# MUTUAL COHERENCE AND SILENCE <br> by <br> Hubert P Debart SINTRA ARCUEIL FRANCE 

## ABSTRACT

In this paper we are developing the theoretical principle of a modification of the acoustical emission by a ship. Its objective is making inefficient the angular measurements achieved by a detector. More precisely, the detector is an antenna using a "high resolution" method for angular determination. The concept underlying this approach is the mutual coherence of narrow-band signals.

## INTRODUCTION

The term of "silent ship" can be understood in its strict signification, i.e as a lowering of the power level sent by the ship.

However, let us develop the idea according to which the acoustical emission of the ship, at a rather high level, lures the detector and induces it to false conclusions ; its ears becoming deaf to the received sound.

In order to make sensible this possibility, we are going to recall the pinciples of the "high resolution" method for angular measurements using an antenna, and to investigate how the emission is to be modified to make it unefficient.

1 MUTUAL COHERENCE AND ITS PROPAGATION
The "high resolution" methods use a treatment of the covariance matrix of narrow band analytic signals sent by the ship to a set of sensors.

If $\left(X_{1}, X_{2}\right)$ is a couple of sensors, $s\left(X_{1}\right) s\left(X_{2}\right)$ the analytic signals received by this couple, the corresponding term of the matrix is

$$
s\left(x_{1}\right) s^{*}\left(x_{2}\right)
$$

(——— time average)

This expression is of use if and only if :
a) the band is intrinsecally narrow, i.e if a signal $s(t)$ is shaped as $s(t)=A(t) \cos \omega_{0} t$ the signal in quadrature $\sigma(t)=A(t)$ sin $\omega_{0} t$ coincides closely with the Hilbert transform of $s(t)$

$$
s(t)+i \sigma(t)=A(t) e^{i \omega_{0} t}
$$

is the associated analytic signal.
b) the band is narrow in the sense of the propagation. The signals are sent by an extended source and reach an extended antenna. The differences of propagation time between couples (sending point - receiving point) are bounded by a quantity $\Delta t$. It corresponds to this quantity a phase shift for the centre of the band equal to :

$$
\Delta \phi=\Delta t \cdot \omega_{0}
$$

It must vary just for a small amount through the band of the signal. If swis its bandwidth,it results in the condition

$$
\Delta t . \Delta \omega \ll 2 \pi
$$

A signal complying with these conditions is said "quasi-monochromatic" and an analytic signal is simply associated to it.

Thus let us consider a sending surface (source) and ( $\xi_{1}, \xi_{2}$ ) two points on it

$$
r\left(\xi_{1}, \xi_{2}\right)=\overline{s\left(\xi_{1}\right) s^{+}\left(\xi_{2}\right)}
$$

is the mutual coherence (MC) of the quasi-monochromatic analytic signals for this couple.

On the other hand, we consider a receiving area (antenna) and ( $X_{1}, x_{2}$ ) a couple of points on it.

$$
c\left(x_{1}, x_{2}\right)=\overline{s\left(x_{1}\right) s^{*}\left(x_{2}\right)}
$$

is the MC associated to this couple.
According to these hypotheses, the MC obeys to a second order wave equation and to a "HUYGHENS principle" associated (fig.l).

It can be written

$$
\begin{equation*}
c\left(x_{1}, x_{2}\right)=\iint \gamma\left(\xi_{1}, \xi_{2}\right) \frac{e^{i K\left(\xi_{1}-\xi_{2}\right)}}{r_{1} r_{2}} d \xi_{1} d \xi_{2} \tag{1}
\end{equation*}
$$

where $K$ is the wave number for the centre of the band, $r_{1}, r_{2}$ the distances $\left(\xi_{1} X_{1}\right)\left(\xi_{2} x_{2}\right)$.

DEBART: Mutual coherence and silence

## 2 APPLICATION TO A SHIP AND AN ANTENNA

The ship and the antenna are represented as two segments, 10 is the straight line joining their centres. Their directions are given with reference to 10 , by the angles $(\alpha, \beta)$. The lengths are $\ell$ (ship), L (antenna)

$$
\begin{aligned}
& P_{1} P_{2} \text { is a couple of points on the ship } \\
& M_{1} M_{2} \text { is a couple of points on the antenna (fig.2) }
\end{aligned}
$$

With reference to a system of coordinates whose origin is $0(0 y \equiv O I)$ the coordinates of $P_{1}, M_{1}$, are

$$
\begin{aligned}
& P_{1}\left(R+\xi_{1} \cos \alpha, \xi_{1} \sin \alpha\right) \\
& M_{1}\left(x_{1} \cos \beta, x_{1} \sin \beta\right)
\end{aligned}
$$

Hence

$$
\begin{align*}
P_{1} M_{1}^{2} & =\left(R+\xi_{1} \cos \alpha-X_{1} \cos \beta\right)^{2}+\left(\xi_{1} \sin \alpha-X_{1} \sin \beta\right)^{2}  \tag{2}\\
P_{1} M_{1} & =\left(R^{2}+2 \xi_{1} R \cos \alpha-2 X_{1} R \cos \beta-2 \xi X_{1} \cos (\alpha-\beta)+\xi_{1}^{2}+X_{1}^{2}\right)^{1 / 2} \\
& =R\left(1+\frac{2 \xi_{1}}{R} \cos \alpha-\frac{2 X_{1}}{R} \cos \beta-\frac{2 \xi_{1} X_{1}}{R^{2}} \cos (\alpha-\beta)+\frac{\xi_{1}^{2}+X_{1}^{2}}{R^{2}}\right)^{1 / 2} \tag{3}
\end{align*}
$$

This expression can be expanded up to second order in ( $\xi_{1}, X_{1}$ )

$$
\begin{align*}
P_{1} M_{1} / R & =1+\frac{\xi_{1}}{R} \cos \alpha-\frac{X_{1}}{R} \cos \beta-\frac{\xi_{1} X_{1}}{R^{2}} \cos (\alpha-\beta)+\frac{\xi_{1}^{2}+x_{1}^{2}}{2 R^{2}}-\frac{1}{8}\left(\frac{2 \xi_{1}}{R} \cos \alpha-\frac{2 x_{1}}{R} \cos \beta\right)^{2} \\
& =1+\frac{\xi_{1}}{R} \cos \alpha-\frac{X_{1}}{R} \cos \beta-\frac{\xi_{1} X_{1}}{R^{2}} \sin \alpha \sin \beta+\frac{\xi_{1}^{2}}{2 R^{2}} \sin ^{2} \alpha+\frac{x_{1}^{2}}{2 R^{2}} \sin ^{2} \beta \quad \text { (4) } \tag{4}
\end{align*}
$$

The quantity of use for introduction in formula (1) is

$$
\begin{align*}
P_{1} M_{1}-P_{2} M_{2}=\left(\xi_{1}-\xi_{2}\right) \cos \alpha-\left(X_{1}-x_{2}\right) \cos \beta-\frac{\xi_{1} X_{1}-\xi_{2} x_{2}}{R} \sin \alpha \sin \beta & +\frac{\xi_{1}^{2}-\xi_{2}^{2}}{2 R} \sin ^{2} \alpha+ \\
& +\frac{x_{1}^{2}-x_{2}^{2}}{2 R} \sin ^{2} \beta \tag{5}
\end{align*}
$$

The MC for ( $X_{1}, X_{2}$ ) can be expressed in terms of the MC for $\left(\xi_{1}, \xi_{2}\right)$ by use of the formula (1).

It results in :

$$
\begin{equation*}
c\left(x_{1} x_{2}\right)=e^{i k\left(x_{1}-x_{2}\right) \cos \beta} \cdot e^{\frac{i k\left(x_{1}^{2}-x_{2}^{2}\right)}{2 R} \sin ^{2} \beta \cdot J} \tag{6}
\end{equation*}
$$

$J=\int\left(e^{i K\left(\xi_{1}-\xi_{2}\right) \cos \alpha} \exp \left(-\frac{\left(\xi_{1} x_{1}-\xi_{2} x_{2}\right)}{R} \sin \alpha \sin \beta i k+\frac{\left(\xi_{1}^{2}-\xi_{2}^{2}\right)}{2 R} \sin ^{2} \alpha i k\right) \times\right.$

$$
\begin{equation*}
\times \gamma\left(\xi_{1}, \xi_{2}\right) d \xi_{1} d \xi_{2} \tag{7}
\end{equation*}
$$

The first and the second factors of $C\left(X_{1} X_{2}\right)$ are the essential quantities of the covariance matrix :
a) $e^{i K\left(X_{1}-X_{2}\right) \cos \beta}$
defines the azimuth of the source and allows its determination

$$
i k\left(\frac{x_{1}^{2}-x_{2}^{2}}{2 R}\right) \sin ^{2} \beta
$$

b) e
defines the curvature of the wavefront and allows the determination of the distance R

## 3. PERFECTLY INCOHERENT SOURCE

If the emission of the ship is seen as ponctual acoustical sources, entirely independant to each other, the situation is exactly the astronomical situation, in which the optical sources in two points of the sum for example are independant.

In this case :

$$
\gamma\left(\xi_{1}, \xi_{2}\right)=I\left(\xi_{1}\right) \delta\left(\xi_{1}-\xi_{2}\right)
$$

I is the acoustical intensity in each point, and the integral J reduces to a simple integral :

$$
\begin{equation*}
J=\int e^{-\frac{\xi\left(x_{1}-x_{2}\right)}{R}} i k \sin \alpha \sin \beta \quad I(\xi) d \xi \tag{8}
\end{equation*}
$$

Hence $J$ is the Fourier transform of the function I "acoustical power on the source". This result is well-known and referred by the astronoms as "Zernike-Vancittert theorem".

To investigate the influence of this factor $J$, it is convenient to suppose a finite number of punctual sources on the ship :
$I\left(\xi_{1}\right), I\left(\xi_{2}\right) \ldots I\left(\xi_{n}\right)$
and $J=\sum_{1}^{n} e^{-\frac{\xi_{m}\left(X_{1}-X_{2}\right)}{R} i K \sin \alpha \sin \beta} I\left(\xi_{m}\right)$
Hence the term of the covariance matrix $\left(X_{1} X_{2}\right)$ can be expended in a sum corresponding to the distinct contributions. If the antenna has a very high resolving power, they are seen as distinct, if not they are seen as an unique source whose angular location is between the extremities of the ship.
4. PARTIALLY COHERENT SOURCE

This situation is modified if the ship bears a repartition of acoustical sources, with a MC function finite between any couple of points.

To study this phenomenon, let us suppose that this MC takes the form

$$
\begin{equation*}
r\left(\xi_{1}, \xi_{2}\right)=e^{-\frac{\left(\xi_{1}-\xi_{2}\right)^{2}}{\rho^{2}}} e^{\frac{i\left(\xi_{1}-\xi_{2}\right)}{\sigma}} \tag{10}
\end{equation*}
$$

Its modules is decreasing with the distance, $\rho$ is a radius of correlation It can be supposed also that the argument is variable (linearly) with a characteristic length $\sigma$ ( $\sigma$ can be infinite if no variation of the argument exists).

Hence it can be written :

$$
J=\iint e^{i K\left(\xi_{1}-\xi_{2}\right) \cos \alpha} e^{\frac{i\left(\xi_{1}-\xi_{2}\right)}{\sigma}} \times \ldots
$$

$x \exp \left(-\frac{\left(\xi_{1} x_{1}-\xi_{2} x_{2}\right)}{R} \sin \alpha \sin \beta i k+\frac{\left(\xi_{1}^{2}-\xi_{2}^{2}\right)}{2 R} \sin ^{2} \alpha i k-\frac{\left(\xi_{1}-\xi_{2}\right)^{2}}{\rho^{2}}\right) d \xi_{1} d \xi_{2}$
In order to evaluate this integral, let us suppose that $\rho$ is very small versus : : $\rho \ll \ell$
and adopt new variables, according to the transformation

$$
\begin{aligned}
& \xi_{1}-\bar{c}_{2}=U \\
& \frac{D\left(\xi_{1}, \xi_{2}\right)}{D\left(U_{1} V\right)}=\frac{1}{2}
\end{aligned}
$$

$$
\xi_{1}+\xi_{2}=v
$$

and use the Tchernov approximation, by removing the limits of integration to $(-\infty,+\infty)$ for the variable U.

Thus $J$ reduces to :
$J=\frac{1}{2} \iint e^{i \mu U} \exp \left(-\left(\frac{U}{2}\left(X_{1}+X_{2}\right)-\frac{V}{2}\left(X_{1}-X_{2}\right)\right) \sin \alpha \sin \beta \frac{i K}{R}+\frac{i K U V}{2 R} \sin ^{2} \alpha-\frac{U^{2}}{\rho^{2}}\right) d u d v$
where $\mu=K \cos \alpha+\frac{1}{\sigma}$
The calculation can be achieved with no trouble and gives :

$$
\begin{gather*}
J=\frac{\sqrt{\pi} \rho}{2} \int \exp \left(-\rho^{2}\left(-\frac{\mu}{2}+\frac{k}{4 R}\left(x_{1}+x_{2}\right) \sin \alpha \sin \beta-\frac{K}{4 R} V \sin ^{2} \alpha\right)^{2}\right) \\
x e^{-\frac{V}{2}\left(x_{1}-x_{2}\right) \sin \alpha \sin \beta \frac{i k}{R}} d V \tag{14}
\end{gather*}
$$

If the source is sufficently remote, the $v^{2}$-term in the exponential can be reglected. Hence, if we drop the constant factor :

$$
J=\frac{e^{-(a+i b) \ell}-e^{(a+i b) \ell}}{a+i b}
$$

where :

$$
\begin{aligned}
& a=\frac{\rho^{2} k}{2 R} \sin ^{2} \alpha\left(-\frac{\mu}{2}+\frac{k}{4 R}\left(x_{1}+x_{2}\right) \sin \alpha \sin \beta\right) \\
& b=\frac{k}{2 R}\left(x_{1}+x_{2}\right) \sin \alpha \sin \beta
\end{aligned}
$$

The variation of the argument of $J$ versus $\left(X_{1}-X_{2}\right)$ is approximately the same as the variation of the argument of $a+i b$

$$
\operatorname{Arg}(a+i b)=\operatorname{Arctan}\left(\frac{b}{a}\right)=\phi \quad \phi \sim-\operatorname{trg} J
$$

$$
\begin{align*}
& \operatorname{tg\phi }=\frac{\frac{k}{2 R}\left(x_{1}-x_{2}\right) \sin \alpha \sin \beta}{\frac{\rho^{2} K^{2}}{2 R} \sin ^{2} \alpha\left(-\frac{\mu}{2}+\frac{k}{4 R}\left(x_{1}+x_{2}\right) \sin \alpha \sin B\right)}  \tag{17}\\
& \operatorname{tg} \phi=-\frac{2 \mu}{\rho}\left(x_{1}-x_{2}\right) \sin \alpha \sin \beta \tag{18}
\end{align*}
$$

in the same conditions (very remote source).
If the radius of correlation is very small, $\phi \sim \pm \pi / 2$ for any distance between the sensors $\left(X_{1}, X_{2}\right)$; it is again the case of a totally incoherent source.

If not, let us recall that $\left(X_{1}-X_{2}\right)$ vary from $-L$ to $+L$ if $L$ is the length of the antenna.

Let us define a radius of correlation from the condition :

$$
\phi= \pm \frac{\pi}{4} \text { for } x_{1}-x_{2}= \pm \frac{L}{2}
$$

Or :

$$
\begin{equation*}
\frac{2}{4 \rho^{2}} \frac{L}{2} \sin \alpha \sin \beta=1 \tag{19}
\end{equation*}
$$

If (usual conditions) $\alpha=\beta=\frac{\pi}{4} \quad \sin \alpha \sin \beta=\frac{1}{2}$

$$
\begin{equation*}
\frac{L}{2 \mu \rho^{2}}=1 \quad \text { or } \quad \rho=\sqrt{\frac{L}{2 \mu}} \tag{20}
\end{equation*}
$$

The argument of $J$ vary quasi-linearly from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ if $X_{1}-x_{2}$ vary from $-L$ to +L and the angular measurement the antenna can achieve is false (the argument of the characteristic factor takes any value according to $x_{1}-x_{2}$ ).
If, for example $L=20 m \quad \mu=K \cos \alpha=\frac{2 \pi}{\lambda} \cos \alpha$

$$
\begin{aligned}
& \lambda=1 \mathrm{~m} \quad \alpha=\frac{\pi}{4} \quad \mu=\pi \sqrt{2} \\
& \rho=\frac{\overline{20}}{2 \sqrt{2 \pi}} \sim 1,5 \mathrm{~m}
\end{aligned}
$$

## 5. REALIZATION

A repartition of acoustical sources whose correlation is shaped as

$$
-\frac{\left(\xi_{1}-\xi_{2}\right)^{2}}{\rho^{2}}
$$

can be achieve by a sum of sources of certain structures and random amplitudes, according to the Loeve-Karhunen theorem

$$
s(\xi)=\Sigma \varepsilon_{n} s_{n}(\xi)
$$

$s_{n}(\xi)$ is an extended source whose repartition is roughly sinusoidal

$$
\alpha_{n} \cos \left(a_{n} \xi\right)+\beta_{n} \sin \left(a_{n} \xi\right)
$$

The correlation function can be approximated by a few sources.

## 6. CONCLUSION

The theoretical possibility to make inefficient the high resolution method is thus shown. However, achieving this result involves a modification of the acoustical emission of the ship, and this problem must be studied from the point of view of its practical realization.


HUYGHENS
Fig. 1 : Secoritorder Huyghens Principe


