RANDOM TEMPERATURE STRUCTURE AS A FACTOR IN LONG RANGE PROPAGATION

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ABSTRACT

A statistical description of horizontal thermal microstructure in anosotropic turbulence is developed for application with an existing acoustic propagation theory. The model predicts a temperature power density spectrum which decays as -5/3 and -3 in the convective and buoyancy ranges, respectively. The relative power in the two ranges is a function of depth and depends on the total rate of energy dissipation, the total rate of dissipation of temperature variance, and the Brunt-Väisälä frequency. Companion experimental data at two widely separated stations of the North Atlantic verify the theoretical predictions.

INTRODUCTION

This paper presents models and measurements both for the horizontal oceanic random temperature field and for the horizontal transverse correlation of the acoustic energy which has propagated through a random environmental field. First the environmental field model is discussed and measurements supporting the model are presented. Then a theoretical solution for the acoustic correlation function which incorporates the environmental field properties is compared with acoustic data.

A considerable amount of effort by a number of researchers has been devoted to the description of the random horizontal temperature structure of the oceans. This effort has produced a variety of single term power law representations of this structure, particularly in the wavenumber range between internal waves and dissipation. This intermediate range is treated both theoretically and experimentally in this paper. It is shown that two physical processes, buoyancy and convection, have a depth dependent effect on certain scale sizes of the temperature microstructure spectrum which can result in a number of single term power law representations. The form of the temperature power spectral density curve, which is developed and observed within the anisotropic buoyancy-convective wavenumber range, is strongly influenced by the Väisälä frequency, N.

ENVIRONMENTAL THEORY

The first step in the theoretical treatment,[Moseley and DelBalzo, 1974], of horizontal spatial temperature fluctuations in an anisotropic ocean throughout the buoyancy-convective range involves a modification of the standard assumptions (Corrsin, 1951) applicable in an isotropic medium. The modified assumptions are listed below.

1. Temperature is not a simple passive additive. Although the temperature fluctuations are still so small as to have no appreciable effect on the velocity field, the mean temperature gradient, through its effect on density, does directly influence the velocity structure.

2. The mean density gradient is statically stable.

3. The statistical properties of both temperature and velocity in the spectral range of interest are homogeneous and isotropic only in horizontal planes and are stationary.

4. With the exception of scaling and dimensionalizing factors, the statistical properties of the temperature fluctuations are

determined solely by (a) D_θ, the temperature variance flow rate through the buoyancy-convective range; (b) ε_o, the viscous energy dissipation rate; and (c) the energy spectrum in the buoyancy-inertial range of velocity fluctuations (Lumley, 1964).

A dimensional analysis, based on the above assumptions, yields

$$T_{BC}(k) = A_1 k^{-5/3} + A_2 k^{-3} \qquad (1)$$

where $A_1 = BD_\theta \epsilon_o^{-1/3}$ and $A_2 = BCD_\theta \epsilon_o^{-1} N^2$ with B and C being dimensionless constants of order one. The first term results from convective turbulence; the second term occurs due to the buoyancy influence on the turbulence. The coefficient of the second term has an explicit dependence on the Brunt-Väisälä frequency.

There are a number of predictions that can be made on the basis of this simple formulation. We shall just treat a couple. First, if the wavenumber interval under analysis is the fixed interval $[k_L, k_u]$ and a single-term power law analysis is applied, then the value of the resulting exponent would be expected to vary in depth in a manner similar to the Väisälä frequency. This occurs because the relative dominance of the second term increases as the Väisälä frequency increases and so the single-term power law approximation begins to approach -3. Conversely, as the Väisälä frequency decreases, the influence of the second term decreases and the single-term approximation provides a power law close to -5/3. This effect is depicted in figure 1 where the transition wavenumbers k_b and k'_b (above which inertial turbulence is the dominant influence) correspond to large and small values of N, respectively.

A second theoretical prediction would be that if one plotted the power law resulting from a single-term analysis versus the square of the local Väisälä frequency, one would expect the power law to be asymptotic towards 3 as the Väisälä frequency increases.

In summary, this development shows a two-term power law for temperature fluctuations in the buoyancy-convective range of wavenumbers. The spectral decay associated with buoyancy is shown to be proportional to k^{-3}, and the decay associated with convection is proportional to $k^{-5/3}$.

ENVIRONMENTAL MEASUREMENTS

A series of environmental measurements (temperature, sound speed, pressure and rate of advance) were taken along 54 straight horizontal tows near Bermuda over a wide range of depths, speeds and distances. Spatial power density spectra of temperature fluctuations after removal of a mean and linear trend were computed and corrected for depth variability for each of the runs.

In all of the treatments that follow, logarithms were taken to equalize the variance of the power density estimates before the least square analysis procedure was applied.

Figure 2 indicates the results of single-term power law analyses over a fixed wavenumber interval versus depth. The symbols denote the power law values obtained and the vertical bars indicate the 90% confidence limits. As predicted, the single-term power law varies between -5/3 and -3. The dependence on depth was strongly suggestive of the Väisälä frequency profile for the area. In figure 3 the exponent resulting from the single-term analysis is graphed as a function of the temperature gradient (which is proportional to the square of the Väisälä frequency) and the predicted asymptotic approach toward -3 for large N should be noted.

Figure 4 illustrates schematically the components of a procedure to check the details of the theory via a separated two-term analysis. If the initial wavenumber interval under analysis is constrained to sufficiently large wavenumbers, the influence of the -3 power law term becomes negligible. In this restricted wavenumber interval, a single-term power law analysis provides experimental estimates of the coefficient and exponent for the -5/3 term. Next an analysis is performed over the entire wavenumber interval throughout which the contributions (as determined from the experimental coefficient and exponent) of the -5/3 term are treated as noise and removed from the total temperature power spectrum. A single-term power law analysis is performed on the residual and this gives experimental estimates of the coefficient and exponent for the -3 term.

Our measurements extended to the large wavenumbers required by this procedure in 19 of the 54 tow runs. The average value of the exponent in the constrained high wavenumber interval was -1.68, extremely close to the theoretical value of -5/3. Having removed the contributions of this term, the power law analysis on the residual gave an average value for the exponent of -2.94, again close to the theoretical value of -3. In figure 5, for each run, the experimental estimate for the exponent of the term dominant in the buoyancy interval is plotted versus the experimental estimate for the exponent of the term dominant in the convective interval. μ indicates the mean power law in the buoyancy interval. The 90 % confidence limits on this mean include the value -3. Thus, we conclude that the mean is not significantly different from -3, and that the experimental data support the environmental theory in the buoyancy-convective wave-number range.

The environmental model which is to be incorporated in a propagation theory should include the range of wavenumbers dominated by internal waves. The power spectrum for this lower wavenumber interval can be obtained from Garrett and Munk (1975).

ACOUSTIC MODELING AND MEASUREMENT

Equipped with a model of random horizontal temperature fluctuations, we now briefly assess an influence of these fluctuations on acoustic propagation. In particular the two-point acoustic correlation function at the receiver is investigated because it is a low-order statistic directly relevant to system performance. Γ, the acoustic correlation function is defined as

$$\Gamma(\underline{x}_1,\underline{x}_2) = \{P(\underline{x}_1)\,P^*(\underline{x}_2)\}$$

the ensemble average of the product of the acoustic pressure at one point in the field and the complex conjugate of the acoustic pressure at a second point in the domain.

Starting from the reduced wave equation satisfied by the acoustic pressure at each of two points in the field, one can derive via the operator-smoothing method an integro-differential equation governing the propagation of the acoustic correlation function:

$$\begin{aligned}
&[\nabla_1^2 + k^2(\underline{x}_1)]\,[\nabla_2^2 + k^2(\underline{x}_2)]\,\Gamma(\underline{x}_1,\underline{x}_2) \\
&- [\nabla_1^2 + k^2(\underline{x}_1)]\left[k^2(\underline{x}_2)\int G(\underline{x}_2,\underline{\rho})\,k^2(\underline{\rho})\,R_\epsilon(\underline{x}_2,\underline{\rho})\,\Gamma(\underline{x}_1,\underline{\rho})\,d\underline{\rho}\right] \\
&- [\nabla_2^2 + k^2(\underline{x}_2)]\left[k^2(\underline{x}_1)\int G(\underline{x}_1,\underline{\eta})\,k^2(\underline{\eta})\,R_\epsilon(\underline{x}_1,\underline{\eta})\,\Gamma(\underline{\eta},\underline{x}_2)\,d\underline{\eta}\right] \\
&- k^2(\underline{x}_1)\,k^2(\underline{x}_2)\,R_\epsilon(\underline{x}_1,\underline{x}_2)\,\Gamma(\underline{x}_1,\underline{x}_2) = 0
\end{aligned}$$

In this equation, G is the Green's function for the reduced deterministic wave equation, k is the deterministic acoustic wave-number, and R_ϵ is the two-point correlation function for the random environmental field. A weak environmental field assumption has been made so truncation of the infinite series of integral operators can be accomplished at the order requiring only the two-point statistic of the environmental field.

Two approaches toward solving for Γ are currently being implemented. In the first approach a parabolic equation approximation is introduced and then a computer based numerical solution is constructed. This technique allows inclusion of actually observed oceanic sound speed profiles as well as the random environmental field.

The second approach involves additional assumptions which provide a strictly differential equation formulation and subsequently allow a closed form solution. It is a solution resulting from the second approach that will be compared with acoustic measurements in this paper. The closed form solution derived by McCoy and Beran (1975) and shown below retains the basic anisotropic nature of the two-point environmental statistic together with the approximate form of the power spectrum of random temperature fluctuations in the horizontal plane.

$$\Gamma(L, F, Y) = I \exp[-E F^{5/2} L Y^{3/2}]$$

This result states that the acoustic correlation function along a horizontal line transverse to the direction of propagation is an exponential function whose argument is proportional to: E, an environmental factor; the frequency F to the 5/2 power; L, the range; and the transverse horizontal separation distance, Y to the 3/2 power. I is the intensity at a point receiver with the same range and depth. The factors of the environmental parameter are given in detail in the next equation.

$$E = 1.76 \left[\frac{1}{C_o} \frac{\partial C_o}{\partial T}\right]^2 A_T \, l_{yM} \left[2\pi/C_o\right]^{5/2}$$

Here C_o is the average value of the sound speed, T is temperature, A_T is the strength of the random horizontal temperature variations, and l_{yM} is a length scale associated with the vertical correlation function of the temperature fluctuations. A_T is determined in the following manner: the average value of the Väisälä frequency is determined along the propagation path; then an extrapolation of measured single term coefficient versus Väisälä frequency gives the value utilized.

The comparison of this theoretical solution with acoustic data taken in the same locale as the random temperature data is given in figure 6. Here for a fixed range and fixed frequency the dependence of the correlation function versus increased receiver separation on a horizontal line transverse to the propagation direction is shown. The transverse separation and the range are scaled in units of the acoustic wavelength. The correlation function is scaled by the intensity at a point receiver. The crosses are the experimental values for the average of the cosine of the phase fluctuations which account for the vast majority of the change in the correlation function for a single transmission path as was the experimental case. The solid curve represents the theoretical predictions.

CONCLUSIONS

The assumptions, postulates, and dimensional analysis lead to the two term formulation

$$T_{BC}(k) = B D_{\theta} \epsilon_o^{-1/3} k^{-5/3} + BC D_{\theta} \epsilon_o^{-1} N^{2} k^{-3}$$

for the horizontal spatial power density spectrum of random oceanic temperature fluctuations in a wavenumber range called the buoyancy-convective range.

Analysis of experimental data obtained during 54 horizontal tows with depths ranging from 100 to 1450 m supports a number of theoretical predictions. Both the -5/3 and -3 predicted power laws were observed; the mean values were -1.68 and -2.94, respectively.

Upon fitting a single-exponent power law formulation over the entire observation range, the expected power law variation between -5/3 and -3 was seen to occur along with the anticipated asymptotic (to -3) nature of the exponent for increasing magnitude of the Väisälä frequency.

The theoretical predictions together with the experimental evidence in this report provide an explanation for the disparity in experimental results in the field of random temperature microstructure. This environmental theory fills the gap between the wavenumber ranges where internal waves and isotropic turbulence dominate the horizontal spatial random temperature spectrum. The environmental model utilized in the propagation theory formulation should contain the wavenumber ranges dominated by internal waves, buoyancy, and convective turbulence depending upon the acoustic frequency of interest.

The comparison of measured acoustic data and the Beran-McCoy solution for the acoustic correlation function indicates that the theoretical formulation does qualitatively describe the transverse acoustic correlation dependence on receiver separation. The inherent anisotropy and vertical inhomogeneity of the random environmental statistics and the form of the power spectrum for horizontal temperature fluctuations should be included in future theoretical formulations. The quantitative agreement between theory and acoustic measurements indicates that scattering caused by the random temperature field can account for the observed degradation in acoustic correlation for single path transmission.

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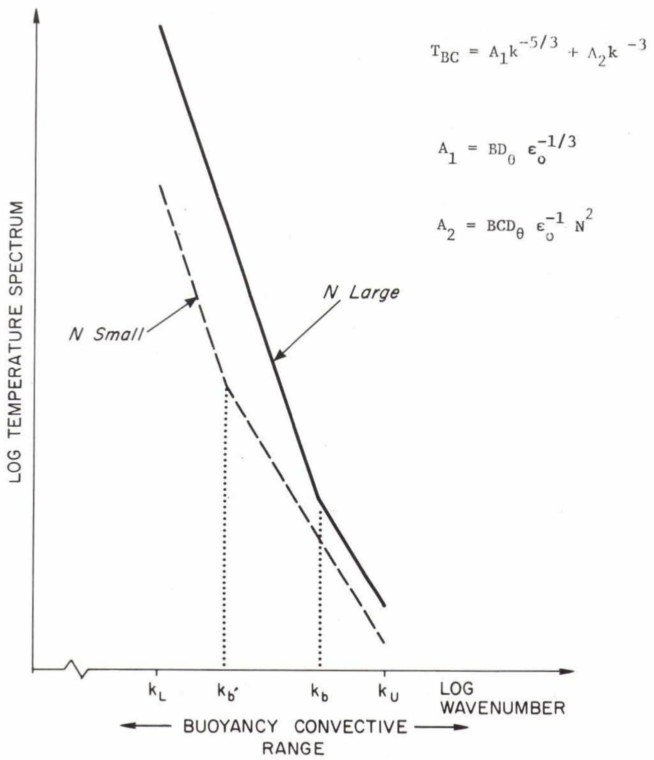

FIG. 1
TEMPERATURE POWER DENSITY SPECTRUM IN BUOYANCY – CONVECTIVE RANGE SHOWING EFFECT OF VAISALA FREQUENCY

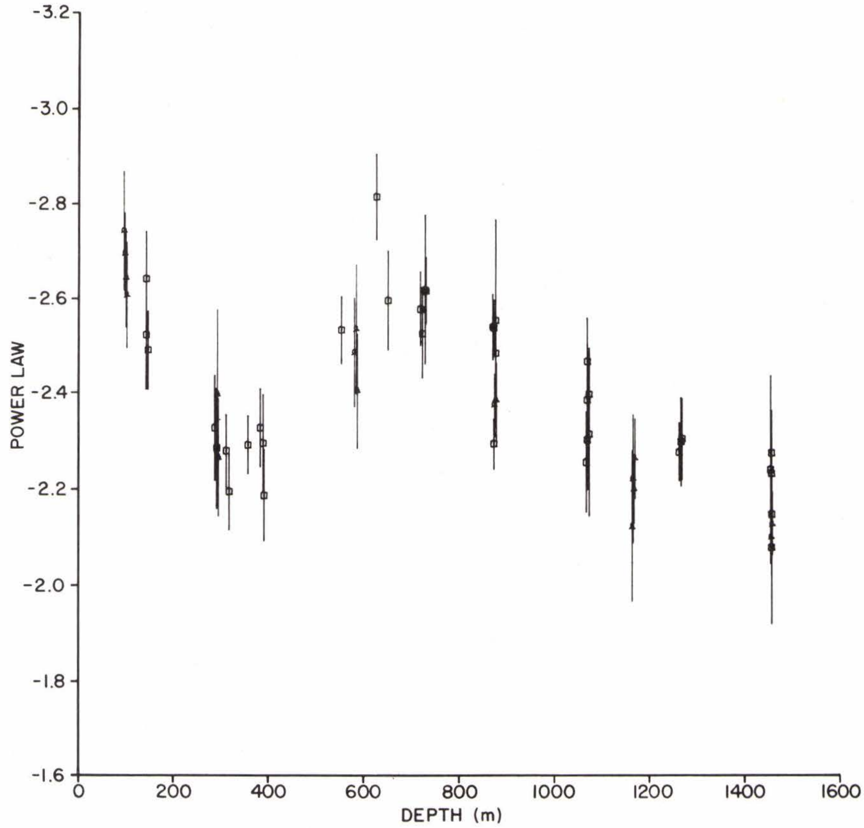

FIG. 2 POWER LAW VS. DEPTH WITH 90% CONFIDENCE LIMITS

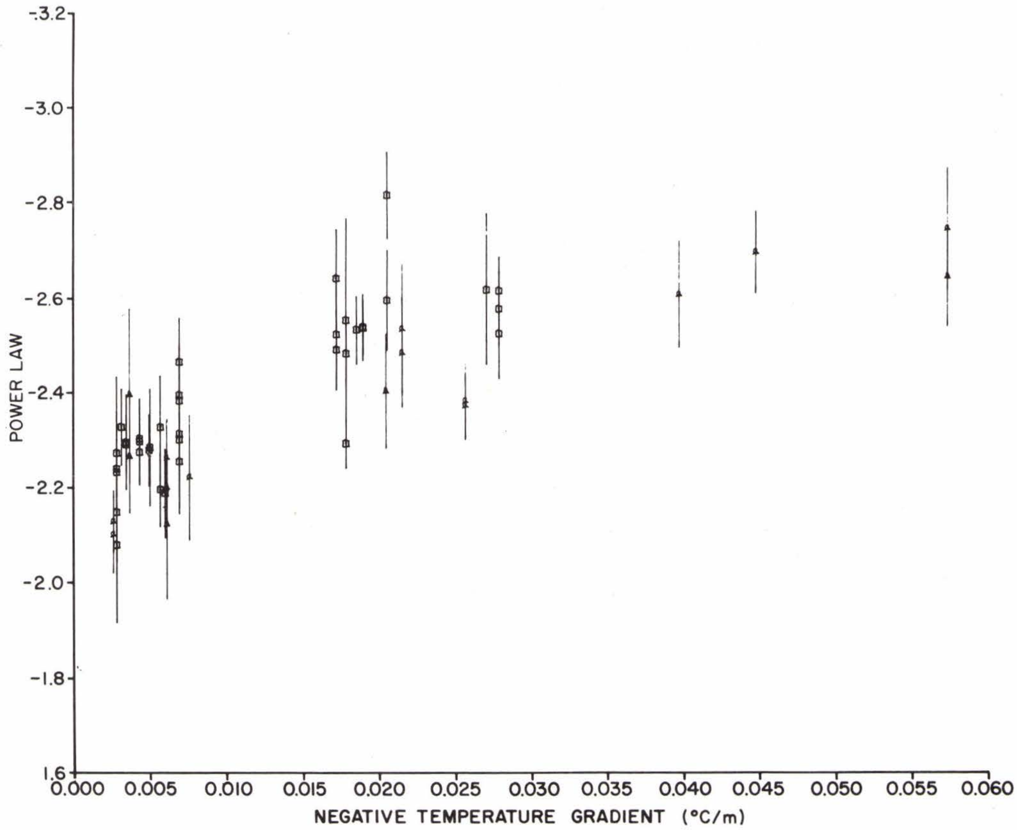

FIG. 3 POWER LAW VS. NEGATIVE TEMPERATURE GRADIENT WITH 90 % CONFIDENCE LIMITS

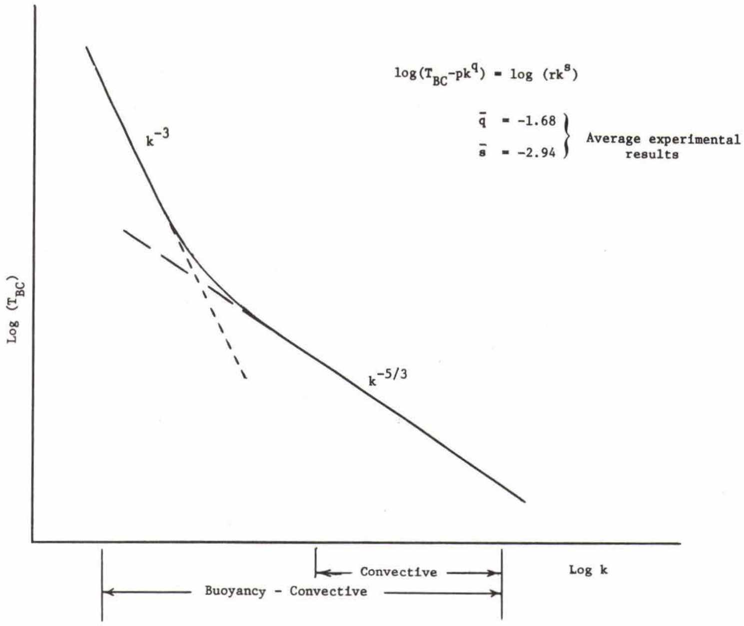

FIG. 4
ANALYZE IN CONVECTIVE RANGE TO OBTAIN ESTIMATES OF p AND q. THEN ANALYZE IN BUOYANCY-CONVECTIVE RANGE WITH ABOVE FORMULA TO OBTAIN ESTIMATES OF r AND s

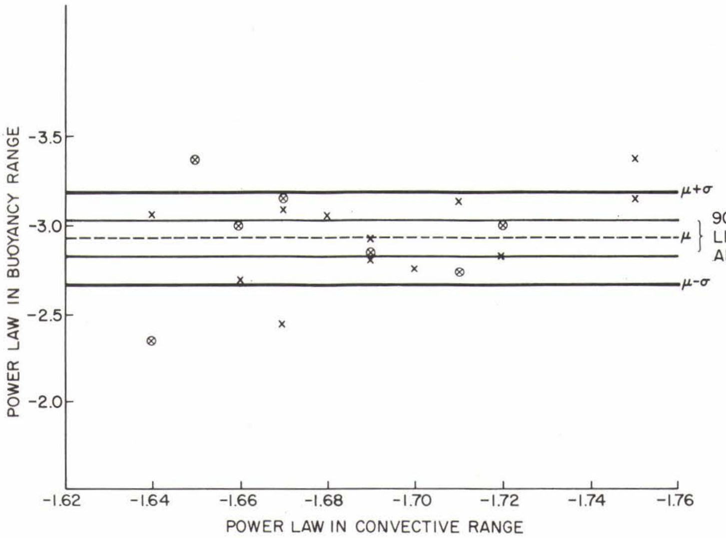

FIG. 5
LEAST SQUARES ANALYSIS ON POWER LAWS IN BUOYANCY AND CONVECTIVE RANGES WITH DISTRIBUTION MEAN, μ, AND ONE STANDARD DEVIATION σ

FIG. 6
VARIATION OF ACOUSTIC CORRELATION FUNCTION VERSUS TRANSVERSE SEPARATION BETWEEN RECEIVING ELEMENTS. MEASUREMENTS GIVEN BY x; THE SMOOTH CURVE IS PREDICTED BY THEORY

$L/\lambda = 0.46 \times 10^5$