at approximately the -2 dB points. The implication is that array gains cannot be improved by using sonar beamwidths smaller than the value of the angular uncertainty. Recognition of this property of the medium enables us to estimate the limits to acoustic array gains.

The procedure for obtaining values of array gain will be: a) to search the literature for measurements of coherence in shallow water; b) to find the value of coherence length corresponding to a coherence or correlation coefficient of 60%; c) to set the angular uncertainty numerically equal to the reciprocal of the coherence length; and d) to substitute the value of angular uncertainty into the expression derived by Wasiljeff[1] given below for the maximum array gain which can be achieved.

$$\text{Array Gain} = 10 \log_{10} 1/\sigma \, [(2/\pi)^{1/2}] \tag{7}$$

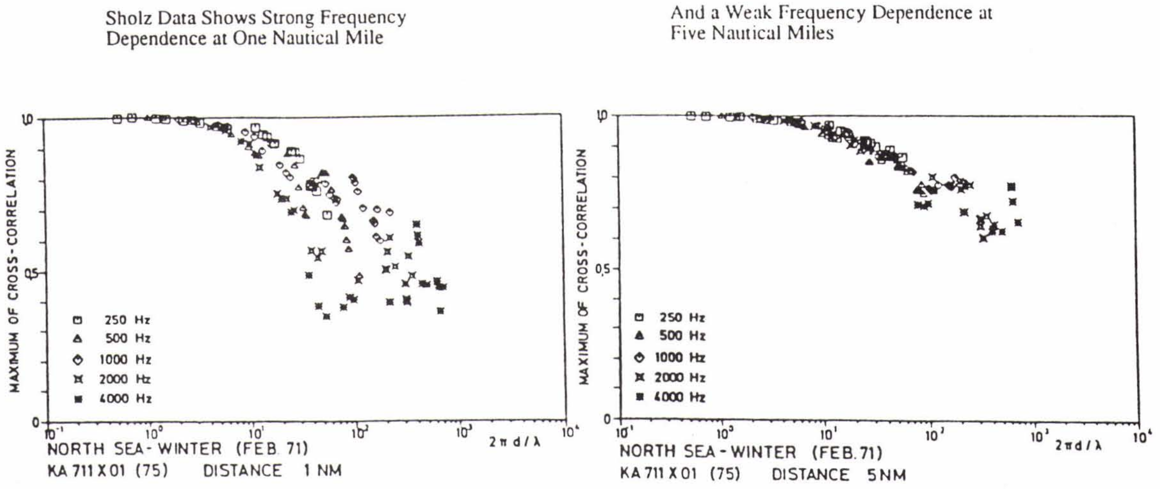
This expression presupposes a Gaussian space correlation function of the type described earlier and a linear array whose omnidirectional hydrophones are uniformly spaced one-half wavelength apart.

Results

The literature was surveyed back to 1970 to uncover measured data on spatial coherence in shallow water. Shallow was interpreted as water depths less than 1000 feet. The search revealed the eight studies listed below. With the exception of the Hug test site, the bottoms were relatively flat and covered by either sand or silt.

Source	Area
Wille-Thiele[2]	North Sea
Ancey[3]	Gulf of Lions
Wasiljeff[1]	Elba, Straits of Sicily
Scholz[4]	North Sea
Herstein, Birtcher, Koenigs[5]	Off Long Island
Hug[6]	Barents Sea
Grandvaux[7]	Gulf of Lions
Zhu, Guan[8]	Yellow Sea

Although these data don't comprise a full variation in bottom roughness, they do offer contrasts between strongly bottom-interacting propagation and upward refracting propagation and cover frequencies form 100 to 4000 Hz. Evidence will be offered to unmask dependencies of area, season, range, and frequency. Because the data were acquired with stationary or slowly moving sources and receivers, a mobile towed array and a moving target might encounter additional degradation due to Doppler effects. The Hug data is unique in two respects. First, the hydrophone arrays were mounted on the bottom. Second, his test sites in the Barents Sea were over regions of salt domes, which introduced geological and perhaps acoustical discontinuities in the bottom structures.



Sholz Data Shows Strong Frequency Dependence at One Nautical Mile

And a Weak Frequency Dependence at Five Nautical Miles

Figure 1. Range and Frequency Effects on Spatial Coherence

Figure 1 and the following two figures will be used to generalize about spatial coherence in shallow water. The work of Scholz in the North Sea[4] will be reproduced because it is typical of the results reported by other investigators for winter sound speed profile conditions where strong ducting is supported. High winds on the order of 30 knots prevailed when this data was being acquired. Up to a distance of one nautical mile (Figure 1), the coherence exhibits erratic behavior with coherence falling off rapidly and coherence lengths being frequency dependent. The coherence length, again, is that value for which the correlation coefficient reaches a value of 60%.

However, when a range of 5 miles is reached (also Figure 1), the coherence increases at all frequencies and there is reason to assert that a single number might be adequate to describe coherence length at all frequencies.

In high wind conditions and ducting propagation, propagation is obviously degraded by surface interactions. The propagation loss can be expected to increase rapidly with range. However, this illustration offers evidence that nature is providing some measure of immunity against the high propagation losses by supporting high coherence.

The trend line on Figure 1 is continued in Figure 2 as range is increased out to 27 nautical miles. The spatial coherence increases monotonically with range. However, remember that this is not a bottom interacting environment. Bottom interactions will be produced whenever a downward refracting profile exists. Generally, the warmer months, especially summertime, will spawn the negative temperature gradients which promote downward refractions. The next figure will present a seasonal contrast in the same North Sea area.

Sholz Data Shows That Spatial Coherence Increases Beyond Five Nautical Miles

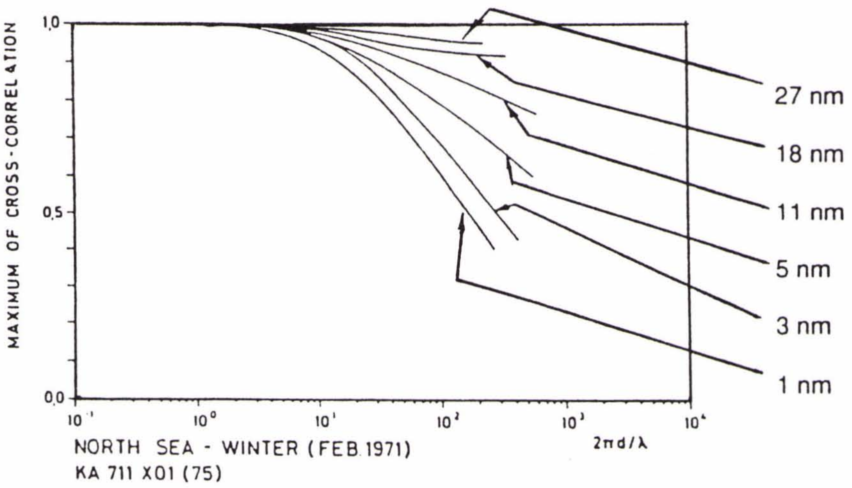


Figure 2. Range Dependence of Spatial Coherence

The display format is changed on Figure 3 to portray angular uncertainty (the reciprocal of coherence length) and hence, array gain in dB versus range for winter and summer. All ranges are adversely affected in summer relative to winter because of bottom interactions. And, array gains achievable in winter will exceed those in summer by a significant number of dB. Seasonal effects are therefore highly significant.

North Sea Spatial Coherence Sengths
as a function of Season (from Scholz)

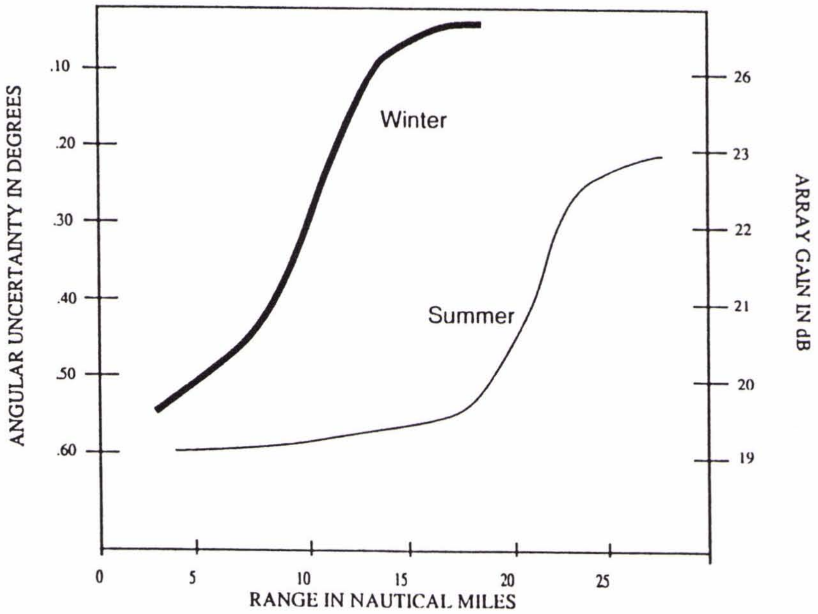


Figure 3. Seasonal Dependence of Spatial Coherence

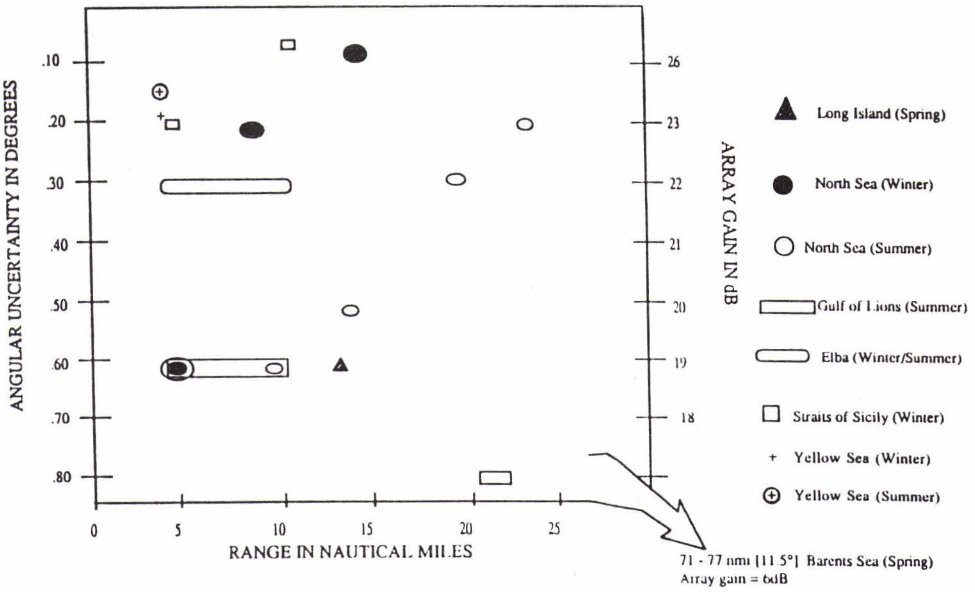


Figure 4. Composite Experience with Spatial Coherence in Shallow Water

Figure 4 is a composite of the seasonal, range, and areal effects on spatial coherence. The representation enables us to specify an upper limit to the array gain that can be achieved in these shallow water sites. Data for high seas, rough sea surfaces, and ducted environments are labeled as Long Island, North Sea winter, Straits of Sicily winter, and Yellow Sea winter. All other data points pertain to a bottom-interacting environment.

The data acquisition procedures by all investigations were similar throughout this collection except:

(1) The Long Island data was obtained with PRN pulses whose phase instability may adversely affect the correlation process. A maximum value of 50% was the highest observed on these tests. This may be due to the signals used or to the limited sampling of hydrophone separations.

(2) The Barents Sea data was gathered with a bottomed array.

The Barents Sea data behaves completely different from all others. Its angular uncertainty is the only reported measurement to exceed 0.8°. Its uncertainty of 11.5 degrees would prevent a linear array from achieving spatial gain in excess of 6 dB in the test area, regardless of the number of hydrophones. Recall that this test site was characterized by salt domes, whereas all the other sites were over relatively flat and occasionally sloping bottoms.

Regrettably, proposed tests in the Barents Sea for simultaneously comparing bottomed and mid-water array depths did not attract the necessary funding. It remains desirable to investigate array depths.

On the basis of these data, array gains of at least 18 dB are generally attainable at ranges in excess of 5 nautical miles with linear arrays in shallow water and up to 26 dB in some areas and seasons.

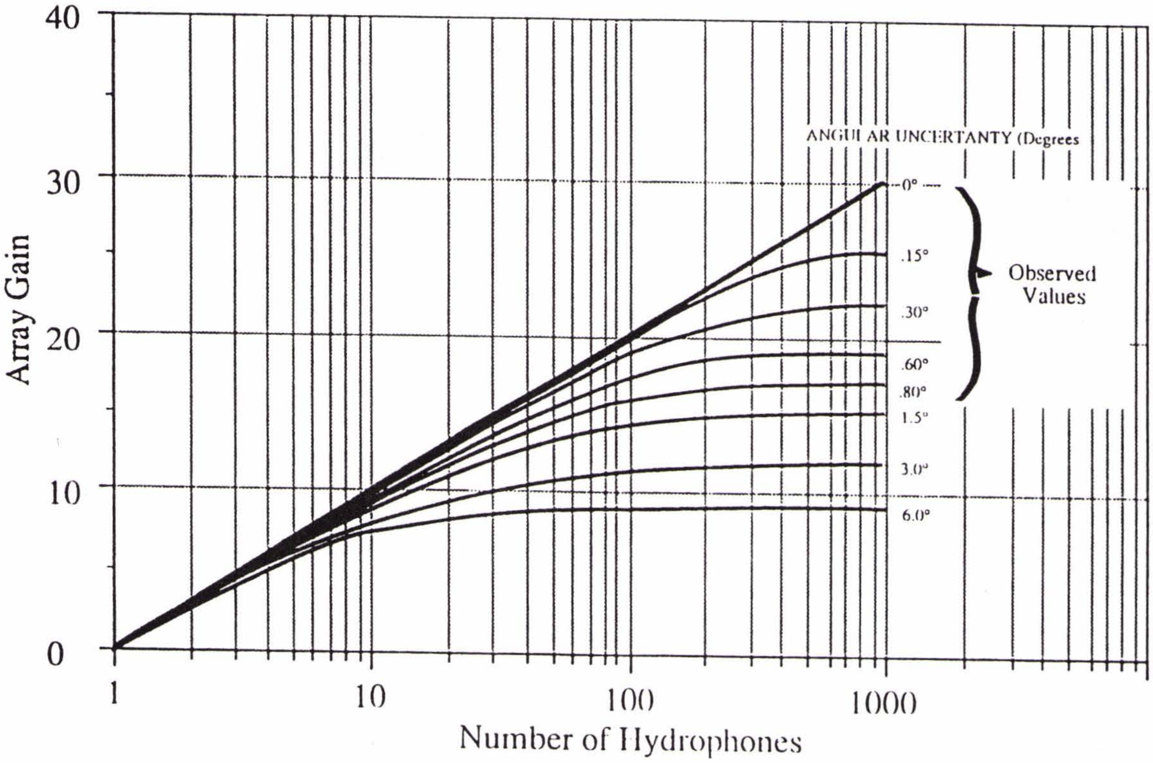
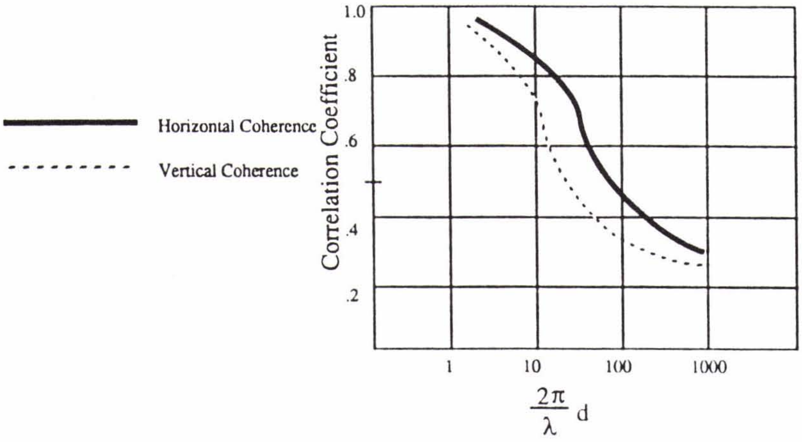


Figure 5. Achievable Array Gains in Shallow Water with a Linear Array

By evaluating the Equations (1) and (5) for one-half wavelength spaced hydrophones, the array gain can be portrayed as a function of a number of hydrophones for specific values of angular uncertainty (see Figure 5). The top curve represents an ideal case where there is no angular uncertainty. Adding hydrophones generally means achieving more array gain. However, should angular uncertainty also increase, the array gain shows lesser increases regardless of number of hydrophones.

With the bulk of the data reported to date falling between 0 and 0.8° angular uncertainties and ignoring the Barents Sea data point, a 200-element array could encounter degradations of up to 7 dB in array gain at times. This suggests that a multiple line towed array, whose individual lines were significantly shorter than 100λ, might offer equal gain and performance.



Gulf of Lions
Summer
Water Depth 100m
Frequency 100-4000Hz
Range ≤ 40000 m

	Coherence Length	Angular Uncertainty	Array Gain
Horizontal Coherence	70	0.8°	17.6dB
Vertical Coherence	18	3.2°	11.5 dB

Figure 6. Comparison of Horizontal and Vertical Coherence in Shallow Water

The possibility of achieving higher gains either with multiple vertically stacked lines or volumetric arrays is offered encouragement in Figure 6, which is taken from the work of Grandvaux. Here, both horizontal and vertical coherence were measured in the Gulf of Lions in 100 meters of water. For frequencies between 100 and 4000 Hz, he reported higher coherence in the horizontal than in the vertical by a factor of 4 to 1. This is roughly comparable to the rule of thumb of 10 to 1 formulated during the NATO-sponsored communications-oriented AFAR experiments around the Azores a number of years ago. This particular example is the only reported measurement of vertical coherence found in our literature review. Obviously, many more measurements are desirable before proceeding to an array design.

The conclusion drawn from this illustration is that degradations in array gain arising from horizontally angular uncertainties may be offset by inserting some vertical aperture.

Summary

There are three major points made in this paper, namely:

(1) A simple methodology has been described for relating spatial coherence to array gain. The literature from 1970-1992 was searched for measurements of spatial coherence in a shallow water (less than 1000 feet) environment. The documented data have been used to determine bounds on the array gain achievable in shallow water.

(2) The well-established conformance of array gain to a 10 $\log_{10}$ (number of hydrophones) relationship for deep water refracted propagation paths does not always apply to shallow water where angular spreading effects from boundary roughness limit the array gains that can be achieved. An example was given where a degradation of up to 7 dB would be incurred with a 100λ linear array. Some limited evidence was presented which supported

the case for using multiple, vertically stacked, or volumetric arrays as a means for satisfying signal gain requirements.

(3) The often quoted phrase "caveat emptor" can be translated as "let the buyer beware since he buys without recourse." A caveat emptor caution needs to be applied to performance assessments of array gains claimed for shallow water operations with long linear arrays. The range, area, and seasonal impacts have yet to be fully evaluated.

References

[1] A. Wasiljeff, 15 May 1975, "Spatial Horizontal Coherence of Acoustical Signals in Shallow Water," SACLANT Memorandum SM-68.

[2] P. Wille and R. Thiele, "Transverse Horizontal Coherence of Explosive Signals in Shallow Water," Journal of the Acoustical Society of America, Vol. 50, No. 1, pp. 348-353.

[3] R. Ancey, 1973, "Coherence Spatiale de Signaux Acoustiques Propages per Petits Fond," in Groupe d'Edude du Traitement du Signal. Proces-Verbaux due Quatriene Collogue sur le Traitement due Signal et ses Applications, Nice, France, pp. 423-441.

[4] R. Scholz, 1977, "Horizontal Spatial Coherence Measurements with Explosives and CW Sources in Shallow Water," in G. Tacconi, "Aspects of Signal Processing, Part 1," Reidel Publishing Co., pp. 94-107.

[5] P.D. Herstein, W.A. Birtcher, and P.D. Koenigs, 24 November 1982, "Cross-Correlation Properties of Band-Limited Signals in Shallow Water (U)," NUSC TD 6801.

[6] E. Hug, 16 December 1985, "Preliminary Rondo Array Gain Analysis," Forsvarets Forskninginstitutt Report FF1/Rapport-85/2008.

[7] B. Grandvaux (undated material), "Sound Propagation Measurement in Shallow Water," SACLANT Shallow Water Symposium Proceedings, pp. 247-250.

[8] R. Zhu and D. Guan, 1992, "Spatial Horizontal Coherence of Sound in Shallow Water," Journal of the Acoustical Society of America, Vol. 92, No. 2, pp. 956-961.